IMPROVED METHOD OF SUCKER ROD STRING DESIGN

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INTRODUCTION

Beam pumping units have been the principal means of artificial lift in the oil field for many years. In recent times, stimulation in secondary recovery projects and encroaching water in water-drive reservoirs have made it necessary to lift more fluid than ever before. There have been several improvements in beam pumping equipment, causing it to remain the best method of artificial lift in most of our wells. A few of these improvements are unique pumping unit geometry, high-slip motors, rolled sucker-rod threads, and sprayed metal couplings.

In several heavily loaded wells, Union Oil Company noted a large percentage of rod failures occurring in the lower section of the tapered rod strings. This triggered an evaluation of the current methods of tapered rod string design to see if improvements could be made in this area.

NOMENCLATURE

- A = Area of rod
- D = Depth to fluid
- F = Impulse factor
- F_D = Impulse Factor (downstroke)
- L = Length of rod string
- L_o = Length of rod section below neutral point
- $L_1,...,L_n$ = Length of each size rod section above neutral point
- L_t = Total length of rods above neutral point
- MRL = Minimum rod load
- N = Strokes per minute
- PRL = Peak rod load
- R = Ratio of maximum to allowable rod stress
- S = Length of stroke
- S_{all} = Allowable stress
- S_{min} = Minimum stress
- S_{max} = Maximum stress
- SF = Service factor
- T = Minimum ultimate tensile strength
- w = Weight of rod string per foot
- w_o = Weight of rod string per foot below neutral point
- w_f = Weight of fluid per foot,
- W_o = Total weight of rods in compression

PRESENT PRACTICES

The current API-accepted method of rod string design entails the use of the Goodman Diagram, shown in Fig. 1, which is based on the formula,

$$S_{all} = (T/4 + 0.5625\ S_{min})SF$$

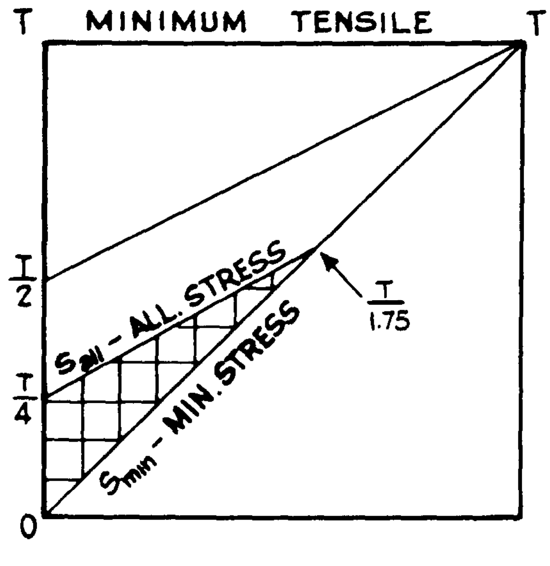


FIG. 1

This formula says the allowable stress on a rod is a function of its load range (MRL/PRL). It was developed by an API committee, based on actual experiences in pumping wells and the best judgment of numerous members on the committee.

The method of designing the rod string is to determine the minimum and maximum polished rod loads and then, utilizing the Goodman Diagram, find a suitable strength rod based on its minimum ultimate tensile strength. While these conditions apply to the top rod only, the tapered string is then designed so that the maximum stress, S_{max} is equal in the top of each size section, when actually the load range is decreasing for each decrease in section size. By applying this existing method, lower rod sections are often overstressed.

Example:

Consider a 3-way taper of 1-in., ⅞-in., and ¾-in. rods. If $S_{max} = 30{,}000$ psi in the top 1-in. rod, then the string is tapered so that $S_{max} = 30{,}000$ psi in the top ⅞ in. and ¾ in. and load range is not considered.

In the new method of rod string design this decrease of load range is accounted for so that the actual stress to allowable stress ratio is the same in the top of each rod size section.

DESIGN METHOD

Theory of Improved Design

Since load range affects the allowable stress in each rod size section, it is desirable that the ratio of S_{max} to S_{all} be the same at the top of each section.

$$R = \frac{S_{max}}{S_{all}} = \frac{PRL}{(PRL)_{all}}$$

If the type rod selected is of adequate strength, it is obvious that $R \leq 1.00$. By setting $R \leq SF$ (assumed service factor), SF can be eliminated from the derivation (if SF = 80%, $R \leq 0.80$). If R, when calculated, is greater than SF, the rods are overloaded and a stronger rod will be needed. By calculating R first, this determination can be made before the entire string is designed.

By breaking down R using the conventional rod design criterion,

$$PRL = Dw_f + Lw\,F, \text{ where } F = 1 + \frac{SN^2}{70500}$$

$$(PRL)_{all} = A_n \frac{T}{4} + 0.5625\,(MRL)$$

$$MRL = \text{Net weight of rods} \times F_D$$

The impulse factor on the downstroke, F_D is $1 - \frac{SN^2}{70500}$ Substituting $\frac{SN^2}{70500} = F - 1$, we find that $F_D = 1 - (F - 1) = 2\text{-}F$.

therefore, $MRL = Lw\,(2\text{-}F)$ and

$$(PRL)_{all} = A_n \frac{T}{4} + 0.5625\,(Lw)\,(2\text{-}F)$$

$$R = \frac{Dw_f + Lw\,F}{A\frac{T}{4} + 0.5625\,(Lw)(2\text{-}F)}$$

For Section No. 1, the bottom section,

$$R_1 = \frac{Dw_f + (L_1 w_1)\,F}{A_1\frac{T}{4} + 0.5625\,L_1 w_1\,(2\text{-}F)}$$

$$R_1 A_1 \frac{T}{4} + R_1\,(0.5625)\,L_1\,w_1\,(2\text{-}F) = Dw_f + L_1\,w_1\,F$$

$$L_1\,(R_1\,(0.5625)\,w_1\,(2\text{-}F) - w_1\,F) = Dw - R_1 A_1 \frac{T}{4}$$

$$L = \frac{Dw_f - R_1 A_1 \frac{T}{4}}{w_1\,(0.5625\,R_1\,(2\text{-}F) - F)}$$

Using these same equations, the equation for length (L_n) of any section can be derived as follows:

$$R = \frac{Dw_f + (L_1\,w_1 + \cdots + L_n\,w_n)\,F}{A_n\frac{T}{4} + 0.5625(L_1\,w_1 + \cdots + L_n\,w_n)(2\text{-}F)} = \frac{L_n\,w_n\,F + (PRL)_{n\text{-}1}}{A_n\frac{T}{4} + 0.5625\,(L_n w_n\,(2\text{-}F) + (MRL)_{n\text{-}1})}$$

$$R_n A_n \frac{T}{4} + 0.5625\ R_n L_n w_n\ (2\text{-}F) +$$

$$0.5625\ R_n\ (MRL)_{n-1} = L_n w_n\ F + (PRL)_{n-1}$$

$$L_n\ w_n\ (0.5625\ R_n\ (2\text{-}F) - F) = (PRL)_{n-1} -$$

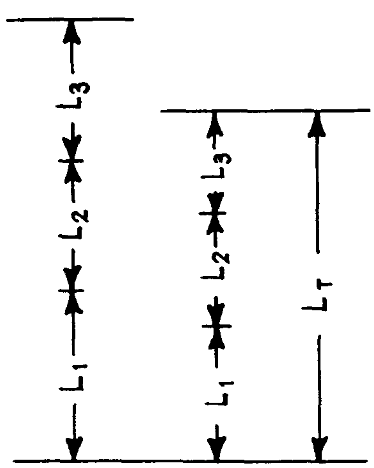
$$R_n\ A_n \frac{T}{4} - 0.5625\ R_n\ (MRL)_{n-1}$$

Since $R = R_1 = R_2 = \cdots = R_n$

$$L_n = \frac{(PRL)_{n-1} - R\left(A_n \frac{T}{4} + 0.5625\ (MRL)_{n-1}\right)}{w_n\ (0.5625\ R\ (2\text{-}F) - F)}$$

If calculated lengths do not equal the design depth, assume a different average weight per foot, w, and re-calculate to converge to correct total length. A higher weight per foot should be used if the sum of L_1, L_2, ..., L_n is less than L_T. Should the total exceed L_T, then use a lower weight per foot. This step does not have to be taken if the designed total length exceeds actual needed and you do not feel that a completely balanced ratio is necessary for the given well.

Example:



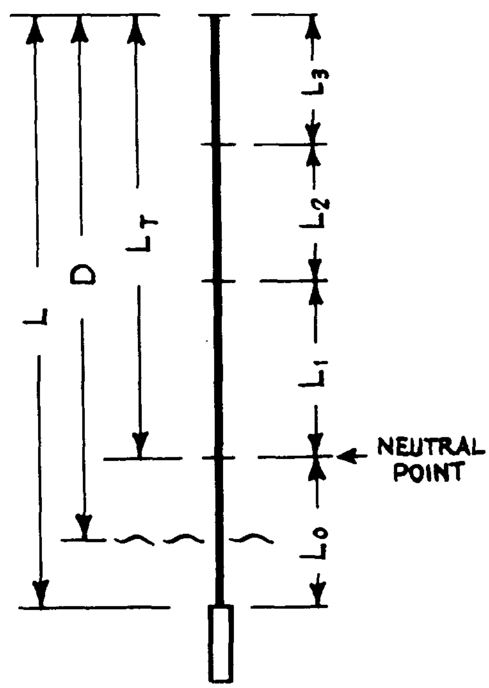
1. An assumed value for w of 2.2 lb/ft was used in the design and the total section lengths calculated is greater than L_t.
2. By reducing w to 2.14 lb/ft, the desired length of rod sections is calculated.

FIG. 2

Compression in Rod Strings

In the API committee's work on modifying the Goodman Diagram, it was concluded that sucker rods should not be placed in compression as their performance in compression is not predictable. However, in actual practice, it is impossible to eliminate compression in the lower portion of a rod string and according to downhole analyses from dynamometer surveys, compression is of high magnitude in some of the faster pumping wells.

In order to account for this lower section of rods in compression the following technique was developed which, when used with the improved sucker-rod design steps, would yield approximately the same rod loadings as indicated by an actual survey.



1. Select rod tensile strength believed to be adequate and an estimated weight per foot of the rod string.
2. Determine the weight of rods in compression and the length, L_o, of rods required to overcome this compressive weight.
3. Starting at the neutral point, proceed with tapered design according to the improved design method.

FIG. 3—OUTLINE OF DESIGN STEPS

Weight of Rods in Compression: In order to determine the minimum rod load at different points in the rod string, the amount of compression on the downstroke of each cycle must be known.

The actual weight in compression is the difference between peak rod load (less fluid load) and the minimum rod load.

$$PRL = Lw\,F + w_f\,D$$

$$\text{where } F = 1 + \frac{SN^2}{70500}$$

$$MRL = Lw\,F_D$$

The fluid load is transferred to the standing valve at the start of the downstroke, therefore, it is *not* a function of compression weight.

The impulse factor on the downstroke, $F_D = 1 - \frac{SN^2}{70500} = 1 - (F - 1) = 2\text{-}F$. Since $MRL = Lw\,F_D$ then $MRL = Lw\,(2\text{-}F)$.

therefore,

$$\begin{aligned} w_o &= PRL - Dw_f - MRL \\ &= (Lw\,F + Dw_f) - Dw_f - Lw\,(2\text{-}F) \\ &= Lw\,F - 2\,L\,w + Lw\,F \\ &= 2\,L\,w\,(F - 1) \end{aligned}$$

Since the average rod weight is not known, a realistic value for w must be assumed.

Length of Rods to Overcome Compressive Weight: The length of rods required to overcome the compressive load is,

$$L_o = \frac{W_o}{w_o}$$

The peak stress of these lower rods must be equal to, or less than their allowable stress when utilizing a zero load ratio, i.e., MRL = 0.

$$S_{max} = \frac{L_o\,w_o\,F + D\,w_f}{A_o}$$

$$S_{all} = \left(\frac{T}{4} + 0.5625\,S_{min}\right) SF$$

Since $S_{min} = 0$ at the neutral point, $S_{all} = \frac{T}{4}(SF)$

If $S_{max} \leq S_{all}$, the rod size is of sufficient strength and you should proceed with the design above the neutral point. However, if $S_{max} > S_{all}$ then let $S_{max} = S_{all}$ and solve for L_{o1}.

$$S_{max} = S_{all}$$

$$\frac{L_{o1}\,W_{o1}\,F + D\,w_f}{A_{o1}} = \frac{T}{4}(SF)$$

$$L_{o1} = \frac{A_{o1}}{w_{o1}\,F}\left(\frac{T}{4}(SF) - \frac{Dw_f}{A_{o1}}\right)$$

This will give an allowable length for L_{o1}. The neutral point will then have to be located in the next larger rod size section.

$$L_{o2} = \frac{W_o - L_{o1}\,w_{o1}}{w_{o2}}$$

Compressive Weight Incorporated into Improved Design: The section of rods in compression on the downstroke does not add anything to the minimum rod load (MRL) and, therefore, alters improved design derivation slightly.

From Theory Section,

$$MRL = (Lw)\,(2\text{-}F)$$

Since the rods below the neutral point are not included,

$$MRL = (Lw - W_o)\,(2\text{-}F)$$

therefore,

$$R = \frac{Dw_f + L\,wF}{A_n\frac{T}{4} + 0.5625\,(Lw\text{-}W_o)(2\text{-}F)}$$

$$\text{and } L_1 = \frac{D\,w_f + L_o\,w_o\,F - R_1\,A_1\frac{T}{4}}{w_1\,(0.5625\,R_1\,(2\text{-}F) - F)}$$

$$\text{or, } L_1 = \frac{(PRL)_o - R_1 A_1 \frac{T}{4}}{w_1 \; (0.5625 \; R_1(2\text{-}F) - F)}$$

The final derivation for any length section (L_n) will be unchanged since $(MRL)_{n-1}$ is a variable in the equations for both R_n and L_n.

COMPARISON OF METHODS

To illustrate the change in tapered design by utilizing the new method, Table 1 compares designs with the API accepted method in two example wells. Also shown is a comparison of load ratios.

TABLE 1—COMPARISON OF IMPROVED DESIGN TO API DESIGN

Example 1
1½" Pump@5500'
120" Str. - 11 SPM
T = 100,000 psi, SF = 80 Per Cent

Rod Size	LR	New Design	LR	API Design
1"	30%	1500'	30%	1275'
⅞"	16%	1550'	18%	1425'
¾"	<0%	2450'	<0%	2800'

Example 2
1¾" Pump@6400'
168" Str. - 8 SPM
T = 120,000 psi, SF = 80 Per Cent

Rod Size	LR	New Design	LR	API Design
1"	37%	1950'	36%	1650'
⅞"	22%	2150'	24%	1850'
¾"	<0%	2300'	<0%	2900'

LR = Load Ratio = MRL/PRL

Conventional formulae for sucker rod pumping design have been used in the development of the improved method. This was used in lieu of the new API formulae for simplification in both developing the new method and in hand calculations. Comparisons of calculated minimum and maximum stresses using the conventional method with the technique for calculating compressive weight versus the API method, which accounts for rod harmonics, are shown in Table 2. Also shown are stresses surveyed by Nabla Corp. and Petro Systems Technology, Inc. These surveys entail the use of a dynamometer for surface recordings and a computer for diagnosis of downhole stresses. The example wells in the table are offered as comparison of the calculation methods and are not tapered according to any particular design.

TABLE 2—COMPARISON OF MINIMUM AND MAXIMUM STRESSES (PSI)

A. Conventional calculation method with technique for calculating compressive weight.
B. Dynamometer survey and computer analysis.
C. API method.

		Example 1 1½" Pump @7950' 192" Str. - 8 SPM			Example 2 1¾" Pump@7450' 168" Str. - 9 SPM		
		A	B	C	A	B	C
1"	S_{max}	32200	36180	26870	35300	38400	25200
	S_{min}	12210	12820	11110	10910	10590	9210
⅞" Rods	S_{max}	28500	32160		28600	33570	
	S_{min}	6510	8970		2470	5000	
¾" Rods	S_{max}	22600	24020		24550	30310	
	S_{min}	-2380	1860		-6400	-1170	

		Example 3 2¼" Pump@6000' 240" Str. - 9 SPM			Example 4 2" Pump@3450' 82" Str. - 15 SPM		
		A	B	C	A	B	C
1" Rods	S_{max}	31400	39440	28990	16530	20550	14580
	S_{min}	5500	3310	5830	3580	2210	3660
⅞" Rods	S_{max}	27700	35920		16300	20270	
	S_{min}	-40	-1200		850	390	
¾" Rods	S_{max}	26900	34880		14300	17860	
	S_{min}	-5950	-3840		-2580	-1500	

DISCUSSION

One main item that is not taken into consideration in this design method, or any other method, is downhole fluid-rod and rod-tubing friction. Since these values are dependent on well conditions and are not easily attainable, an allowance has been incorporated into the work. This was handled by utilizing the weight of rods in air thereby neglecting all bouyancy effects. This tends to counteract the friction load effects.

Due to the converging steps involved, the new method has applicability to a computer solution and additional work is planned to enable a program to be written.

IMPROVED SUCKER ROD STRING DESIGN

Example:

Pump depth	5500 ft
Fluid level	5200 ft
Pump size	1½ in.
Unit speed	11 SPM
Stroke length	120 in.

¾ in., ⅞ in., 1 in. taper
100,000 psi minimum tensile rod
SF required - 80%

Compressive Weight

Assume w = 2.2 lb/ft

$$W_o = 2Lw\ (F\text{-}1) = 2\,(5500)(2.2)(0.206) = 4980 \text{ lb}$$

$$R = \frac{Dw_f + LwF}{A_n \frac{T}{4} + 0.5625\ (Lw\text{-}W_o)\ (2\text{-}F)}$$

$$= \frac{5200\ (0.765) + 5500\ (2.2)\ (1.206)}{0.785\,(25{,}000) + 0.5625\,(5500(2.2) - 4980)\,0.794}$$

$$= \underline{0.815} \text{ (slightly over 80\%)}$$

$$L_{o(¾")} = \frac{4980 \text{ lb}}{1.64 \text{ lb/ft}} = 3040 \text{ ft}$$

$$S_{max} = \frac{Dw_f + L_o\ w_o\ F}{A_{¾"}}$$

$$= \frac{3980 + 6010}{0.442} = 22{,}600 \text{ psi}$$

$$S_{all} = \frac{T}{4}\ (SF) = 25{,}000\ (0.80)$$

$$= 20{,}000 \text{ psi} < S_{max}$$

$$\therefore L_{o(¾")} = \frac{A_{¾}\ S_{all} - Dw_f}{w_{¾} F} = \frac{0.442\ (20{,}000) - 3980}{1.64\ (1.206)}$$

$$= 2460 \text{ ft} \simeq 2450 \text{ ft}$$

$$L_{o(⅞")} = \frac{W_o - (L_o\, w_o)_{¾"}}{w_{⅞}} = \frac{4980 - 4020}{2.2} = 436 \text{ ft}$$

$$(PRL)_o = Dw_f + L_o\ w_o\ F = 3980 + 4980\ (1.206)$$

$$= 9990 \text{ lb}$$

$$L_{⅞} = \frac{(PRL)_o - R\,A_{⅞}\,\frac{T}{4}}{w_{⅞}(0.5625\,R\,(2\text{-}F) - F)}$$

$$= \frac{9990 - 0.815\ (0.601)\ (25{,}000)}{2.2\ (0.5625\ (0.815)\ (0.794) - 1.206)}$$

$$= 1220 \text{ ft}$$

Total ⅞ in. = 436 ft + 1220 ft = 1656 ft ≃ 1650 ft

$$(PRL)_{⅞} = 9990 + 1214\ (2.2)\ (1.206) = 13{,}210 \text{ lb}$$

$$(MRL)_{⅞} = 1214\ (2.2)\ (0.794) = 2120 \text{ lb}$$

$$L_{1\text{-in.}} = \frac{(PRL)_{n\text{-}1} - R(A_1\ \frac{T}{4} + 0.5625\ (MRL)_{n\text{-}1})}{w_1\ (0.5625\ R\ (2\text{-}F) - F)} =$$

$$\frac{13{,}210 - 0.815\,(0.785(25{,}000) + 5625\ \ (2120)}{2.88\ (0.5625\ (0.815)\ (0.794) - 1.206)}$$

$$= 1550 \text{ ft}$$

$$\text{Total Length} = 2450 \text{ ft} + 1650 \text{ ft} + 1550 \text{ ft}$$

$$= 5650 \text{ ft} > 5500 \text{ ft}$$

$$w = \frac{2450\,(1.64) + 1650\,(2.2) + 1550\ \ (2.88)}{5650}$$

$$= 2.14 \text{ lb/ft} < 2.2$$

use:

$$w = 2.14 \text{ lb/ft:}$$

$$W_o = 2\ (0.206)\ (2.14)\ (5500) = 4850 \text{ lb}$$

$$R = \frac{3980 + 5500\ (2.14)\ (1.206)}{0.785(25{,}000) + 0.5625(5500\,(2.14) - 4850)0.794}$$

$$= 0.80 \leq \text{SF, (O.K.)}$$

$$L_{o(3/4)} = \frac{0.442\ (20{,}000)\ -\ 3980}{1.64\ (1.206)} = \underline{\underline{2450}}$$

$$L_{o(7/8)} = \frac{4850 - 4020}{2.2} = 377 \text{ ft}$$

$$(PRL)_o = 3980 + 4850\ (1.206) = 9830 \text{ lb}$$

$$L_{7/8} = \frac{9830 - 0.80\ (0.601)\ (25{,}000)}{2.2\ (0.5625\ (0.80)\ (0.794) - 1.206)}$$

$$= 1173 \text{ ft}$$

$$\text{Total } 7/8\text{-in.} = 377 + 1173 = \underline{\underline{1550 \text{ ft}}}$$

$$(PRL)_{7/8} = 9830 + 1173\ (2.2)\ (1.206) = 12{,}940 \text{ lb}$$

$$(MRL)_{7/8} = 1173\ (2.2)\ (0.794) = 2050 \text{ lb}$$

$$L_{1''} = \frac{12{,}940 - 0.80\ (0.785)\ (25{,}000) + (0.5625)\ (2050)}{2.88\ (0.5625\ (0.80)\ (0.794) - 1.206)}$$

$$= 1500 \text{ ft}$$

ROD DESIGN:

1"	1500'
⅞"	1550'
¾"	2450'
TOTAL	5500'

BIBLIOGRAPHY

1. Gibbs, S.G. and Nolen, K.B.: Well Site Diagnosis of Pumping Problems Using Mini-Computers, SPE of AIME Paper No. 4068, San Antonio, Texas, Oct. 8, 1972.
2. API Recommended Practice for Care and Handling of Sucker Rods, API RP-11 BR, Mar. 1969, p. 8.