THE ULTIMATE ROD STRING DESIGN PROCEDURE

Gabor Takacs and Mihaly Gajda, Jr.
University of Miskolc, Hungary

ABSTRACT

Available sucker-rod string design models calculate rod taper lengths that ensure proper operation without premature fatigue failures. Their common design problems are (a) defining the principle of taper length determination, and (b) calculating the true mechanical stresses along the string. The universally accepted principle of taper length calculations is to provide the same level of safety against fatigue failure in each taper section. Mechanical loads and stresses, on the other hand, are found from highly approximate calculations in most of the design procedures. These loads, therefore, can greatly deviate from the true mechanical loads that would be measured in the rod string run in the well. The paper discusses the development of a novel procedure that estimates rod loads from the predictive solution of the damped wave equation when designing the rod string. Since loads calculated that way imitate very accurately the actual loads, the most important limitation of previous rod string design procedures is eliminated. Strings designed using the proposed model, therefore, have a much enhanced safety against fatigue failures as compared to previous designs.

INTRODUCTION

The sucker-rod string is the most vital part of the rod pumping system because it connects the prime mover situated at the surface to the subsurface pump that provides the useful work of the installation. A properly designed sucker-rod string should provide failure-free pumping operations for an extended period. Improper design of rod tapers can lead to early mechanical failures (rod breaks) with a complete termination of pumping action and an inevitable loss of production.

Due to the usually great length of the string dictated by the typical depths of oil wells, a single rod size cannot be used for the full length. The general solution is the use of tapered rod strings made up of sections (tapers) of increasing diameters toward the surface. This construction very efficiently matches the shape of the ideal rod string which is an inverted cone continuously tapering from top to bottom. The mechanical design of tapered rod strings is highly complicated because of the type of loading the rods are subjected to. Investigation of the possible loads that occur during the pumping cycle shows that the following distributed and concentrated loads act on the rod string:

- Weight of rods in air; it is a distributed load along the string.
- Buoyancy forces oppose the rod weight and are the result of the immersion of the rods into the produced liquid.
- Fluid load on the plunger of the downhole pump is a concentrated force acting during the upstroke only.
- Dynamic loads are the results of changes in acceleration of the moving masses (rods, fluid column).
- Frictional forces are: (1) fluid friction between the rods and the produced liquid, and (2) mechanical friction between the rods and the tubing string.

After considering the variation of these forces during a complete pumping cycle, one can easily conclude that the rod string is exposed to a cyclic mechanical loading. During the downstroke, the string carries the buoyant weight of the rods minus dynamic and friction forces only, while on the upstroke it also carries the load of the fluid lifted. Mechanical stresses follow the variation of rod loads and are cyclic, too; they are typically tension stresses with the tension level considerably increasing during the upstroke as compared to the downstroke. This is the reason why the loading of the rod string can be classified as pulsating tension.

The typical failure mechanism of sucker-rod strings is the consequence of the type of loading: mostly fatigue failure. Fatigue failures occur at much lower levels of mechanical stresses than the tensile strength or even the yield point of the material and are basically caused by the extremely high number of repetitions of the variable loads. This type of failure is absolutely different from tensile (overpull) failures and is the root cause of the great majority of rod string breaks. Therefore, rod string design procedures must inevitably take into account the cyclic nature of rod loading; this is why, in order to ensure a sufficiently long service life, the string has to be designed for fatigue endurance.

AVAILABLE ROD STRING DESIGN MODELS

Fatigue Endurance Limits

The maximum stress allowed in sucker-rod materials a.k.a. the fatigue endurance limit that ensures a failure-free operation for a sufficiently high number of cycles (usually 10 million) under pulsating tension loads typical for pumping operations is calculated from the modified Goodman formula [1]. This formula, in a generalized form, is valid for different available rod materials and shows that the fatigue endurance limit varies with the strength of the steel material as well as with the minimum stress that occurs in the rod.

$$S_a = SF\left(\frac{T}{A} + B\ S_{min}\right) \qquad \mathbf{1}$$

where:
- S_a = fatigue endurance limit (allowable stress), psi,
- SF = service factor, **-,**
- T = minimum tensile strength of the rod material, psi,
- S_{min} = minimum rod stress, psi, and
- A, B = empirical constants specific for the rod material, -.

The constants ***A*** and ***B*** featuring in the Goodman formula were empirically determined for different steel materials using experiments on material samples. Their values are listed, along with the minimum tensile strengths of available rods in **Table 1**. The variable ***SF*** represents the effects of the environment where the rod string is operating and can be considered as the inverse of a safety factor; it is mainly used to allow for the corrosivity of the well fluid and is usually held constant in a given field.

Overview of Design Models

The earliest tapered rod string design procedure was proposed in the Bethlehem Handbook [2] in 1953; it utilized the simplifying assumption that the string was exposed to a static tension loading. The design goal was to set the maximum stress at the top of each taper equal. Strings designed according to this principle became inadequate, as deeper and deeper wells had to be produced because rod breaks in the bottom taper section became dominant. [3] It is easy to prove that this behavior is a direct consequence of using static loads in the design procedure instead of the actual pulsating tension loads of the rod string.

Based on the negative experience with the early design, today's rod string designs include measures to account for fatigue loading of the string. West [4, 5] developed a taper design to attain the same ratio of maximum stress to allowable stress (represented by the Service Factor) in each taper. Rod strings designed this way have the same safety factor included in every taper section and will not have any weak points. The Neely design [6] aims at reaching the same values of the modified stress (defined by Neely) at the top of each taper section. In 1976 the American Petroleum Institute adopted this design model and published pre-calculated taper percentages in API RP 11L (today TL 11L [7]). The Gault-Takacs [8] model ensures that every taper section has the same degree of safety against fatigue failure by setting the service factor (***SF***) values equal at the top of each taper.

Evaluation of Design Models

If the objectives of rod string designs, except the Bethlehem model, are compared then it is easy to see that all designs try to reach the same safety against fatigue failure in each rod taper. This is ensured when service factors (***SF***s), based on the maximum and minimum stresses found at the top of each taper section and calculated from **Eq. 1** are equal. Rod strings designed according to this principle have the same safety factors included in every taper and do not have any weak points.

Based on the previous discussion it can be concluded that a proper rod string design procedure should produce strings whose tapers are identically safe against fatigue failure; it follows from this that the design should be based on the modified Goodman diagram. This defines the generally accepted principle to be used for the determination of taper lengths. The required design calculations, however, necessitate the determination of true mechanical loads in the different tapers during the pumping cycle, which is a complicated task because the loads in a string about to be designed can only be estimated. The approaches of the different available design procedures to the calculation of rod loads (minimum, and maximum, as well as dynamic loads) are illustrated in **Table 2**; as seen, in the different models rod loads are estimated by using widely different assumptions.

The present authors proved [9] that the loads and stresses estimated by the different design models do not represent actual pumping conditions. This means that the mechanical stresses in any string, when run in the well will be different from those computed during the design process. Since actual stresses at the top of rod tapers are different from their designed values the fatigue loading represented by the relevant service factors will also be different. As a consequence, the basic objective of the design procedure, i.e. having identical service factors at the top of each taper, will not be met. This is a general problem with all known rod string designs that stems from the fact that rod loads required in the design process are only estimated from approximate formulas.

DEVELOPMENT OF A NEW DESIGN PROCEDURE

Basic Considerations

As discussed previously, one faces two basic problems when designing a sucker-rod string. The first is the principle to be used for the determination of taper lengths; as detailed before, the proper objective is to select tapers that have the same level of safety against fatigue failure. To achieve this goal, one has to use the modified Goodman diagram and select taper lengths so that they have the same service factor (***SF***) values. This is the final objective of the design procedure developed in this paper.

The second problem in rod string design involves the calculation of rod loads during the pumping cycle. If loads in the different tapers just designed could be measured during the design process then the final rod string would surely meet the objectives of the design and would have tapers with identical safety against fatigue failures. In reality, however, this approach is impossible to follow and some other solution must be found to estimate the loading conditions of the rod string. A feasible answer to this problem is provided by the calculation of rod loads from the solution of the one-dimensional damped wave equation. The reason for this is that the predictive solution of the wave equation gives loads that match actual measurements very closely, as proved by universal experience gained since the introduction of the wave equation by Gibbs [10] in the late 1960s. The rod string design procedure developed in this paper, therefore, relies on “true” mechanical loads including fluid buoyancy that are predicted from the solution of the wave equation.

Design Fundamentals

The solution of the problem i.e. finding a taper combination with identical service factors is surely an iterative process. To reach the final rod string design one has to calculate rod loads and stresses in many cases of assumed or calculated taper combinations. Therefore it is important to have an idea on the possible distribution of rod loads and stresses along a rod string. **Fig. 1** presents the distribution of rod loads with well depth as calculated from the solution of the wave equation for a 6,070 ft deep well using an API 86 rod string, 120 in polished rod stroke and different pumping speeds. The two families of curves represent downstroke (the left-hand curves) and upstroke (the right-hand curves) loads; their difference is basically equal to the fluid load on the plunger. Based on this and similar cases one can conclude that the variation of maximum and minimum loads in rod tapers can be approximated very accurately by straight lines.

According to the previous observation rod loads along a given taper, found from the solution of the wave equation, can be fitted in function of taper length, ***l***, with straight lines as follows:

$$F_{\max i} = a_i \; l + b_i \qquad \mathbf{2}$$

$$F_{\min i} = c_i \; l + d_i \qquad \mathbf{3}$$

where: $F_{min\,i}$, $F_{max\,i}$ = maximum and minimum loads in the i^{th} taper, lbs,
l = taper length, ft, and
$a_i \dots d_i$ = parameters of the best-fitting lines.

For a given taper in the string, one can determine, based on the modified Goodman diagram, the safety of the taper against fatigue failure. In principle, the safety of any taper section against fatigue failure is defined as the ratio of the actual maximum stress and the allowable stress. Therefore, expressing the service factor (***SF***) valid at the top of any taper from the formula describing the modified Goodman diagram, i.e. **Eq. 1**, provides a way to indicate the taper's safety level. Introducing rod loads instead of mechanical stresses into **Eq. 1** and solving the formula for the service factor, we receive:

$$SF_i = \frac{F_{\max i}}{\frac{T}{A} A_i + B\, F_{\min i}} \quad \mathbf{4}$$

where:	SF_i	= service factor for the i^{th} taper, **-,**
	$F_{min\ i}$, $F_{max\ i}$	= maximum and minimum rod loads in the i^{th} taper, lbs,
	T	= minimum tensile strength of the rod material, psi,
	A_i	= metal area of the rod in the i^{th} taper, sq in, and
	A, B	= empirical constants specific for the rod material, -.

Substituting in this formula the linear expressions received from curve fitting of the rod load vs. depth functions, **Eqs. 2** and **3**, the following equation is found:

$$SF_i = \frac{a_i\ l + b_i}{\frac{T}{A} A_i + B\left(c_i\ l + d_i\right)} \quad \mathbf{5}$$

This formula represents the relationship between the length of the taper, ***l***, and the service factor, SF_i, which can be calculated from the maximum and minimum stresses along the taper. It will be used in the design procedure to investigate the effect of changing the length of the taper for a fixed service factor. For that reason, let's solve **Eq. 5** for the taper length, denoted L_i:

$$L_i = \frac{SF\ \frac{T}{A} A_i + B\ SF\ d_i - b_i}{a_i - B\ SF\ c_i} \quad \mathbf{6}$$

where:	L_i	= length of the i^{th} taper, ft,
	SF	= assumed service factor, **-,**
	$a_i \ldots d_i$	= parameters of the best-fitting lines, -,
	T	= minimum tensile strength of the rod material, psi,
	A_i	= metal area of the rod in the i^{th} taper, sq in, and
	A, B	= empirical constants specific for the rod material, -.

Eq. 6 forms the cornerstone of the new rod string design procedure developed in this paper because it allows one to calculate the required length L_i of any taper based on the required or assumed service factor (***SF***) value. As will be shown later, taper lengths during the iteration process will be changed according to this formula, while considering the actual values of the variables involved.

Description of the Calculation Procedure

Although the developed design can handle any number of tapers, calculation steps are detailed for the case of a three-taper rod string in the following. The description of the design procedure is accomplished with reference to **Fig. 2**.

At the initial conditions (iteration number J = 0) the total rod string length is divided into three equal parts and the following taper lengths are calculated:

$$L_{1,0} = L_{2,0} = L_{3,0} = \frac{L_{total}}{3} \quad \mathbf{7}$$

where:	$L_{i,j}$	= length of the i^{th} taper in the j^{th} iteration, ft, and
	L_{total}	= total required length of the string, ft.

Taper lengths being known, one can use the predictive solution of the wave equation to find the distribution of minimum and maximum loads along the entire length of the string. The input parameters to the wave equation are the usual operating parameters as plunger size, polished rod stroke length, pumping speed, fluid parameters, etc. Similarly to other string design models, pump-off conditions and pumping of water are assumed to increase the safety of the design. The main parameters of the actual pumping unit are needed also for the solution of the wave equation.

Based on the calculated rod loads the minimum and maximum loads in the first taper are fitted with straight lines, according to **Eqs. 2**, and **3** and the parameters of the best-fitting lines $\boldsymbol{a_1 \dots d_1}$ are found. Service factors at the top of each taper $\boldsymbol{SF}_{1,0}$, $\boldsymbol{SF}_{2,0}$, $\boldsymbol{SF}_{3,0}$ are calculated according to **Eq. 4** and their average, $\boldsymbol{SF}_0$ is determined.

In the first iteration step (J = 1) the length of the bottom taper (i =1) is modified while keeping the combined length of taper one and two constant. To find the bottom taper's length **Eq. 6** is utilized using the target service factor $\boldsymbol{SF}_0$ and the parameters $\boldsymbol{a_1 \dots d_1}$; the expressions to find adjusted taper lengths are the following:

$$L_{1,1} = \frac{SF_0 \frac{T}{A} A_1 + B\, SF_0\, d_1 - b_1}{a_1 - B\, SF_0\, c_1} \qquad \mathbf{8}$$

$$L_{2,1} = L_{2,0} + \left(L_{1,0} - L_{1,1}\right) \qquad \mathbf{9}$$

$$L_{3,1} = L_{3,0} \qquad \mathbf{10}$$

Using the modified taper lengths, the wave equation is solved again and the distribution of minimum and maximum loads in taper two is utilized to find the parameters of the best-fitting lines, $\boldsymbol{a_2 \dots d_2}$.

The second iteration step (J = 2) starts with the adjustment of the length of the second taper and keeping the length of the first taper. The third taper length is adjusted also and the relevant formulas are:

$$L_{1,2} = L_{1,1} \qquad \mathbf{11}$$

$$L_{2,2} = \frac{SF_0 \frac{T}{A} A_2 + B\, SF_0\, d_2 - b_2}{a_2 - B\, SF_0\, c_2} \qquad \mathbf{12}$$

$$L_{3,2} = L_{total} - \sum_{i=1}^{2} L_{i,2} \qquad \mathbf{13}$$

Now the wave equation is solved again using the adjusted taper lengths; service factors at the top of each taper $\boldsymbol{SF}_{1,2}$, $\boldsymbol{SF}_{2,2}$, $\boldsymbol{SF}_{3,2}$ are calculated according to **Eq. 4** and their average, $\boldsymbol{SF}_2$ is determined. Since the aim of the design is to reach identical service factors in each taper, the deviation from the average, $\boldsymbol{SF}_2$, of the individual $\boldsymbol{SF}$ values is evaluated from the formula:

$$Error = \frac{\sum_{i=1}^{3} \left(SF_2 - SF_{i,2}\right)^2}{3} \qquad \mathbf{14}$$

If the error is greater than the required accuracy previously set then the string received in the second iteration step must be adjusted further using the procedure described in iteration step one, and the whole process is repeated. Otherwise, if the error is below the required accuracy, the process has converged and the final string design is reached. At this point a final check is made where the final service factor is compared to its value required in the given field. In case the converged $\boldsymbol{SF}$ is below the accepted value then the design is final; otherwise the calculations must be repeated using different (a) rod materials, (b) numbers of tapers, or (c) rod sizes.

EXAMPLE PROBLEMS

To illustrate the operation of the developed rod string design procedure an example problem is presented. The task is to design an API 86 rod string in a 6,070 ft deep well with a 2 in pump. The C-456D-256-120 pumping unit runs at 10 SPM with a 120 in polished rod stroke length.

Since the API C and API K materials turned out to be weak, the string was designed for Grade D material and the taper lengths presented in **Table 3** were received. The average value of the service factors is SF_{avg} = 0.944 which indicates that the rods are very heavily loaded. In order to find strings with lighter overall loading, the design was repeated for stronger materials: high-strength (HS) rods and rods with Tenaris premium connections. As shown in **Table 3**, use of these materials resulted in much lightly loaded strings with average service factors of SF_{avg} = 0.543 and SF_{avg} = 0.587, respectively.

The merits of the string design procedure introduced in this paper are easily seen in **Fig. 3**, where minimum and maximum rod stresses in the tapers are plotted in a dimensionless form of the modified Goodman diagram. For every kind of material used the three points belonging to the three tapers fall on lines representing the average service factors in the string. This proves that all tapers have the same service factor, consequently the same safety included in their design. In comparison to other designs available today, rod loads during the whole design procedure were found from the solution of the wave equation. Since the wave equation predicts rod loads with the highest possible accuracy the design developed in this paper can be considered as the ultimate tool for designing sucker rod strings.

CONCLUSIONS

Based on the detailed evaluation of the available procedures for rod string design and the development of a new design model the following conclusions were drawn:

- Available rod string design procedures estimate rod loads from approximate formulas which result in loads that are not a true measure of the actual conditions; this is the main cause of their inaccuracy.
- The ultimate rod string design model presented here calculates rod loads from the solution of the one-dimensional wave equation and, therefore, results in strings with a much enhanced safety against fatigue failures as compared to previous designs.

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Table 1

Rod Type	T	A	B
	psi	-	-
API K steel	90,000	4	0.5625
API C steel	90,000	4	0.5625
API D steel	115,000	4	0.5625
High Strength (HS) rods	140,000	2.8	0.3750
Tenaris rods	125,000	2.3	0.3750

Table 2

Year	Model	Min. Load	Max. Load	Dyn. Loads	Design Goal
1953	Bethlehem	-	Fluid load plus rod weight in air	-	Equal max. stresses
1973	West	Rod weight in air	Fluid load plus rod weight in air plus dynamic loads	Mills acceleration factor	SF = const.
1976	Neely	Buoyant rod weight	Fluid load plus buoyant rod weight plus dynamic loads	Special formula	Equal modified stresses
1990	Gault - Takacs	Buoyant rod weight	Fluid load plus buoyant rod weight plus dynamic loads	From RP 11L	SF = const.

Table 3

Rod Size	API Grade D		HS Rods		Tenaris	
	L_i	SF_i	L_i	SF_i	L_i	SF_i
in	ft	-	ft	-	ft	-
1"	2,283	0.944	1,983	0.588	1,965	0.544
7/8"	1,807	0.947	1,695	0.587	1,680	0.543
3/4"	1,980	0.943	2,391	0.586	2,424	0.542

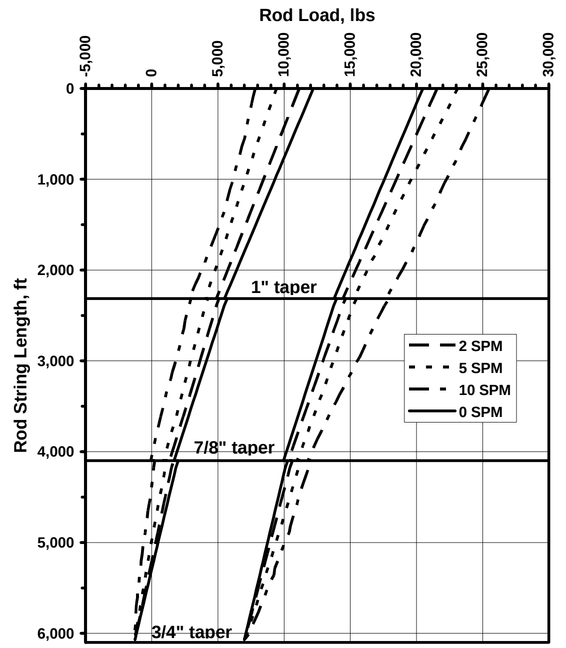

Figure 1

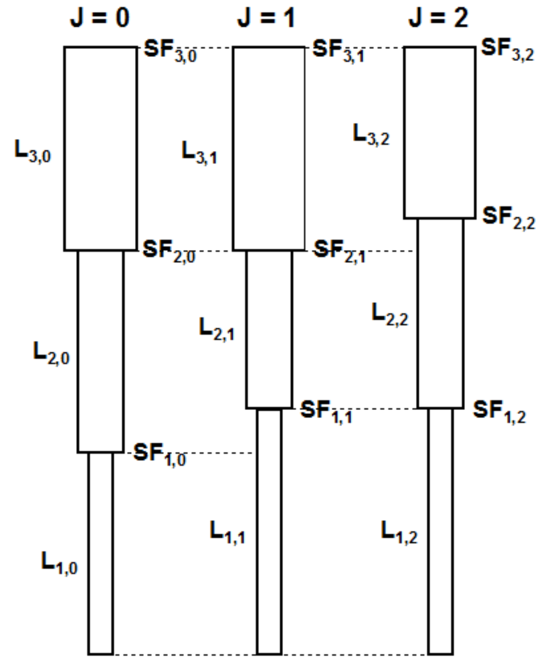

Figure 2

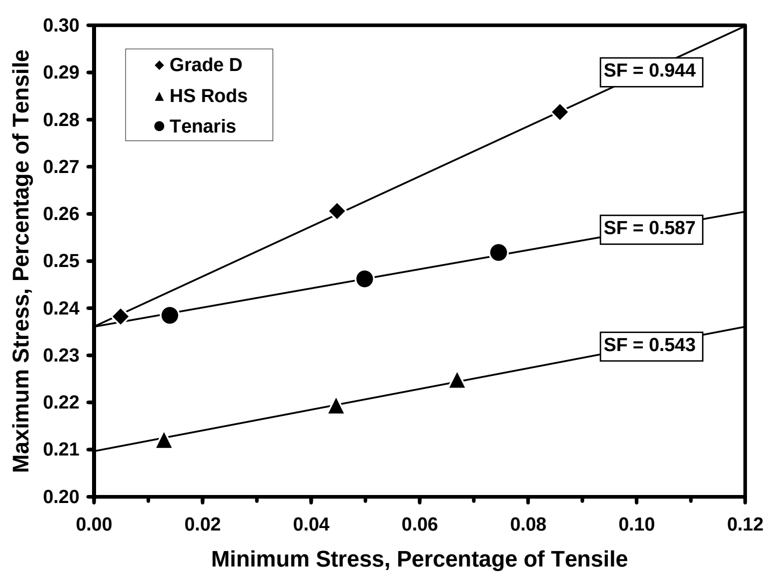

Figure 3