MODIFIED EVERITT-JENNINGS ALGORITHM WITH DUAL ITERATION ON THE DAMPING FACTORS

Victoria Pons-Ehimeakhe
Weatherford

ABSTRACT
In rod pumping wells, the downhole data can be computed from the surface data by solving the one dimensional damped wave equation. Currently, the modified Everitt-Jennings method uses finite differences and an iteration on the damping factor to solve the one dimensional damped wave equation. The iteration on the damping factor allows for an automatic damping factor adjustment, which can be a valuable tool when dealing with large fields of wells. The damping factor pertaining to this method is a common damping factor for both the upstroke and downstroke. In this paper, a new iteration on the damping factor is presented, in which the algorithm is split such that the upstroke and the downstroke damping factors are refined separately. Results are presented.

1. INTRODUCTION AND MOTIVATIONS.

Over the past years, sucker rod pumping has represented a major part of artificial lift. In sucker rod pumping, a surface dynamometer system is used to record the position and load data at the surface. Ideally a downhole dynamometer can be used to record the position and load data at the pump. However, the use of downhole dynamometer is an expensive and impractical commodity. Therefore, most companies today use some type of software analysis to compute the downhole data. The most accurate and popular of these methods is to compute the downhole data from the surface data by solving the one dimensional damped wave equation.
Solving the one dimensional wave equation is ideal for this problem since it most accurately models the behavior of the stress waves traveling down the rod string. The stress waves are a result of the elastic behavior of the rod string, see [12]. Since the rod string has the physical characteristics of an ideal slender bar, the propagation of stress waves is a one-dimensional phenomenon.
Originally Snyder solved the wave equation using the method of characteristics, see [11], while Gibbs employed separation of variables and Fourier series; see [4, 5, 7]. In 1969, Knapp, see [8], introduced finite differences to solve the wave equation, which is also the method used by Everitt and Jennings in 1976, see [3, 10]. The Everitt-Jennings method has been implemented and modified by Weatherford International, see [2].

As stated above, in the modified Everitt-Jennings method, finite differences are used to solve the wave equation. The rod string is discretized into M finite difference elements. Position and load including stress is computed at each increment down the wellbore. The modified Everitt-Jennings method also includes an iteration on the net stroke and damping factor, which automatically selects a damping factor for each stroke. Results of the modified Everitt-Jennings method and the iteration on the net stroke and damping factor are presented in [2].

Even though the results of the modified Everitt-Jennings method proved to be accurate with the use of a single damping factor, there has been some controversy about the use of separate upstroke and downstroke damping factors.
The conditions are very different between the upstroke and downstroke, mainly due to the direction of the fluid flowing with or against the rods, which would create more viscous friction on the downstroke than the upstroke.

The purpose of this paper is to introduce a new method for splitting and iterating on the upstroke and downstroke damping factor separately. The dual iteration on the damping factors combines a two step process for damping factor modification along with a bisection-like algorithm for the accuracy of the damping factor selection.

In Section 2, an overview of the modified Everitt-Jennings algorithm as well as details on the modified iteration on the net stroke and damping factor are presented.
In Section 3, the dual iteration on the net stroke and damping factor is presented.
Results are presented in Section ,4 with conclusions in Section 5.

2. OVERVIEW OF THE MODIFIED EVERITT-JENNINGS ALGORITHM.

2.1. The Everitt-Jennings method.

The Everitt-Jennings method uses finite differences to solve the one dimensional wave equation. The algorithm is reinforced by an iteration on the net stroke and damping factor as displayed in Figure 1. Let $u = u(x,t)$ be the displacement of position x at time t.
The condensed one-dimensional wave equation reads:

$$v^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + D\frac{\partial u}{\partial t}, \tag{1}$$

where the acoustic velocity is given by: $v = \sqrt{\frac{144Eg}{\rho}}$.
The above equation only considers friction forces of a viscous nature. The friction particular to this problem are of viscous and mechanical nature. The mechanical friction is a result of the friction between the rods, tubing and couplings. In the past, mechanical friction has generally been ignored but has since been addressed by Gibbs in [6] and Lukasiewicz in [9]. The viscous friction mentioned above is the results of the viscous forces arising in the annular space during the pumping cycle, which are proportional to the velocity of u.

In the Everitt-Jennings model, the first and second derivatives with respect to time are replaced by the first-order-correct forward differences and second-order-correct central differences. The second derivative with respect to position is replaced by a slightly rearranged second-order-correct central difference.

For more details on the algorithm, see [2, 3].

2.2. The Iteration on the net stroke and damping factor.

Everitt and Jennings presented a method for the automatic selection of the damping factor through an iteration on the net stroke and the damping factor. Through the use of finite differences, the rod string is divided into M finite difference nodes of length L_i (ft), density ρ_i (lbm/ft^3) and area A_i (in^2).
The damping factor can be computed through the equation:

$$D = \frac{(550)(144g)}{\sqrt{2}\pi}\frac{(H_{PR}-H_H)\tau^2}{(\sum \rho_i A_i L_i)S^2}, \tag{2}$$

Where H_{PR} is the polished rod horsepower (hp), S is the net stroke (in), τ is the period of one stroke (sec.) and H_{HYD} is the hydraulic horsepower (hp) obtained as follows:

$$H_{HYD} = (7.36 \cdot 10^{-6})Q\gamma F_l, \tag{3}$$

Where Q is the pump production rate (B/D), γ is the fluid specific gravity and F_l is the fluid level (ft). The pump production rate is given by:

$$Q = (0.1166)(SPM)\mathrm{S}D^2, \tag{4}$$

Where SPM is the speed of the pumping unit in strokes/minute and D is the diameter of the plunger.

For more details on the derivation of (2) and the original iteration on the net stroke and damping factor algorithm, see [3].

The original iteration on the net stroke and damping factor was modified in order to provide more accurate results in the field. As seen from above the calculation, the damping factor relies on the net stroke and the hydraulic horsepower. Both of which are unknown or approximated at the beginning of the calculation. The hydraulic horsepower is a factor of the fluid level, which reflects the amount of fluid in the wellbore annulus. Therefore unless calculated for that particular stroke, the fluid level may be highly inaccurate. Indeed, for most wells, a fluid level is taken typically at the beginning of well's life and stored in the software database without being updated on a regular basis. Therefore the value available for the fluid level might not reflect the current downhole state of the well.

The original iteration on the net stroke and the damping factor was therefore modified to incorporate a fluid level calculation and initialization step of the variables needed for the iteration. Using a fixed damping factor of 0.5, the downhole data is computed in order to produce a realistic value for the net stroke and compute a more accurate fluid level. By using this initialization step, the speed of convergence of the modified iteration on the net stroke and damping factor was seen to increase dramatically, see [2].

The modified iterative process for determining the net stroke and damping factor is as follows:

(1) Calculate downhole data using a fixed damping factor of 0.5.
(2) Determine the net stroke and compute the fluid level.
(3) Compute the damping factor and calculate the downhole data using the damping factor.
(4) Determine the new net stroke.
(5) If the new net stroke is within tolerance of the previous net stroke, move to the iteration on the damping factor (step 6), otherwise, go back to (step 2).
(6) Compute the damping factor using the converged net stroke value and calculate downhole data.
(7) Compute pump horsepower.
(8) If the pump horsepower value is within desired tolerance of the hydraulic horsepower, the iteration is complete. Otherwise, update the damping factor and go back to (step 7).

The results of the above algorithm depend on the quality of the input variables for the fluid level calculation. A flowchart of the process is displayed in Figure 1.

The great advantage of the automatic iteration on the net stroke and the damping factor is the fact that the damping factor is adjusted automatically without human intervention. Users, when managing a medium to large group of wells, do not have to spend time manually adjusting the damping factor as may be required by other methods.

In the next section, the separation of the upstroke and downstroke damping is presented as well as the dual iteration on the damping factors algorithm.

3. DUAL ITERATION ON THE DAMPING FACTORS.

3.1. Splitting the damping factor into upstroke and downstroke damping.

Realistically, the upstroke and downstroke damping can be very different. On the downstroke the rods moves against the fluids, while on the upstroke the rods moves with the fluids. Therefore, it is necessary to account for that difference by allocating a separate damping factor for the upstroke and for the downstroke.

For each stroke, the top of stroke is calculated. The top of stroke is the turning point at which the upward movement of the pumping unit stops and the downstroke begins, making it the delimiting factor between the downstroke and upstroke. The top of stroke is computed by calculating the zero of the first derivative of the position $u = u(x, t)$, i.e. the velocity. For more details on that computation, see [1].

The modified Everitt-Jennings finite difference algorithm, including the changes associated with treating two damping factors, reads as follows:

- Initialization: For $j = 1, \cdots, N : \ u_{0,j} = \ g_{PR,j}$.

- From Hooke's law: For $j = 1, \cdots, N : \ u_{1,j} = \frac{f_{PR,j} \cdot \Delta x}{EA} + u_{0,j}$.

- For $i = 2, \cdots, M$:

  For $j = 1, \dots, TOS$:

$$u_{i+1,j} = \frac{1}{\left(\frac{EA}{\Delta x}\right)^+}\left\{\left[\alpha\left(1 + D_{up}\Delta t\right)\right] \cdot u_{i,j+1} - \left[\alpha\left(2 + D_{up}\Delta t\right) - \left(\frac{EA}{\Delta x}\right)^+ - \left(\frac{EA}{\Delta x}\right)^-\right] \cdot u_{i,j} + \alpha \cdot u_{i,j-1} - \left(\frac{EA}{\Delta x}\right)^- \cdot u_{i-1,j}\right\}, \tag{5}$$

For $j = TOS + 1, \dots, N$:

$$u_{i+1,j} = \frac{1}{\left(\frac{EA}{\Delta x}\right)^+}\left\{\left[\alpha(1 + D_{down}\Delta t)\right] \cdot u_{i,j+1} - \left[\alpha(2 + D_{down}\Delta t) - \left(\frac{EA}{\Delta x}\right)^+ - \left(\frac{EA}{\Delta x}\right)^-\right] \cdot u_{i,j} + \alpha \cdot u_{i,j-1} - \left(\frac{EA}{\Delta x}\right)^- \cdot u_{i-1,j}\right\}, \tag{6}$$

where $\alpha = \frac{\overline{\Delta x}}{\Delta t^2}\left[\frac{\left(\frac{\rho A}{144g}\right)^+ + \left(\frac{\rho A}{144g}\right)^-}{2}\right]$ and $\overline{\Delta x} = \frac{1}{2}(\Delta x^+ + \Delta x^-)$.

- At the pump:
  For $j = 1, \dots, TOS$:

$$u_{pump,j} = \left(1 + D_{up}\Delta t\right) \cdot u_{M-1,j+1} - D_{up}\Delta t \cdot u_{M-1,j} + u_{M-1,j-1} - u_{M-1,j}, \tag{7}$$

For $j = TOS + 1, \dots, N$:

$$u_{pump,j} = (1 + D_{down}\Delta t) \cdot u_{M-1,j+1} - D_{down}\Delta t \cdot u_{M-1,j} + u_{M-1,j-1} - u_{M-1,j}, \tag{8}$$

For $j = 1, \dots, N$:

$$F_{pump,j} = \frac{EA}{2\Delta x}\left(3u_{M,j} - 4u_{M-1,j} + u_{M-2,j}\right).$$

In equation (5-8), the upstroke damping factor D_{up} is used for $j = 1, \dots, TOS$ while the downstroke damping factor D_{down} is used for $j = TOS + 1, \dots, N$.

In the next sub section, the dual iteration on the damping factors algorithm is presented.

3.2. Iteration on the upstroke and downstroke damping.

The dual iteration on the damping factors is composed of two parts. Similarly to the modified iteration on the damping factor, the iteration on the net stroke and the dual iteration on the damping factors are treated separately. Due to the fact that the net stroke varies only little with the variation of the damping factors, the iteration on the net stroke is done first. Once the net stroke is accurate the dual iteration on the damping factors begins.

The dual iteration on the damping factors is essentially a more complicated method than for the single damping factor.
Initially, the top of stroke must be determined before the dual iteration can begin. As mentioned above the top of stroke is the delimitation point for the dual iteration.

Secondly, the convergence criteria are two-fold. Similarly to the single iteration on the damping factor the absolute value of the difference between the pump horsepower and the hydraulic horsepower is monitored closely. Ideally the pump horsepower should equal the hydraulic horsepower. Therefore the difference between the pump horsepower and the hydraulic horsepower is determined. If that difference is less than a certain tolerance, the damping factors are assumed to be correct and the algorithm converged.

Moreover, if the pump horsepower and the hydraulic horsepower are not within the set tolerance, and if the pump horsepower is greater than the hydraulic, this is assumed to mean there is not enough damping when solving the wave equation. Similarly if the pump horsepower is less than the hydraulic horsepower, it is assumed that there is too much damping when solving the wave equation.

Ideally, the graphical representation of the upstroke and the downstroke should be horizontal lines. If the downhole card is concave, too much damping is used when solving the one-dimensional damped wave equation. Similarly, if the downhole card is convex, too little damping is used when solving the one-dimensional damped wave equation.

In the event that the pump horsepower and the hydraulic horsepower are not within tolerance and the damping needs to be adjusted, a second set of tests is set in place. This test compares the statistical distributions of the upstroke points and the downstroke points to the computed values of the fluid load lines. Essentially, the concavity of the upstroke and downstroke lines is tested.

The fluid load lines are determined by using the first and second derivatives of the position. The upstroke fluid load line referred in this paper as $F0_{up}$ while the downstroke fluid load line is referred to as $F0_{down}$.
The actual fluid load lines $F0_{up}actual$ and $F0_{down}actual$ are computed using the first and second derivatives of the downhole data. The upstroke actual fluid load line $F0_{up}actual$ is calculated as the load corresponding to the top of stroke. The top of stroke is computed by finding the zero of the first derivative of the downhole position data. In order to compute the downstroke actual fluid load line $F0_{down}actual$, the location of the transfer point must be calculated. The transfer point is the point at which the load is transferred from the traveling valve to the standing valve. The transfer point is computed using a pump fillage calculation, see [12]. The actual fluid load line $F0_{down}actual$ is taken to be the load of the absolute minimum of the second derivative of the downhole position data after the transfer point, i.e. the lower right corner of the downhole card.

The upstroke and downstroke data is statistically ordered by load in order to produce a probability density function. The maximums of the probability functions yield a set of load ranges in which most of the upstroke and downstroke data reside. The maximum of the probability density function for the upstroke data is referred to as the calculated fluid load line $F0_{up}calc$ while the maximum of the probability density function for the downstroke data is referred to as the calculated fluid load line $F0_{down}calc$.

The iteration on the net stroke and the dual iteration on the damping factors read as follows:

1) Guess net stroke (NS_0)
Guess top of stroke (TOS)
Compute daily production rate (Q)
Compute hydraulic horsepower (H_{hyd})
Set upstroke damping factor D_{up} to 0.25 and downstroke damping factor D_{down} to 0.5

2) Compute downhole data
Get top of stroke (TOS)
Compute actual fluid load lines ($F0_{up}$actual, $F0_{down}$actual)
Calculate net stroke (NS)
Calculate fluid level (Fl)
Calculate upstroke and downstroke damping factors (D_{up}, D_{down})

3) Is the absolute value of the difference between the guess net stroke and the calculated net stroke within tolerance? If yes, proceed to dual iteration on the damping Step 6. If no, continue iterating on the net stroke and proceed to Step 4.

4) Calculate daily production rate (Q)
Calculate hydraulic horsepower (H_{hyd})
Calculate upstroke and downstroke damping factors (D_{up}, D_{down})
Compute downhole data

5) Is the absolute value of the difference between the previous net stroke and new net stroke within tolerance? If yes, proceed to dual iteration on the damping Step 6. If no, continue iterating on the net stroke and go back to Step 4.

6) Calculate daily production rate (Q)
Calculate hydraulic horsepower (H_{hyd})

Calculate upstroke and downstroke damping factors (D_{up}, D_{down})

7) Compute downhole data
Compute calculated fluid load lines ($F0_{up}calc$, $F0_{down}calc$)
Compute pump horsepower (H_{pump})

8) Enter Damping Variation Algorithm. If successful exit, else go back to Step 7.

The above algorithm is given in the form of a flowchart in Figure 2.

The values for the calculated and actual fluid load lines are compared to determine if the upstroke and downstroke damping factors should be updated. This process serves as a test of concavity of sorts.

In the damping variation algorithm, the direction of the variation on the damping factors is determined by the sign of the difference between the pump horsepower and the hydraulic horsepower. If the pump horsepower is greater than the hydraulic horsepower, the data is deemed to not have enough damping. Consequently, the damping factors need to be increased. Similarly, if the pump horsepower is less than the hydraulic horsepower, the data is deemed to have too much damping and the damping factors need to be decreased. However, another test is set in place to determine if the damping factors should be increased.

The algorithm to determine the change in the damping factors is as follows:

1) Is the pump horsepower H_{pump} within tolerance of the hydraulic horsepower H_{hyd}? If yes, the iteration is successful and exits. If no, go to Step 2.

2) Is the calculated upstroke fluid load within range of the actual upstroke fluid load? And, is the calculated downstroke fluid load within range of the actual downstroke fluid load? If yes, the iteration is successful and exits. If no, go to Step 3 or Step 4.

3) Is the pump horsepower greater than the hydraulic horsepower? If yes,
   a) Is the calculated upstroke fluid load greater than the actual upstroke fluid load? If yes, update upstroke damping factor. If no, no update is necessary.
   b) Is the calculated downstroke fluid load less than the actual downstroke fluid load? If yes, update downstroke damping factor. If no, no update is necessary.

4) Is the pump horsepower less than the hydraulic horsepower? If yes,
   a) Is the calculated upstroke fluid load less than the actual upstroke fluid load? If yes, update upstroke damping factor. If no, no update is necessary.
   b) Is the calculated downstroke fluid load greater than the calculated downstroke fluid load? If yes, update downstroke damping factor. If no, no update is necessary.

Lastly a bisection-like method was set in place to refine the selection of the damping factors. Once the direction of the variation of the damping changes, i.e. the sign of the variation, a bisection-like method commences in order to zero in on the correct damping factor. The above damping factors are increased or decreased by a set factor. Once the bisection-like method is started, the factor will progressively be lowered until the change in the damping factor is within 0.1. A flowchart of the variation on the damping factors is presented in Figure 3.
In the next section, results are presented.

4. RESULTS.

In this section, results are presented. In Figures 4-13, results for the comparison of the dual iteration on damping factors with the single iteration on the damping factor and the Delta program are presented. The Delta program uses the Gibbs' method as coded in the LOWIS software. In the figures below, on the top left corner, the downhole card resulting from the dual iteration on the damping factors is depicted. On the right hand side, the downhole card resulting from the single iteration on the damping factor is depicted, while on the lower left side, the downhole card resulting from the Delta program is depicted. All of the downhole cards presented as results have been shifted to the zero line to offer a better comparison between the methods. Also on the bottom right corner, details about the iterations are presented. These details include for the dual iteration on the damping factors, the actual and calculated fluid load lines, the polished rod horsepower, the hydraulic horsepower, the pump horsepower, the damping factors as well as the number of iterations. For the single iteration on the damping factor, the polished rod horsepower, the hydraulic horsepower, the pump horsepower, the damping factor as well as the number of iterations are included. A set of two numbers is presented for the number of iterations. The first number is the number of iterations for the iteration on the net stroke, while the second number is the number of iterations for the damping. The computer code for the iteration on the net stroke is identical for both methods.

The damping factors are values designed to remove and add "noise" to the wave equation. Therefore the range for a damping factor is from 0 to 1. As a precaution, the damping factors are corrected so that they do not exceed the value 1 or go below 0. Therefore a limit value for each end of the range is set in place with D_{max} = 0.99 and D_{min} = 0.01.

For the examples presented below, the numerical value for the damping factors used for the Delta program do not compare to the damping factors used for the modified Everitt-Jennings algorithm. The damping factors used for the Delta are computed using rod information provided by the rod manufacturers and require manual adjustment.

In example 1, depicted in Figure 4, a well from San Joaquin Valley is examined. The original stroke length for the well is 100 inches. The iteration on the net stroke converged in 2 iterations to a value of 61.78 inches. The polished rod horsepower is 2.52 hp, while the hydraulic horsepower is 2.04 hp. For both the dual iteration on the damping factors and the single iteration on the damping factor, the pump horsepower is 2.22 hp. The $F0_{up}actual$ and $F0_{down}actual$ are 2311 and 26 lbs respectively, while the $F0_{up}calc$ and $F0_{down}calc$ are 2265 and 163 lbs respectively.
The dual iteration on the damping factors exited after only 1 iteration, since the $F0_{up}actual > F0_{up}calc$ and the $F0_{down}actual < F0_{down}calc$, even though $|H_{pump} - H_{hyd}| > 0.1$. The corresponding damping factors are D_{up} = 0.25 and D_{down} = 0.5. The single iteration on the damping factor also stopped at the first iteration since the $H_{pump} > H_{hyd}$, with a damping factor of 0.5. The card obtained through the Delta program shows over-damping.

In example 2, depicted in Figure 5, a well from West Texas is examined. The original stroke length for the well is 144.05 inches. The iteration on the net stroke converged in 2 iterations to a value of 119.44 inches. The polished rod horsepower is 7.98 hp, while the hydraulic horsepower is 6.9 hp. For the dual iteration on the damping factors, the pump horsepower is 7.23 hp, while for the single iteration on the damping factor, the pump horsepower 6.96 hp. The $F0_{up}actual$ and $F0_{down}actual$ are 4734 and 10 lbs respectively, while the $F0_{up}calc$ and $F0_{down}calc$ are 4746 and -16 lbs respectively.
The dual iteration on the damping factors exited after 1 iteration since $H_{pump} > H_{hyd}$ and the $F0_{up}actual$ and $F0_{down}actual$ are within tolerance of the values for the $F0_{up}calc$ and $F0_{down}calc$. The corresponding damping factors are 0.25 and 0.5.
The single iteration on the damping factor also exited in 1 iteration since $H_{pump} > H_{hyd}$, resulting in a damping factor of 0.5. The card obtained through the Delta program displays a loop on the right hand side.

In example 3, depicted in Figure 6, a well from West Texas is examined. The original stroke length for the well is 64.00 inches. The iteration on the net stroke converged in 2 iterations to a value of 37.56 inches. The polished rod horsepower is 2.38 hp, while the hydraulic horsepower is 1.56 hp. For the dual iteration on the damping factors the pump horsepower is 1.6 hp, while for the single iteration on the damping factor the pump horsepower is 1.47 hp. The $F0_{up}actual$ and $F0_{down}actual$ are 1502 and -753 lbs respectively, while the $F0_{up}calc$ and $F0_{down}calc$ are 1603 and -624 lbs respectively.
The dual iteration on the damping factor converged in 4 iterations, with $|H_{pump} - H_{hyd}| < 0.1$ with damping factors D_{up} and D_{down} equal to 0.1 and 0.2 respectively. Moreover, the actual fluid load lines are within tolerance of the calculated fluid load lines, which further confirms that the appropriate damping factors were selected. For the single iteration on the damping factor, the iteration converged after 2 iterations with a damping factor of 0.88. However from the presented downhole card, the card is clearly over-damped. The downhole card obtained through the Delta program also displays a "loop".

In example 4, depicted in Figure 7, a well from Bakersfield is examined. The original stroke length for the well is 86.3 inches. The iteration on the net stroke converged in 2 iterations to a value of 10.06 inches. The polished rod

horsepower is 1.26 hp, while the hydraulic horsepower is 1.19 hp. For the dual iteration on the damping factors the pump horsepower is 1.16 hp while for the single iteration on the damping factor, the pump horsepower is 1.01 hp. The actual fluid load lines are 2612 and 0 lbs, while the calculated fluid load lines are 2613 and 67 lbs.
The dual iteration of the damping factors converged in 7 iterations satisfying $\left|H_{pump} - H_{hyd}\right| < 0.1$ with damping factors D_{up} = 0.1 and D_{down} = 0.5. For this example the values for the actual fluid load lines are within tolerance of the values for the calculated fluid load lines.
The single iteration on the damping factor converged in 1 iteration also satisfying $\left|H_{pump} - H_{hyd}\right| < 0.1$ with a damping factor of 0.5. The downhole card obtained with the Delta program, shows a "loop", suggesting over-damping.

In example 5, depicted in Figure 8, a well from the Permian Basin is examined. The original stroke length for the well is 123.2 inches. The iteration on the net stroke converged in 2 iterations to a value of 59.84 inches. The polished rod horsepower is 5.37 hp while the hydraulic horsepower is 3.8 hp. For the dual iteration on the damping factors, the pump horsepower is 4.45 hp while for the single iteration on the damping factor, the pump horsepower is 3.72 hp. The values for the actual fluid load lines are 5783 and 343 lbs respectively, while the values for the calculated fluid load lines are 5626 and -371 lbs.
The dual iteration on the damping factors exited after 4 iterations even though $\left|H_{pump} - H_{hyd}\right| > 0.1$ since $F0_{up}actual > F0_{up}calc$ and $F0_{down}actual > F0_{down}calc$ with damping factors equal to 0.1 and 0.4 respectively.
For the single iteration on the damping factor, the algorithm converged in 5 iterations satisfying $\left|H_{pump} - H_{hyd}\right| < 0.1$ with a damping factor of 0.41. Both downhole cards from the modified Everitt-Jennings with single iteration on damping factor and the downhole card obtained through the Delta program show a "loop".

In example 6, depicted in Figure 9, a well from the Permian Basin is examined. The original stroke length for the well is 120.00 inches. The iteration on the net stroke converged in 2 iterations to a value of 85.21 inches. The polished rod horsepower is 8.45 hp, while the hydraulic horsepower is 5.35 hp. For the dual iteration on the damping factor, the pump horsepower is 6.65 hp, while for the single iteration on the damping factor, the pump horsepower is 7.0 hp. The $F0_{up}actual$ and $F0_{down}actual$ are 3268 and -1113 lbs respectively, while the $F0_{up}calc$ and $F0_{down}calc$ are 3210 and -1086 lbs respectively.
The dual iteration on the damping factors exited in 6 iterations with $H_{pump} > H_{hyd}$, $F0_{up}actual \approx F0_{up}calc$ and $F0_{down}actual < F0_{down}calc$. The corresponding damping factors are $D_{up} = 0.75$ and $D_{down} = 0.5$.
For the single iteration on the damping factor, the algorithm exited after 1 iteration since $H_{pump} > H_{hyd}$ with a damping factor of 0.5. The downhole card associated with the Delta program show under-damping.

In example 7, depicted in Figure 10, a well from the Permian Basin is examined. The original stroke length for the well is 86.1 inches. The iteration on the net stroke converged in 2 iterations to a value of 75.91 inches. The polished rod horsepower is 8.55 hp, while the hydraulic horsepower is 4.38 hp. For the dual iteration on the damping factors, the pump horsepower is 6.59 hp while for the single iteration on the damping factor, the pump horsepower is 5.84 hp. The values for the actual fluid load lines are 2424 and -1197 lbs respectively, while the values for the calculated fluid load lines are 2732 and -972 lbs respectively.
The dual iteration on the damping factors exited after 9 iterations since $H_{pump} > H_{hyd}$, $F0_{up}actual < F0_{up}calc$ but $D_{up} = D_{max} = 0.99$ and $F0_{down}actual < F0_{down}calc$. The corresponding damping factors are $D_{up} = 0.99$ and $D_{down} = 0.5$.
For the single iteration on the damping factor, the algorithm exited after 1 iteration since $H_{pump} > H_{hyd}$ with a damping factor of 0.99. The associated downhole card, however shows over-damping.
The downhole card associated with the Delta program shows under-damping.

In example 8, depicted in Figure 11, a well from Bakersfield is examined. The original stroke length for the well is 57.10 inches. The iteration on the net stroke converged in 2 iterations to a value of 13.05 inches. The polished rod horsepower is 0.74 hp, while the hydraulic horsepower is 0.6 hp. For the dual iteration on the damping factors the pump horsepower is 0.67 hp, while for the single iteration on the damping factor the pump horsepower is 0.56 hp. The values for the actual fluid load lines are 2234 and 311 lbs respectively, while the values for the calculated fluid load lines are 1909 and 338 lbs respectively.
The dual iteration on the damping factors converged in 4 iterations with $\left|H_{pump} - H_{hyd}\right| < 0.1$ with damping factors of $D_{up} = 0.01$ and $D_{down} = 0.2$. Furthermore $F0_{up}actual > F0_{up}calc$ and $F0_{down}actual \approx F0_{down}calc$.

The single iteration on the damping factor converged in 1 iteration to a damping factor of 0.99. The downhole card obtained through the Delta program shows over-damping.

In example 9, depicted in Figure 12, a well from the Permian Basin is examined. The original stroke length for the well is 119.90 inches. The iteration on the net stroke converged in 2 iterations to a value of 95.92 inches. The polished rod horsepower is 17.04 hp, while the hydraulic horsepower is 10.44 hp. For the dual iteration on the damping factors the pump horsepower is 11.6 hp, while for the single iteration on the damping factor the pump horsepower is 10.87 hp. The values for the actual fluid load lines are 4831 and 676 lbs respectively, while the values for the calculated fluid load lines are 4832 and -1034 lbs respectively.
The dual iteration on the damping factors exited after 7 iterations with $F0_{up}actual \approx F0_{up}calc$ and $D_{up} = 0.75$, while $F0_{down}actual > F0_{down}calc$ with $D_{down} = D_{max} = 0.99$ even though $H_{pump} > H_{hyd}$.
For the single iteration on the damping factor, the algorithm exited after 1 iteration since $H_{pump} > H_{hyd}$ with a damping factor of 0.9. The downhole card associated with the Delta program shows under-damping.

In example 10, depicted in Figure 13, a well from the Permian Basin is examined. The original stroke length for the well is 144.00 inches. The iteration on the net stroke converged in 2 iterations to a value of 123.33 inches. The polished rod horsepower is 24.00 hp, while the hydraulic horsepower is 16.20 hp. For the dual iteration on the damping factors the pump horsepower is 15.13 hp, while for the single iteration on the damping factor the pump horsepower is 16.15 hp. The values for the actual fluid load lines are 7220 and 2223 lbs respectively, while the values for the calculated fluid load lines are 7346 and 2329 lbs respectively.
For the dual iteration on the damping factors, the algorithm exited after 7 iterations with $F0_{up}actual < F0_{up}calc$, $F0_{down}actual < F0_{down}calc$ and $H_{pump} < H_{hyd}$ with damping factor values of 0.75 and 0.8 respectively.
For the single iteration on the damping factor, the algorithm converged in 6 iterations with a damping factor of 0.6845 to satisfy $|H_{pump} - H_{hyd}| < 0.1$.
The downhole card associated with the Delta program suggests under-damping.

For example 1, 2, 4, 8 and 9 corresponding to Figures 4, 5, 7, 11 and 12, the results from the dual iteration on the damping factors and the single iteration on the damping factor, are very similar. Indeed for both methods, the downhole cards show appropriate damping. However, on these examples, the downhole card obtained through the Delta program shows inappropriate damping. For example 9, depicted in Figure 12, the downhole card obtained through the Delta program displays not enough damping as seen by the “fat” appearance of the card, whereas for examples 1, 2, 4 and 8, depicted in Figures 4, 5, 7 and 11, the downhole card obtained through the Delta programs shows too much damping since the cards loop over themselves.

For examples 3 and 5, depicted in Figures 6 and 8, the results from the dual iteration on the damping factors and the single iteration on the damping factor are different. In both cases, the single iteration on the damping converged, however the damping factors are not appropriate since the downhole cards loop on themselves. The same “looping” phenomenon is also observed for the downhole cards obtained through the Delta program, which is a sign of over-damping. For both examples, the dual iteration on the damping factors did not converge through the horsepower test but rather through testing the concavity of the fluid load lines.

For examples 6, 7 and 10, depicted in Figures 9, 10 and 13, each downhole card obtained through all three methods vary in “thickness”. Even though for each example, it appears that the downhole card obtained through the dual and single iterations show more appropriate damping than for the “fatter” Delta card, it is hard to determine which damping is most accurate. The above examples clearly come from wells with mechanical friction, which can complicate the selection of the damping, since the viscous damping is not meant to overcome the mechanical friction.

In the next section, conclusions are presented.

5. CONCLUSIONS.

Handling the damping of the one dimensional damped wave equation can be done using a single damping factor or by using both an upstroke damping factor and a downstroke damping factor. As can be shown by the above results, in most instances, the difference between the results of both methods is small. However in some instances, the single iteration on the damping factor is not enough to overcome the downhole conditions present in the well, and selects, even upon convergence, an inappropriate damping factor.

Consistently however, the results from either iterations on damping prove better than the calculation of the damping factor as present in the Delta program. The damping factors in the Delta program can be manually adjusted to match the results from the dual iteration on the damping factors or the single iteration on the damping factor.

The concavity test added to the dual iteration on the damping factors is a useful tool, which really completes the iteration. Depending on what downhole conditions are present in a well, the use of the more robust dual iteration on the damping factor can prove essential for the user.

ACKNOWLEDGEMENTS
Special thanks to Lynn Rowlan and the team here at Weatherford.

BIBLIOGRAPHY.

1. Ehimeakhe, V. : “Calculating Pump Fillage for Well Control using Transfer Point Location”, SPE Eastern Regional Meeting, 12-14 October 2010, Morgantown, West Virginia, USA.
2. Ehimeakhe, V. : “Comparative Study of Downhole Cards Using Modified Everitt-Jennings Method and Gibbs Method”, Southwestern Petroleum Short Course 2010.
3. Everitt, T. A. and Jennings, J. W.: “An Improved Finite-Difference Calculation of Downhole Dynamometer Cards for Sucker-Rod Pumps,” paper SPE 18189 presented at the 63rd Annual Technical Conference and Exhibition, 1988.
4. Gibbs, S. G., and Neely, A. B. : “Computer Diagnosis of Down-Hole Conditions in Sucker Rod Pumping Wells,” JPT (Jan. 1996) 91-98; *Trans.*, AIME,237
5. Gibbs, S. G.: “A Review of Methods for Design and Analysis of Rod Pumping Installations,” SPE 9980 presented at the 1982 SPE International Petroleum Exhibition and Technical Symposium, Beijing, March 18-26.
6. Gibbs, S. G.: “Design and Diagnosis of Deviated Rod-Pumped Wells”, SPE Annual Technical Conference and Exhibition, Oct 6-9, 1991, Dallas, USA.
7. Gibbs, S.G.:”Method of Determining Sucker-Rod Pump Performance,” U.S. Patent No. 3,343,409 (Sept. 26, 1967).
8. Knapp, R. M.: “A Dynamic Investigation of Sucker-Rod Pumping,” MS thesis, U. of Kansas, Topeka (Jan. 1969).
9. Lukasiewicz, S. A.: “Dynamic Behavior of the Sucker Rod String in the Inclined Well”, Production Operations Symposium, April 7-9, 1991, Oklahoma City, Oklahoma, USA.
10. Schafer, D. J. and Jennings, J. W.: “An Investigation of Analytical and Numerical Sucker-Rod Pumping Mathematical Models,” paper SPE 16919 presented at the 1987 SPE Annual Technical Conference and Exhibition, Dallas, Sept. 27-30.
11. Snyder, W. E.: “A Method for Computing Down-Hole Forces and Displacements in Oil Wells Pumped With Sucker Rods,” paper 851-37-K presented at the 1963 Spring Meeting of the API Mid-Continent District Div. of Production, Amarillo, March 27-29.
12. Takács, G.: “Sucker-Rod Pumping Manual.” PennWell, 2002.

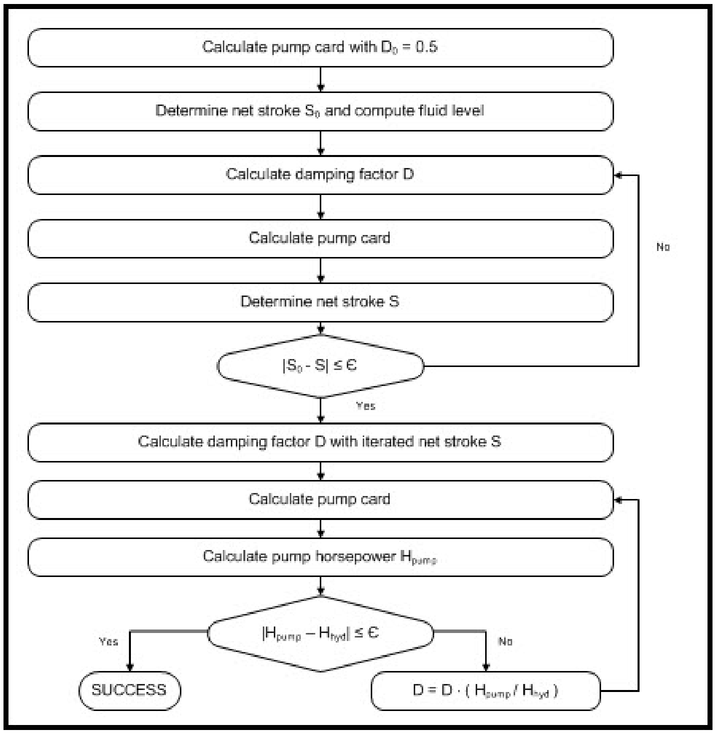

Figure 1 - Flowchart for the iteration on the net stroke and damping factor for the modified Everitt-Jennings algorithm.

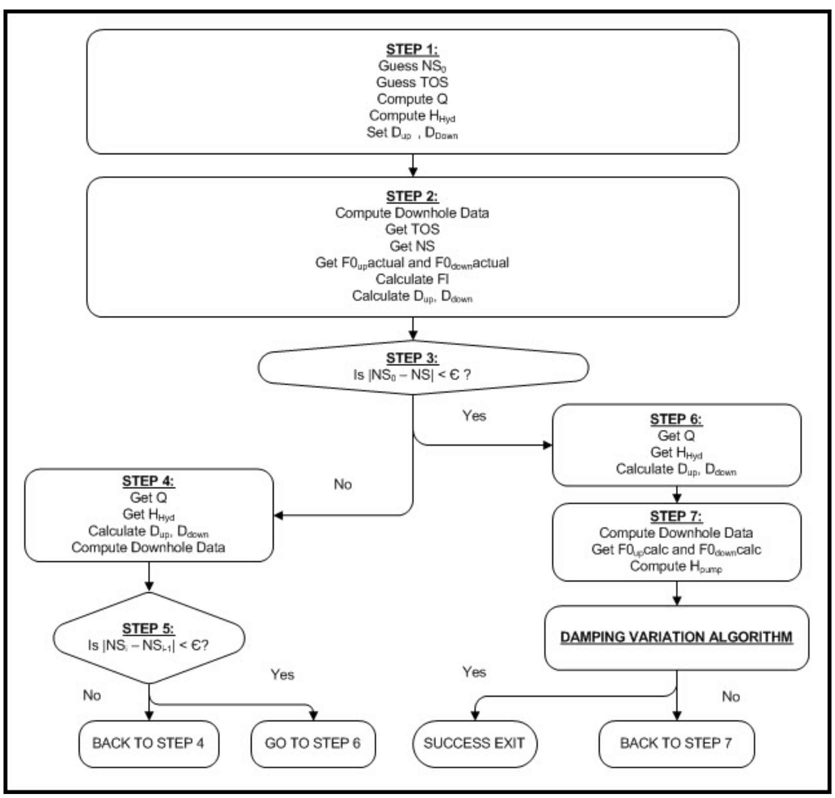

Figure 2 - Flowchart for the dual iteration on the net stroke and damping factors for the modified Everitt-Jennings algorithm.

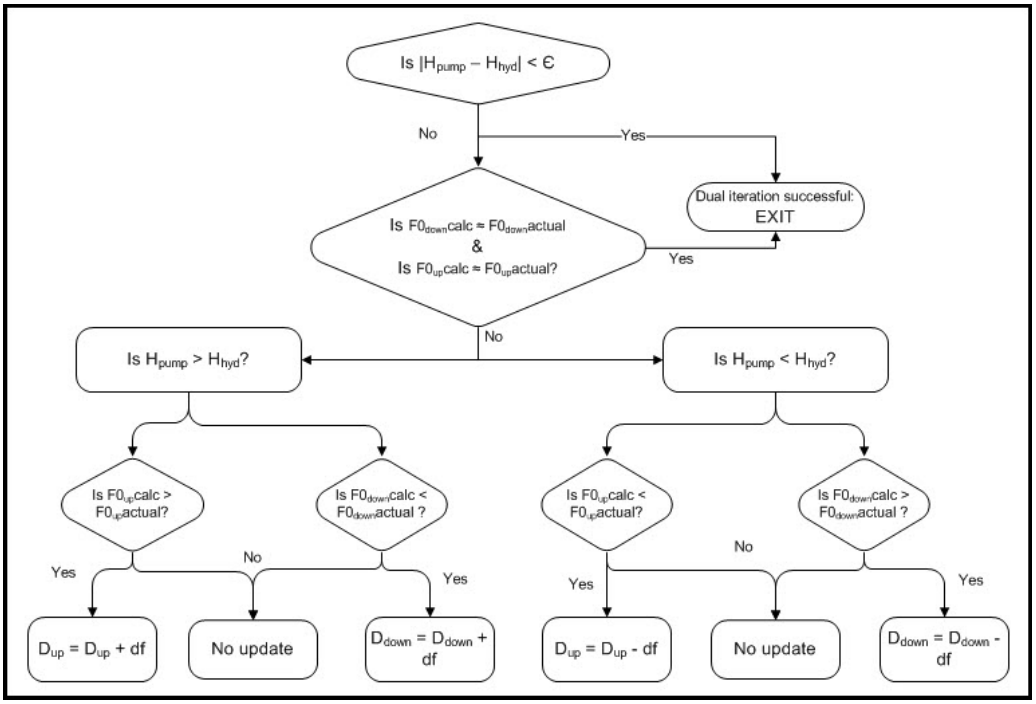

Figure 3 - Flowchart variation on the upstroke and downstroke damping criteria.

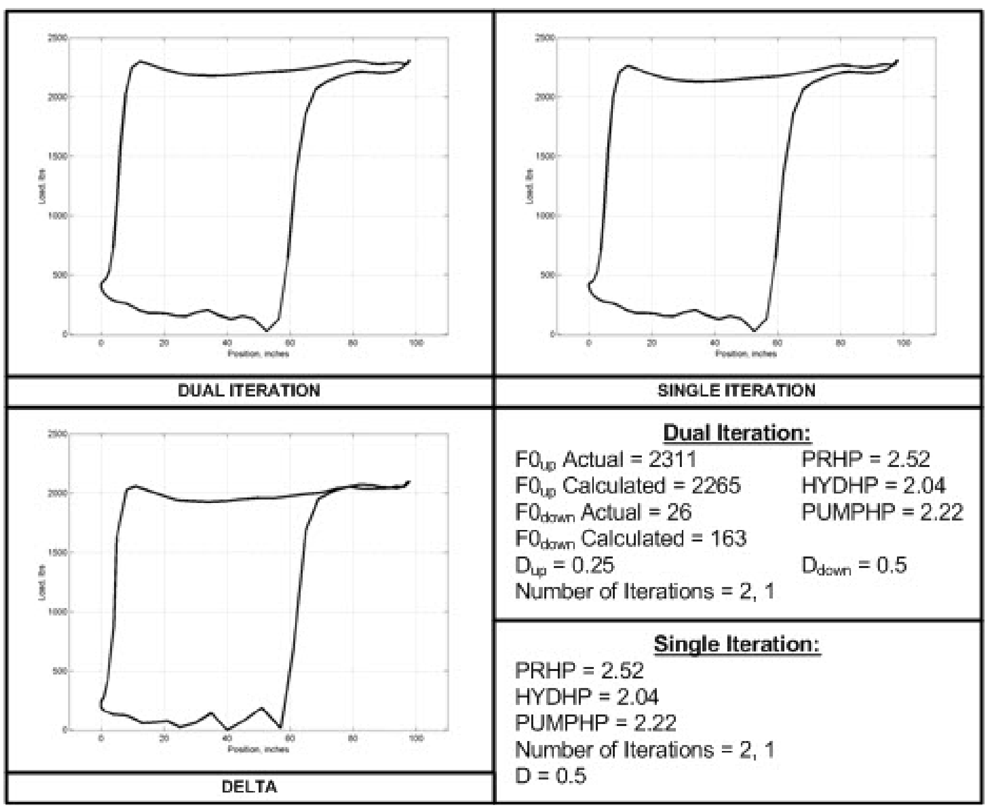

Figure 4 – Example 1.

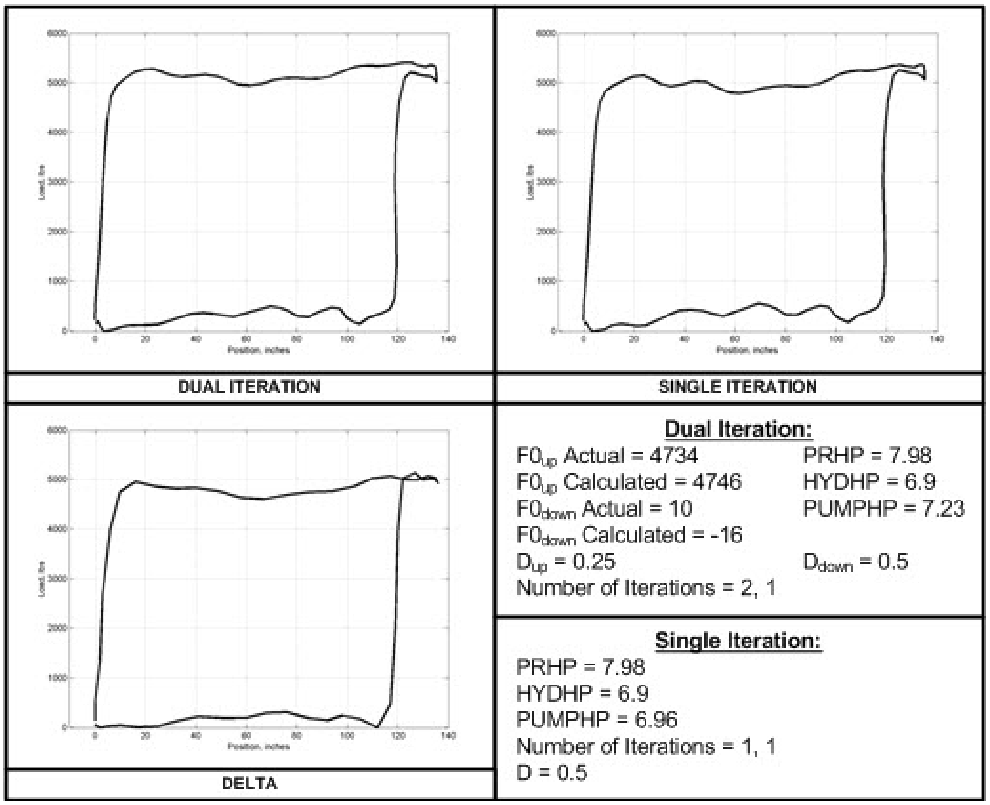

Figure 5 – Example 2.

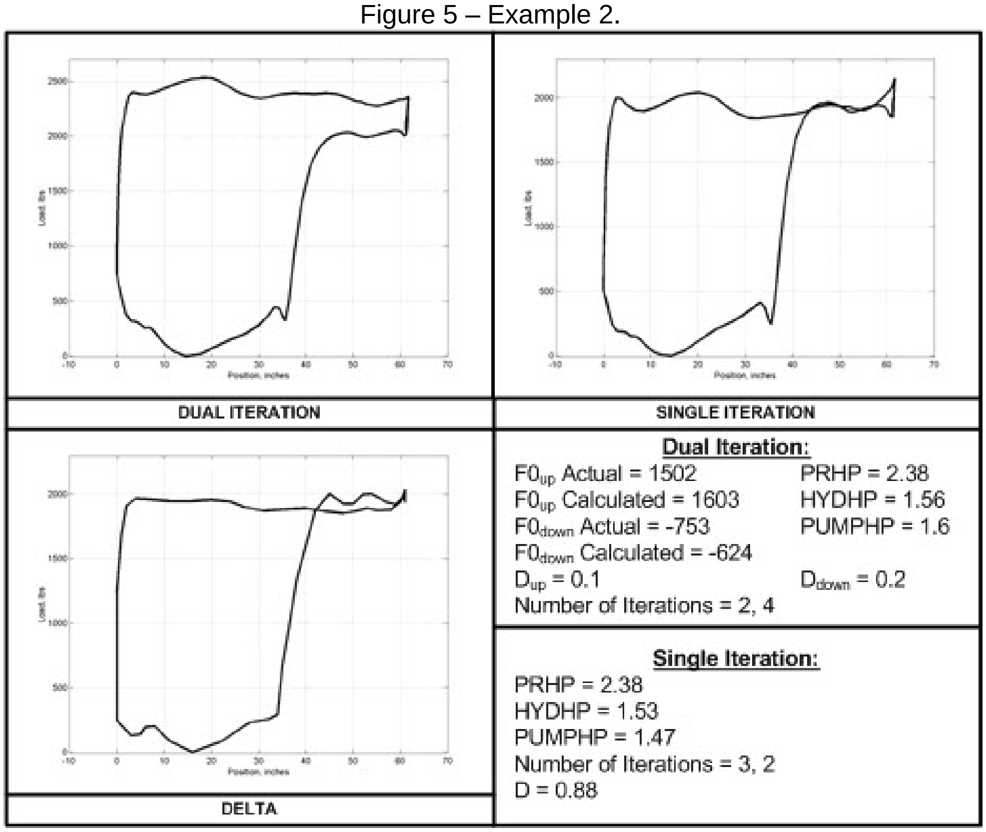

Figure 6 – Example 3.

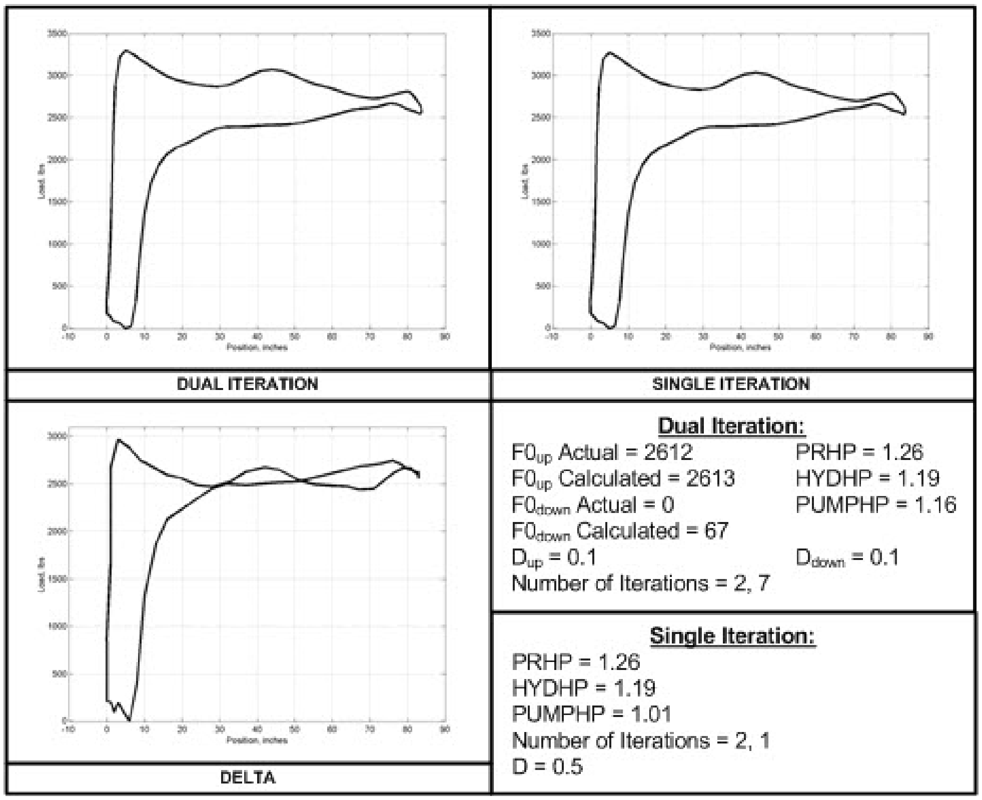

Figure 7 – Example 4.

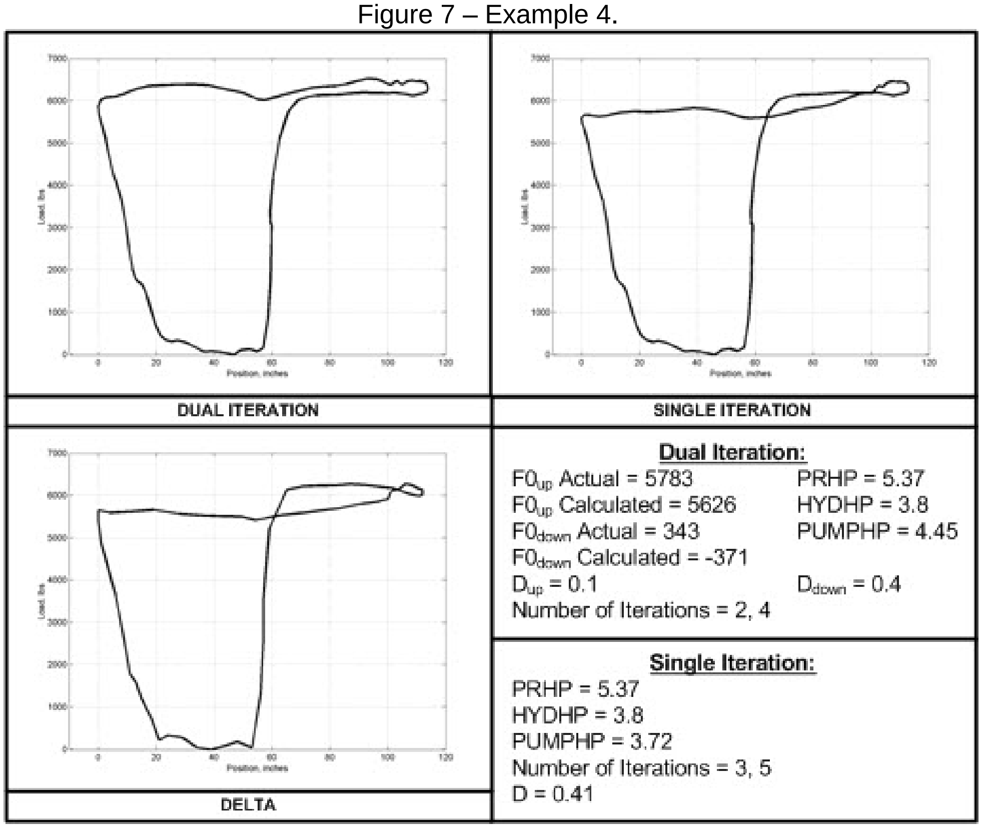

Figure 8 – Example 5.

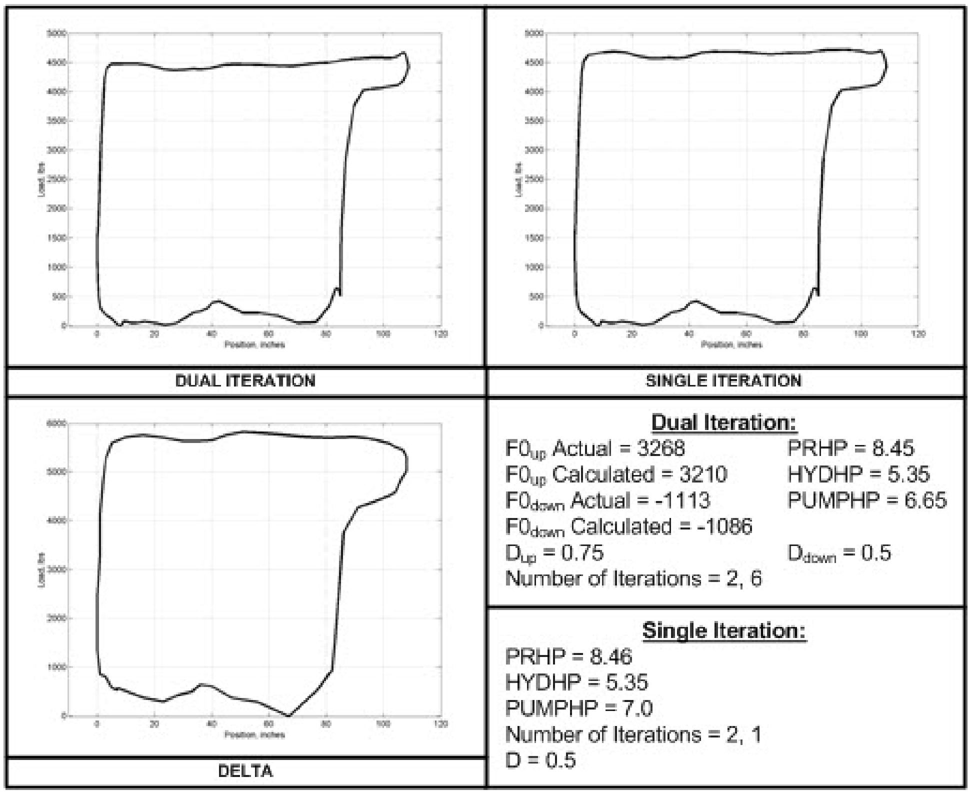

Figure 9 – Example 6.

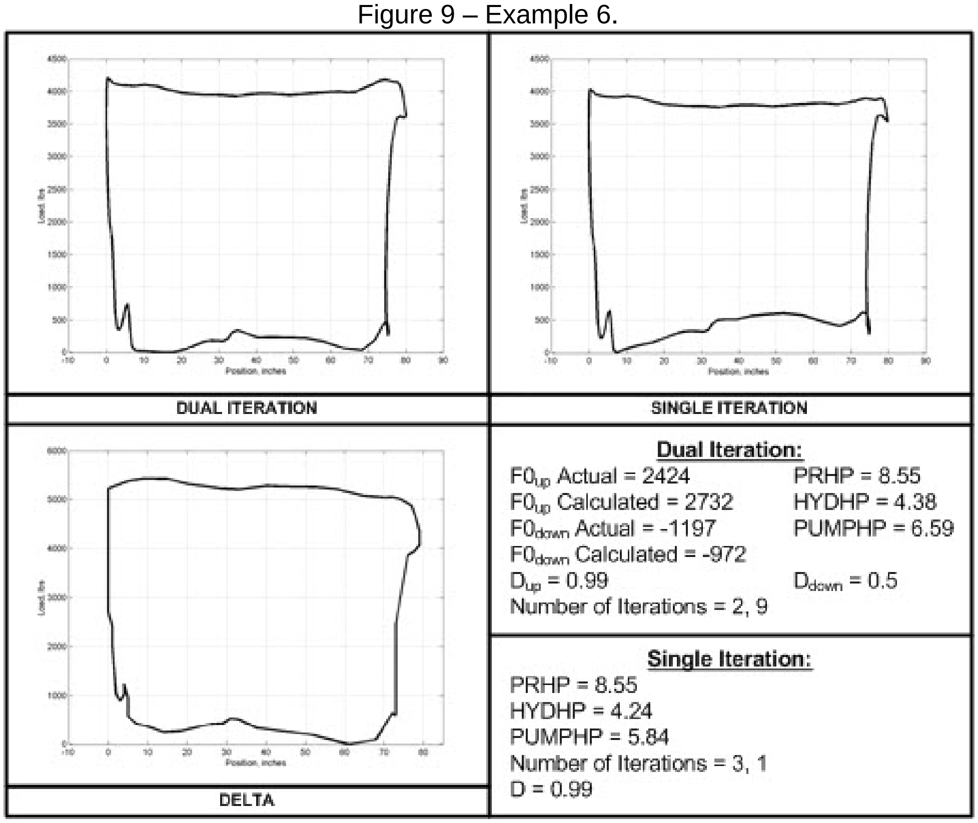

Figure 10 – Example 7.

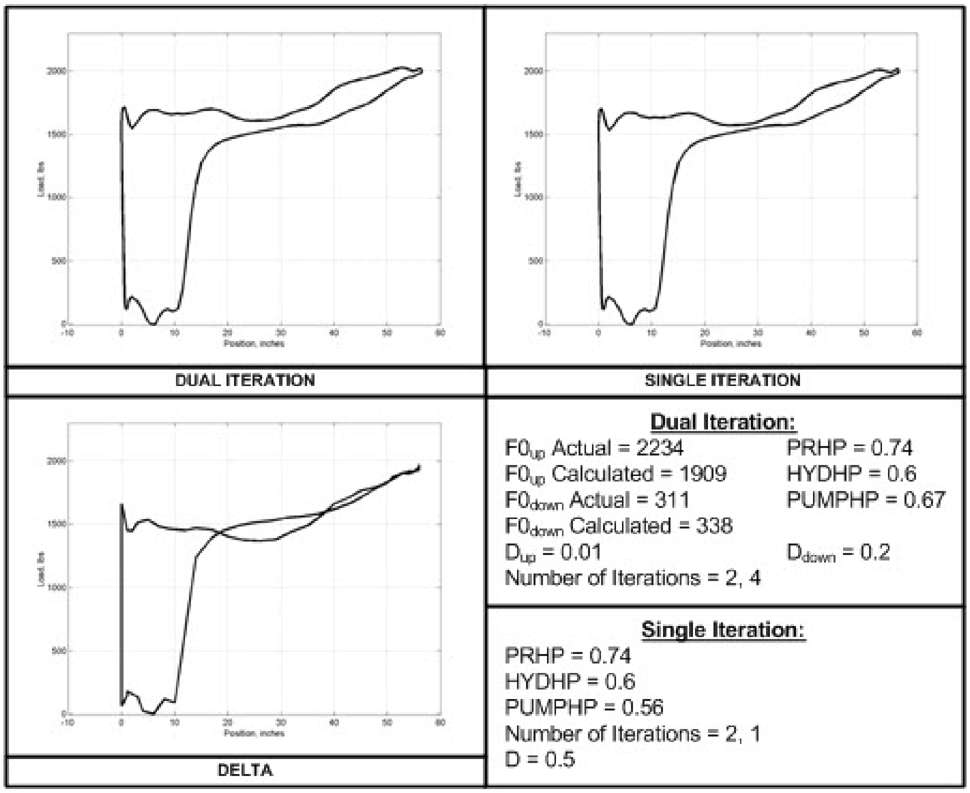

Figure 11 – Example 8.

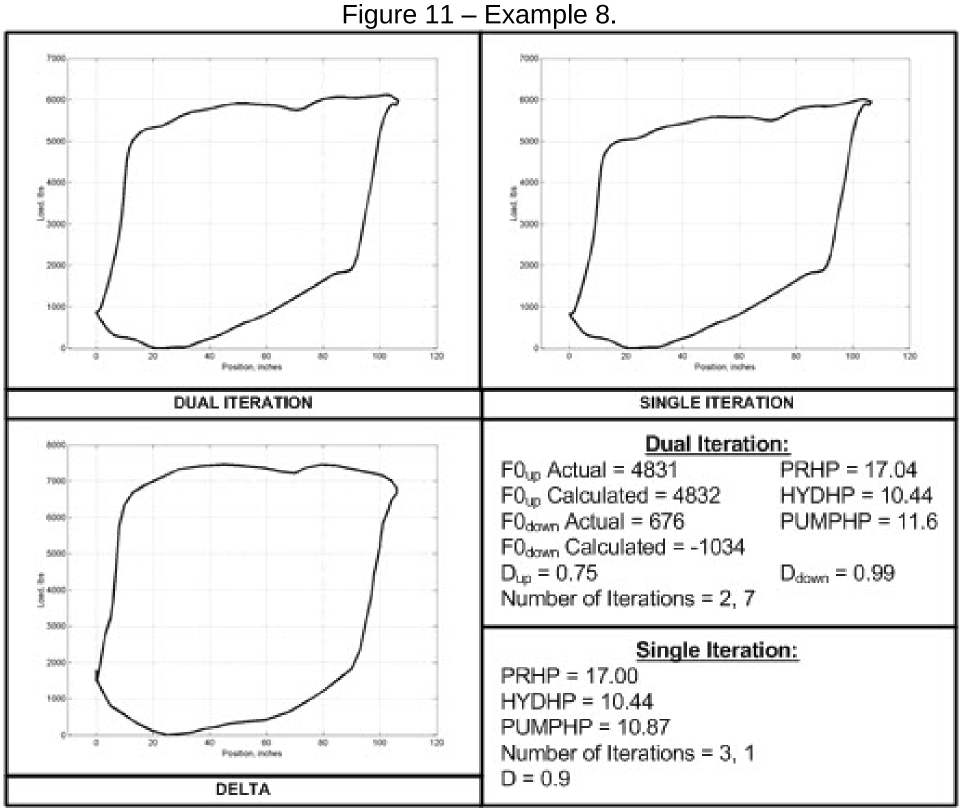

Figure 12 – Example 9.

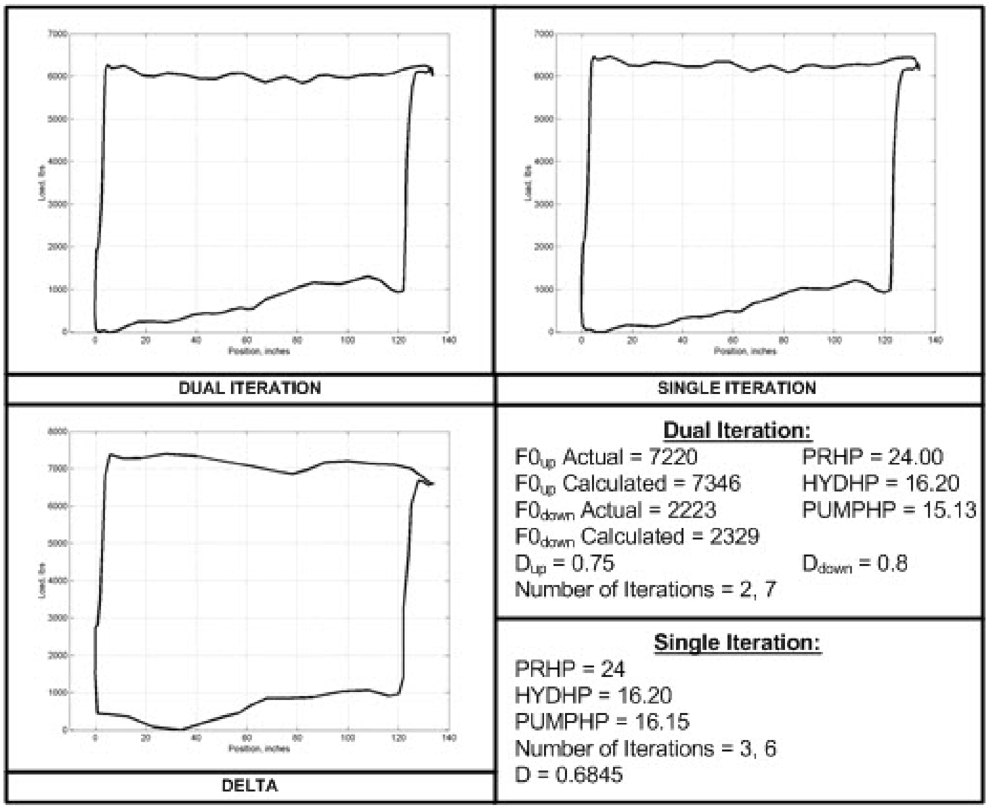

Figure 13 – Example 10.