NEW TECHNIQUE FOR DETERMINING SAND BANK GEOMETRY IN VERTICAL FRACTURES PROVIDES FOR A MORE EFFECTIVE STIMULATION DESIGN

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INTRODUCTION

The placement of proppants in vertical fractures has long been subject to a great deal of uncertainty. As long ago as 1959, it was realized that far from perfect distribution was being achieved. Vertical fracturing models have invariably shown that during fracturing a bank of sand begins to form along the bottom of the fracture and that all the proppant pumped will eventually be contained in this bank. Prior to this, most methods of predicting productivity increases had assumed an even distribution of sand in the fracture. It has since been realized that productivity increases were strongly influenced by the shape and dimensions (length and height) of the bank.

All investigators have agreed that the bank starting at the wellbore would grow vertically to an equilibrium height and then extend horizontally at this same height across the fracture. It was further agreed that this height was controlled by an equilibrium velocity which was primarily a property of the specific fluid being pumped and the size of proppant used.

This paper will not present any experimental evidence, but rather will deal with an analytical approach for predicting bank shape and dimension.

STATE OF THE ART

Kern, Perkins, and Wyant[1] reported work with a model in 1959. This investigation first identified equilibrium height and equilibrium velocity. Expressions for rate of bank height growth and equilibrium height were developed.

Babcock et al[2], in 1967 reported a more extensive model study and further refined the calculation of equilibrium velocity.

Both of the above approaches considered only the conditions in the model and would result in an even bank equilibrium height from the wellbore to the end of the bank.

Alderman and Wendorff[3] conducted further studies in 1970 and reported further equilibrium velocity data. This paper went further than previous work and considered variable fracture velocity due to leak-off along the fracture wall. The bank shape predicted was roughly of triangular shape (viewed horizontally) with a minimum height at the wellbore. In this analytical approach, leak-off was assumed to occur uniformly across the entire fracture area with no allowance made for negligible leak-off that would occur in the area occupied by the sand bank. This paper did however, lend itself to a more rational design approach.

These various modeling studies resulted in widely differing predictions of equilibrium velocities for similiar fluids.These differences can be accounted for by modeling differences and correlations used.

PREDICTION OF SAND BANK GEOMETRY

Assumptions used in the development are:

1. It includes only fluids with measurable proppant fall rates.
2. The equilibrium velocity is known or can be measured.
3. It considers only restricted vertical fractures.
4. It assumes that the Howard, Fast, and Carter[4] equation adequately describes the leak-off of fluid from a fracture.

Consider one wing of a vertical fracture (Fig. 1) with an inlet flow rate of Q_0. Between the inlet (x=0) and a point x, a measurable flow occurs through the fracture walls which we define as Q_{lo}. From this we see:

$$Q_l = Q_0 - Q_{lo}$$

and from the relationship:

$$Q = AV, \text{ then } V_0 > V_l$$

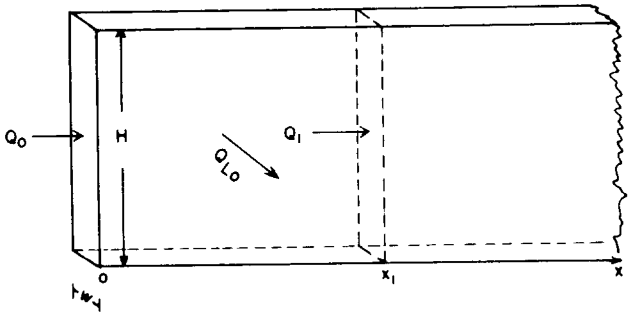

FIG. 1—DEPICTION OF ONE WING OF VERTICAL FRACTURE

When a sand bank begins to form, the fluid velocity is controlled by:

1. The fluid flow rate Q.
2. The cross-sectional area open to flow.

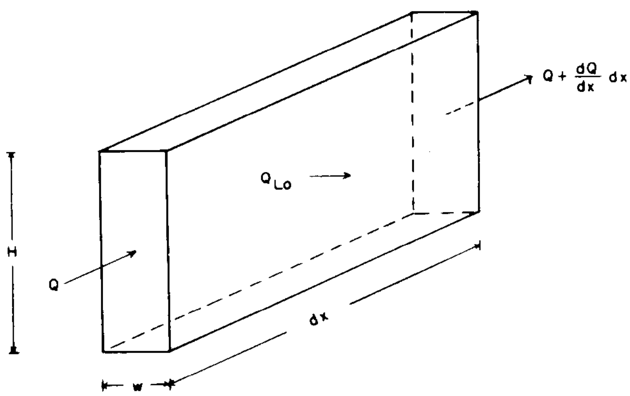

FIG. 2—MASS BALANCE CONTROL VOLUME

Referring to Fig. 2, the fluid flow rate can be determined as follows:

$$Q = Q + \frac{dQ}{dx}dx + Q_{lo} \tag{1}$$

Noting that $Q_{lo} = 2H(x)V_\phi dx$

Where:

Q_{lo} = leak-off rate (ft^3/min)
$H(x)$ = fracture height open to flow (ft)
V_ϕ = leak-off velocity (ft/min)
Q = fluid flow rate (ft^3/min)

we have:

$$\frac{dQ}{dx}dx = -2V_\phi H(x)dx \tag{2}$$

Since the function H(x) is not known, a relationship must be written which describes H as a function of penetration x. This can be done as follows:

$$Q = AV \tag{3}$$

Where:

A = area open to flow (ft^2)
V = fluid velocity (ft/min)

$$A = wH(x) \tag{4}$$

Where:

w = fracture width (ft)

$$Q = wH(x)V \tag{5}$$

Now, when the velocity "V" in Eq. (5) is equal to the equilibrium velocity 'V_e", H(x) becomes the equilibrium height ($H_e(x)$) (See Fig. 3) and Eq. (5) is written as:

$$Q = wH_e(x)V_e \tag{6}$$

and

$$H_e(x) = \frac{Q}{wV_e} \tag{7}$$

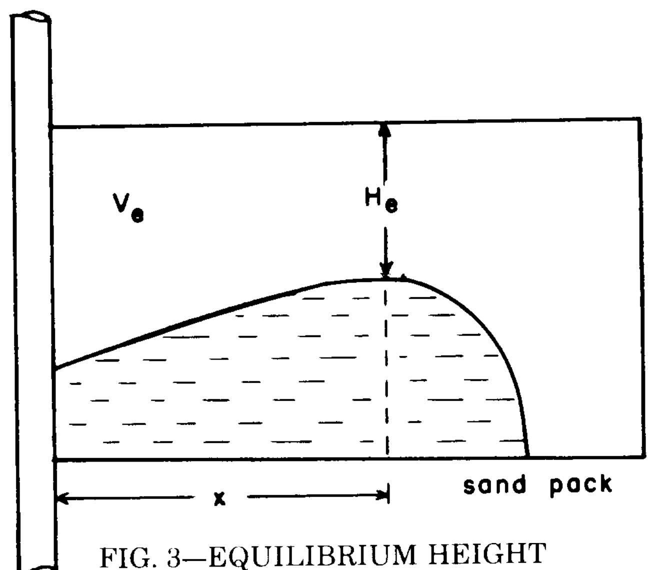

FIG. 3—EQUILIBRIUM HEIGHT OF VERTICAL FRACTURE

Substituting Eq. (7) into Eq. (2) yields,

$$dQ = -2V_\phi \frac{Q}{wV_e} dx \tag{8}$$

It has been assumed that a major portion (in fact, all) of the leak-off occurs only in the unbanked portion of the fracture. Equation (8) may now be integrated noting the following boundary conditions:

at $x = O; Q = Q_0$
$x = x; Q = Q$

Where:

Q_0 = flow rate in one fracture wing at the wellbore

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{2V_\phi}{wV_e} \int_0^x dx \qquad (9)$$

Which yields:

$$Q = Q_0 \, e^{\left(\frac{-2V_\phi}{wV_e}\right) x} \qquad (10)$$

Now, substituting Eq. (10) into Eq. (7), we may determine the sand bank height at any position "x" noting that $H_s(x) = H - H_e(x)$.

Where:

$H_x(x)$ = sand bank height at x

H = total fracture height

$$H_x(x) = H - \frac{1}{wV_e} Q_0 \, e^{\left(\frac{-2V_\phi}{wV_e}\right) x} \qquad (11)$$

Noting that the leak-off velocity can be approximated closely by[5]:

$$V_\phi = \frac{\sqrt{2}\, C}{\sqrt{t}} \qquad (12)$$

Equation (11) becomes:

$$H_s(x) = H - \frac{1}{wV_e} Q_0 \, e^y \qquad (13)$$

Where:

C = combined fluid loss coefficient

t = pumping time

$y = -2\sqrt{2}\, Cx/wV_e \sqrt{t}$

Equation (13) describes the sand bank geometry once the bank is established.

Now it remains to develop an expression for propped fracture length. The volume of sand in the fracture is equal to:

$$v_s = \frac{D}{2(100)} \qquad (14)$$

Where:

v_s = volume of sand (ft^3)

D = total quantity of sand pumped in lbs.

and the bulk sand density is 100 lb /ft^3. Also:

$$v_s = w A_s \qquad (15)$$

where A_s is the area occupied by the sand bank. We note that:

$$A_s = H(x)dx \qquad (16)$$

where L is the propped sand bank length. Therefore:

$$v_s = w \int_0^L H(x)dx \qquad (17)$$

Substituting Eq. (13) into Eq. (17), we have:

$$v_s = w \int_0^L H - \frac{1}{wV_e} Q_0 \, e^y \, dx \qquad (18)$$

Integrating between the limits:

$$v_s = wHL + \frac{wQ_0\sqrt{t}}{2\sqrt{2C}} (e^y - 1) \qquad (19)$$

It is seen that Eq. (19) is implicit in L and must be solved by iteration. While this equation is somewhat tedious for hand calculation, it lends itself well to computer applications.

CONCLUSIONS

This paper has presented a means of predicting sand bank height and propped fracture length. This information can be used to better predict the outcome of hydraulic fracturing treatments by making suitable adjustments in the predicted productivity increases. Several methods of predicting these increases have been presented in the literature including McGuire and Sikora[6], Smith[7], and Prats.[8] These methods have assumed an even proppant distribution in the fracture and that the bank length was equal to the fracture penetration. One method for correcting productivity increases for bank height has been presented by Tinsley et al.[9] The method presented lends itself directly to the information provided by Eqs. (13) and (19).

The ultimate aim of this work is to provide the individual charged with stimulation design with the tools required to design treatments that are near optimum for the individual well case.

Future work in this field will require further modeling studies to reconcile the divergence of data so far presented. Better definition of equilibrium velocities is needed using better models and improved correlations. As in the past, success will come from a combination of realistic experimental work and rigorous analytical techniques.

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