MODIFICATION OF VOGEL'S IPR CURVE FOR SATURATED OIL RESERVOIRS

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ABSTRACT

Vogel's inflow performance relationship relates the flowing well pressure to production rate for solution-gas drive reservoirs. Because two-phase flow exists, the graph of bottom-hole flowing pressures versus oil production rate results in a curved line. This trend accounts for the decrease in production as more gas comes out of the solution.
Vogel assumes the initial reservoir pressure is the same as the bubble point pressure for the starting point of the IPR curve. This implies no gas has initially come out of the solution, i.e. the reservoir is at bubble point pressure. Saturated reservoirs, as studied in this paper, are initially undersaturated reservoirs with average reservoir pressure below the bubble point pressure. Traditionally, Vogel's inflow performance relationship has been applied to these reservoirs using the reservoir pressure as the starting point for the curve. However, due to the presence of gas at the reservoir pressure, this is not an accurate assumption. This paper modifies the Vogel IPR curve for use in wells within reservoirs that are below the bubble point pressure.

INTRODUCTION

Oil productivity is generally assumed to be proportional to the drawdown, or the pressure differential between the reservoir and the wellbore. This holds true for a single phase, incompressible fluid. For undersaturated reservoirs, or reservoirs above the bubble point, a plot of the bottom-hole flowing pressure versus the rate results in a straight line.

Figure 1 illustrates the straight line IPR for undersaturated reservoirs. The productivity index (PI) is equal to the inverse of the slope of the line.

$$PI = \frac{4}{P_R - P_{wf}} \qquad (1)$$

In 1942, Evinger and Muskat[1] stated the straight-line relationship for oil productivity is not valid when two phases, oil and gas, are flowing in the reservoir. They suggested a curved relationship between pressure and flow rate, based on theoretical calculations using Darcy's equation for radial flow.
Vogel[2] expanded on the ideas of Evinger and Muskat in 1968. Vogel used a computer to simulate solution gas drive reservoirs with two-phase flow of oil and gas. He created a plot of bottom hole flowing pressure versus oil production rate from the simulated data. These plots had a curved shape, as suggested by Evinger and Muskat. Vogel discovered the curves had a similar shape, and when normalized, they could be represented with a single curve. Figure *2* is a demonstration of Vogel's IPR curve. Based on the normalized curve, Vogel developed an empirical inflow performance relationship (IPR) for saturated solution gas drive reservoirs that includes the flow of two phases, oil and gas, in the reservoir. Vogel's IPR equation is

$$\frac{q}{4_{\max}} = 1 - 0.2\frac{P_{wf}}{P_R} - 0.8\left(\frac{P_{wf}}{P_R}\right)^2 \qquad ..(2)$$

In developing his equation, Vogel assumed the initial reservoir pressure was the same as the bubble point pressure. He used this pressure as the starting point for his IPR curve.

As P_R falls beneath P_b , the fluid saturations within the reservoir change. Since relative permeability is dependent on saturation, the flow of fluid entering the wellbore changes **as** the reservoir depletes. Therefore, inflow performance relationships are dependent on depletion. To obtain IPR curves for solution gas drive reservoirs depleted beneath the bubble point, Standing[4] used current productivity index values to develop furture IPR curves based on fluid and **rock** properties. This method allowed him to modify inflow performance relationships for changing fluid saturations as the well is produced.
In his textbook on artificial lift methods, Brown' discusses Fetkovich's contribution to inflow performance relation-

ships. Fetkovich conducted isochronal tests to examine the flow rate as a function of pressure. He noted wells producing below the bubble point should behave more like gas wells. Therefore, Fetkovich proposed a back pressure equation for oil wells similar to that of gas wells

$$q_o = J_o^{'}(P_R^2 - P_{wf}^2)^n \qquad (3)$$

where J_o represents the productivity index or the back pressure coefficient. For an exponent of $n = 1$, Fetkovich's method predicts rates similar to those of Vogel.

In 1992, Wiggins, Russell, and Jennings[3] developed an analytical IPR based on the physical nature of the multiphase flow system to present a theoretical basis for Vogel's IPR. Their analytical IPR equation is

$$\frac{q}{q_{max}} = 1 + \frac{C_1}{D}\frac{P_{wf}}{P_R} + \frac{C_2}{D}\left(\frac{P_{wf}}{P_R}\right)^2 + \frac{C_3}{D}\left(\frac{P_{wf}}{P_R}\right)^3 + \frac{C_4}{D}\left(\frac{P_{wf}}{P_R}\right)^4 \qquad (4)$$

where the coefficients are a function of the mobility function, k_{ro}/μ_o. The mobility term is dependent on the pressure and saturation. Therefore, the analytical IPR is largely affected by reservoir depletion. Because the analytical IPR has the same form as Vogel's IPR, Wiggins, et al. suggest the coefficients in Vogel's IPR have a physical basis and are more than fitting parameters.

Reservoir depletion and changing mobility due to a saturation variation affect the IPR curves. Because inflow performance relationships indicate well productivity at a given reservoir pressure, it is necessary to have a range of IPR equations to model reservoir conditions as P_R decreases.

ANALYTICAL DEVELOPMENT

Traditionally, Vogel's IPR equation has been applied to saturated reservoirs (i.e. $P_R < P_b$) using the reservoir pressure, P_R, as the starting point for the curve. This method does not consider the restricted oil flow due to the gas saturation at P_R. This paper proposes a modified IPR to account for the presence of gas in solution gas drive reservoirs beneath the bubble point pressure.

In solution gas drive reservoirs below the bubble point, gas is released from the solution with depletion. **As** the gas saturation within the reservoir increases, the oil flow meets a greater resistance. Therefore, the productivity for a well in a solution gas drive reservoir decreases as the reservoir pressure is reduced. To include the effect of the gas on oil productivity, this paper adjusts Vogel's IPR curve to place the bubble point pressure P_b at the starting point of the curve. Figure 3 demonstrates this adjustment on Vogel's IPR curve.

The shift places the IPR curve in terms of theoretical rates to determine what the well could produce had the effects of the gas saturation been considered. The theoretical rate q_T is equal to the actual rate q_A plus the distance shifted q_{xx} This adjustment to Vogel's IPR equation gives

$$\frac{q_T}{q_{max(T)}} = \frac{q_A + q_{xx}}{q_{max(A)} + q_{xx}} = 1 - 0.2\frac{P_{wf}}{P_b} - 0.8\left(\frac{P_{wf}}{P_b}\right)^2 \qquad (5)$$

Therefore, the actual rate equals

$$q_A = (q_{\max(A)} + q_{xx})\left[1 - 0.2\frac{P_{wf}}{P_b} - 0.8\left(\frac{P_{wf}}{P_b}\right)^2\right] - q_{xx} \quad \text{.......(6)}$$

Assuming the bubble point pressure is known from PVT analysis or correlations, the above equation can he solved if a test point with a bottom hole flowing pressure and rate is known along with the average reservoir pressure

(for $P_b > P_R > P_{test}$).

If $P_{wf} = P_R$, then $q_A = 0$ and equation (6) becomes

$$0 = q_{xx}\left[-0.2\frac{P_R}{P_b} - 0.8\left(\frac{P_R}{P_b}\right)^2\right] + q_{\max(A)}\left[1 - 0.2\frac{P_R}{P_b} - 0.8\left(\frac{P_R}{P_b}\right)^2\right] \quad \text{.......(7)}$$

If $P_{wf} = P_{test}$, then $q_A = q_{test}$ and equation (6) becomes

$$q_{test} = q_{xx}\left[-0.2\frac{P_{test}}{P_b} - 0.8\left(\frac{P_{test}}{P_b}\right)^2\right] + q_{\max(A)}\left[1 - 0.2\frac{P_{test}}{P_b} - 0.8\left(\frac{P_{test}}{P_b}\right)^2\right] \quad \text{.......(8)}$$

Simultaneously solving equations **(7)** and (8) gives values for $q_{\max(A)}$ and q_{xx}. If these two values are known, then equation (6) can be used to calculate the anticipated rate at any bottom hole flowing pressure.

TEST CASES FOR MODIFIED IPR **EQUATION**

Since field data was unavailable, the Modified IPR equation ***(6)*** was compared to the Vogel IPR using test cases based on data from Figure ***2*** in Vogel's paper. Vogel's Figure ***2*** is a plot of bottom hole flowing pressure versus production rate and illustrates how the IPR curves change with depletion. The purpose of the test cases was to determine how accurately the Modified IPR and Vogel IPR could predict q_{max} compared to the absolute open flow, or q_{max}, from Vogel's Figure ***2.***

The values for P_R, q_{max}, P_{test}, and q_{test} were read off Figure ***2*** for the depletion curves. (This figure is very small and values from this graph were estimated visually.) Three separate cases were run by varying the percent drawdown in order to examine the sensitivity of the Modified IPR and Vogel IPR to the test point. Depletion refers to the average reservoir pressure decreasing as fluids are produced from the reservoir. Drawdown is the difference between P_R and P_{wf}. Test points were taken for 5%, 10%, and 20% drawdown for each depletion curve considered. The q_{max} for Vogel's IPR was calculated using equation ***(2).*** Simultaneously solving equations (7) and (8) determined the value of q_{max} for the Modified IPR. P_b was considered to be a constant for all cases at 2130 psi (per Vogel's data set). Table 1 lists the test data used and the q_{max} values calculated for each case.

RESULTS OF TEST CASES

In every case, the Modified IPR predicted a higher q_{max} than the Vogel IPR. In three of fourteen cases, the Modified q_{max} was closer to the given q_{max} than the Vogel. For the low depletion cases where the P_b approximately equaled P_R, the Modified IPR and Vogel IPR produced the same curve. In all cases, the q_{max}, predicted by both equations was very close to the same value and within 4% difference for the low depletion cases and within 10% difference for the high depletion cases. However, using a test point at a higher drawdown resulted in a closer approximation of q_{max} for each case.

DEPENDENCE OF MAXIMUM FLOW ON THE BUBBLE POINT PRESSURE

In each of the test cases described above, there is only a small difference between the Modified q_{max} and the Vogel q_{max}. However, large bubble point pressure relative to the reservoir pressure will cause significant changes in q_{max}. Figure 4 is a plot $q_{max}/Vogel\, q_{max}$ versus P_b/P_R and illustrates how the bubble point pressure influences the maximum flow rate.

For undersaturated reservoir conditions (i.e. $P_R > P_b$), the ratio of maximum flow rates decreases linearly as P_b approaches P_R. This indicates the q_{max} changes at the same rate as Vogel's q_{max} as the P_b increases.

As the reservoir becomes saturated, the ratio of the maximum flow rates increases as P_b becomes greater than P_R.

Therefore, the Modified IPR predicts a higher q_{max} than the Vogel IPR. The Modified IPR has a 20% increase in q_{max} over the Vogel IPR when the ratio of P_b to P_R is 5 or greater. This ratio of pressures is more likely to occur in highly depleted reservoirs. In Vogel's paper, equation (2) does not match the highly depleted cases of approximately greater than 10% depletion.

As P_b becomes increasingly greater than P_R, the gas saturation of the reservoir will increase. At some point, the GOR will become so large the well will be considered a gas well and not an oil well. Vogel's IPR and the Modified IPR were developed for solution gas drive reservoirs and may not be valid for gas wells. The limiting value of GOR for inflow performance relationships to apply was not studied as part of this paper but should be considered in further studies.

IMPORTANCE IN ACCURATE TEST POINTS

While creating data sets to test the Modified IPR equation, slight changes in P_{wf} with little drawdown caused significant variations in q_{max}. Consequently, a sensitivity study was conducted to verify the importance of the selection of the single test point. Vogel's IPR equation requires a test point to extrapolate to the maximum pressure drawdown to determine q_{max}. Therefore, the maximum flow rate is dependent on P_{test} and q_{test}.

Selection of P_{test} is crucial due to the squared term in equation *(2)*. To eliminate all non-reservoir (wellbore) and reservoir transient pressure effects, the well must reach pseudo-steady state flow before the test point is taken. Even a slight error in pressure measurement or selection of the correct bottom hole flowing pressure to use as P_{test} results in a large error in the calculation of q_{max}. Vogel IPR is very sensitive to variations in pressures as seen in Figure 5. The solid line in Figure 5 represents a data set with without error in P_{test}. **All** of the other IPR curves are the prediction made with Vogel, if the P_{test} was unknowingly recorded or measured with +/-1% and +/-10% error at 10 and **20%** pressure drawdown. The prediction of q_{max} with these modest errors is incredible. **As** an example, the -1% and +1% error in P_{test} for the 10% pressure drawdown case results in a -7.9% and +9.4% error in the calculated q_{max}. In other words a 1% error in P_{test} results in almost a +/-10% error in q_{max} for this example. Figure 5 shows other more dramatic errors in q_{max} based on more significant errors in the test pressure.

The drawdown of the test point should also be considered. Test points with a high drawdown require less extrapolation to reach absolute open flow or q_{max}. **A** higher drawdown therefore predicts a more accurate q_{max}.

CONCLUSIONS

As IPR equations only represent the reservoir productivity at a certain point in time, three separate IPR equations are necessary to model the inflow performance over the life of the reservoir as the average reservoir pressure decreases.

When $P_R > P_b$, the straight line IPR combined with Vogel's IPR by matching slopes at the bubble point should be used. For $P_R = P_b$, Vogel's IPR equation, equation (2), applies. When the average reservoir pressure falls beneath the bubble point (i.e. $P_R < P_b$), calculate q_{max} using the Modified IPR equation, equation (6), to account for the increased gas saturation within the reservoir.

Highly depleted reservoirs or those with a large P_b compared to P_R, have the largest difference in the q_{max} predicted by the Modified and Vogel IPR. Less difference is seen in slightly depleted reservoirs with P_b closer to P_R.

When $P_R = P_b$, the Modified IPR reduces to Vogel's IPR.

Therefore, the Modified IPR equation is an option for determining the well performance in reservoirs beneath the bubble point pressure.

NOMENCLATURE

PI Productivity index(bpd/psi)

P_R Reservoir pressure(psi)

P_{wf} Bottom hole flowing pressure(psi)

q_{max} Maximum flow rate(bpd)

$J_o{}'$ Back pressure curve coefficient(bpd/psi^{2n})

q_T Theoretical flow rate(bpd)

q_A Actual flow rate(bpd)

q_{st} Shifted rate(bpd)

$q_{max(T)}$ Theoretical maximum flow rate(bpd)

$q_{max(A)}$ Actual maximum flow rate(bpd)

P_{test} Bottom hole flowing test pressure(psi)

P_b Bubble point pressure(psi)

q_{test} Test rate(bpd)

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Table 1
Test Cases from Vogel's Figure 2

Case A: 5% Drawdown Test Point

		Data from Vogel's Figure 2				Calculated values	
	Depletion	P_R (psi)	P_{test} (psi)	q_{test} (bpd)	q_{max} (bpd)	Vogel q_{max} (bpd)	Modified q_{max} (bpd)
A1	Np/N=0.1%	2130	2024	10	185	114.2	114.2
A2	Np/N=2%	1960	1862	14.5	151	164.8	166.0
A3	Np/N=4%	1810	1720	11	118	125.7	127.6
A4	Np/N=8%	1500	1425	5.4	66	61.4	63.5

Case B: 10%Drawdown Test Point

		Data from Vogel's Figure 2				Calculated values	
	Depletion	P_R (psi)	P_{test} (psi)	q_{test} (bpd)	q_{max} (bpd)	Vogel q_{max} (bpd)	Modified q_{max} (bpd)
B1	Np/N=0.1%	2130	1917	27	185	157.0	157.0
B2	Np/N=2%	1960	1764	26	151	151.2	152.2
B3	Np/N=4%	1810	1629	21	118	122.1	123.9
B4	Np/N=8%	1500	1350	11	66	64.0	66.1
B5	Np/N=12%	920	828	4	20	23.3	25.5

Case C: 20% Drawdown Test Point

		Data from Vogel's Figure 2				Calculated values	
	Depletion	P_R (psi)	P_{test} (psi)	q_{test} (bpd)	q_{max} (bpd)	Vogel q_{max} (bpd)	Modified q_{max} (bpd)
C1	Np/N=0.1%	2130	1704	60	185	182.9	182.9
C2	Np/N=2%	1960	1568	50	151	152.4	153.5
C3	Np/N=4%	1810	1448	40	118	122.0	123.6
C4	Np/N=8%	1500	1200	20	66	61.0	62.9
C5	Np/N=12%	920	736	6.4	20	19.5	21.2

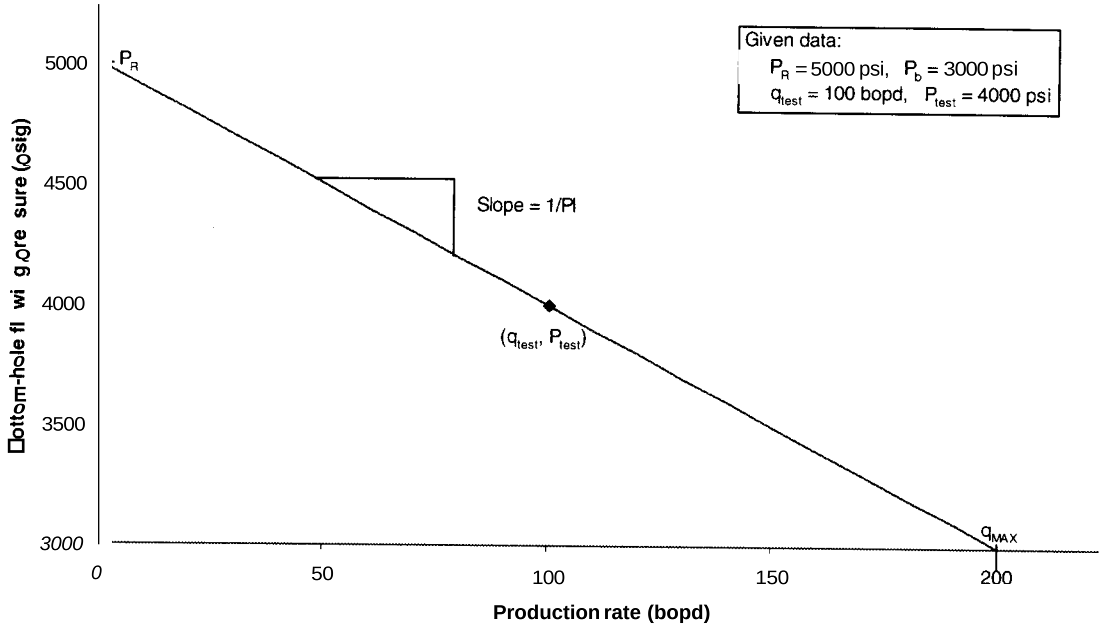

Figure 1 - Straight Line IPR

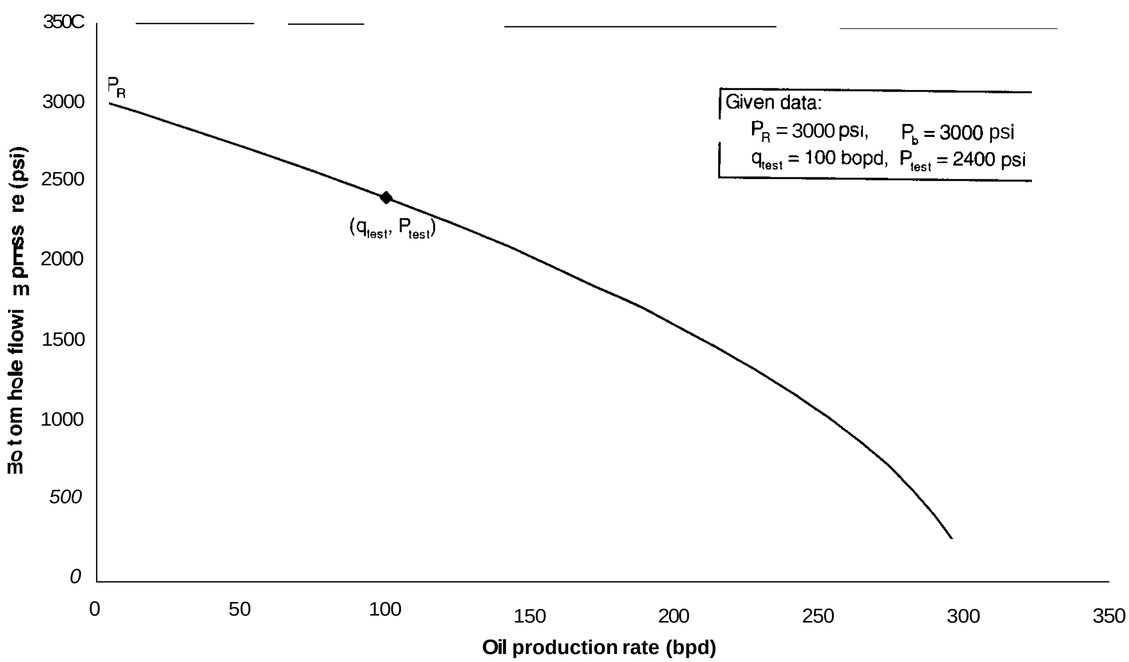

Figure 2 - Vogel's IPR Curve

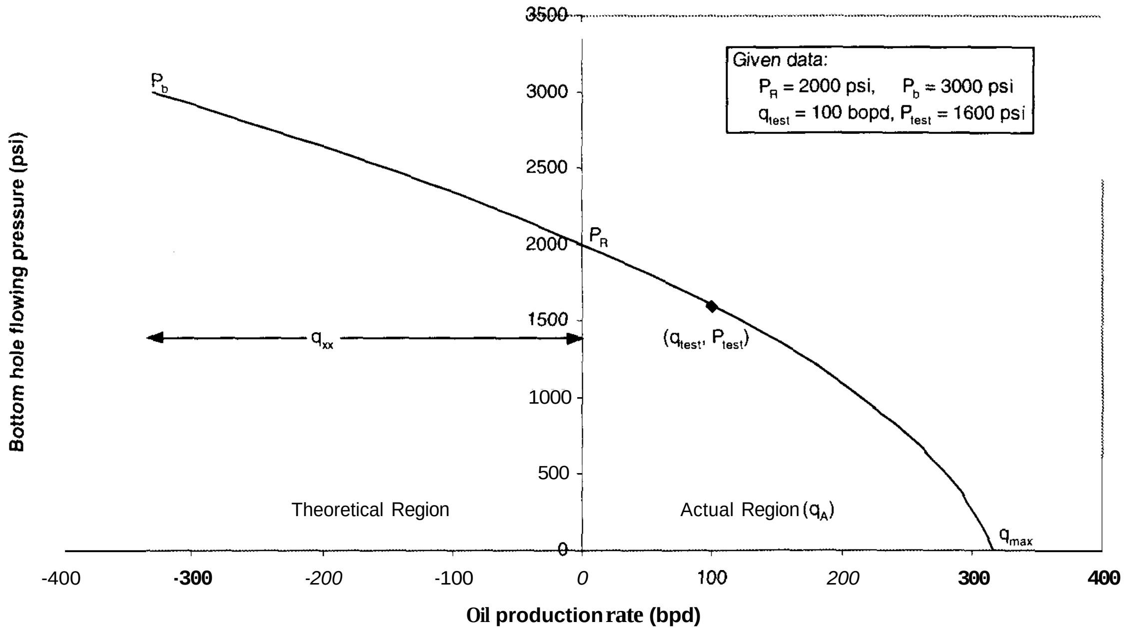

Figure 3 - Modified IPR Curve

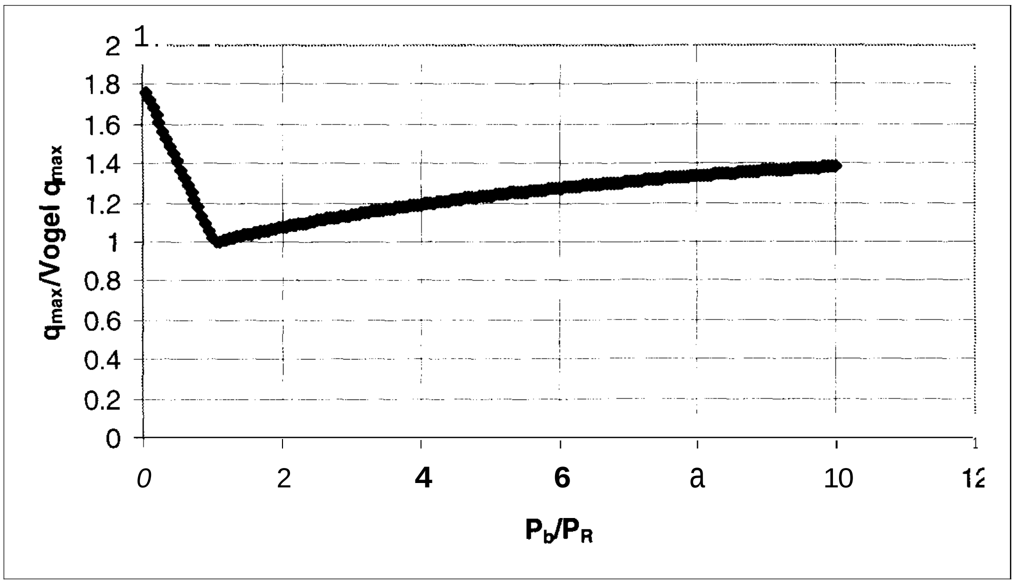

Figure 4 - Maximum Flow Rate as Related to Bubble Point Pressure

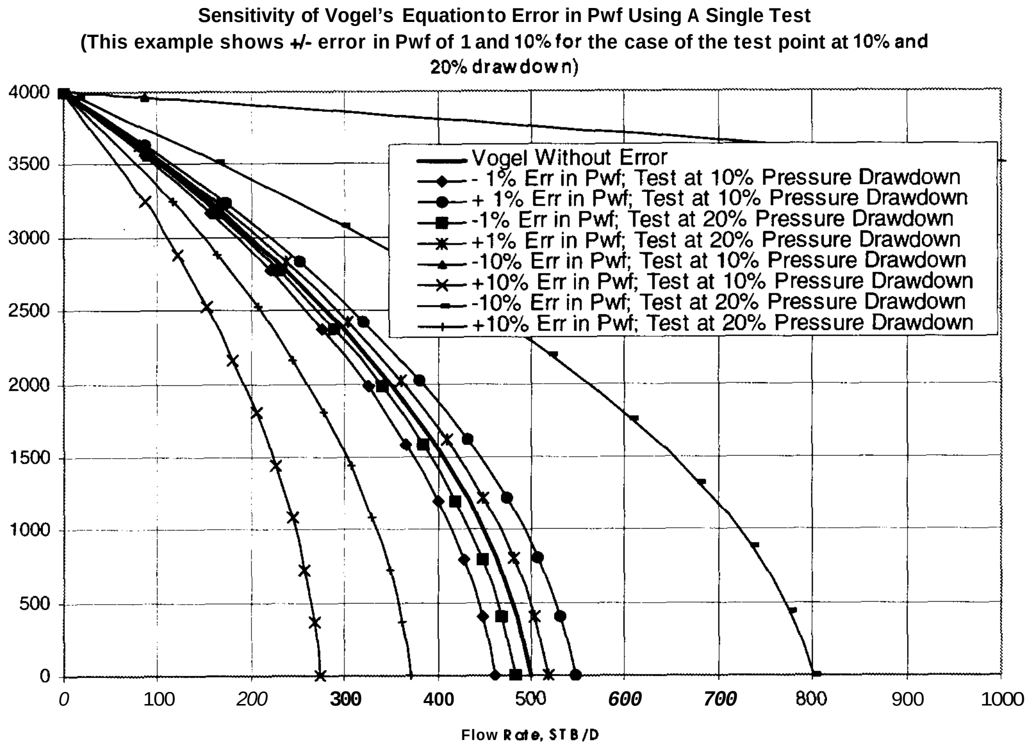

Figure 5 - Sensitivity of Vogel to Pressure