Hydraulic Fracture Treatment Design

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INTRODUCTION

In 1948, Stanolind Oil and Gas Company (now Pan American Petroleum Corporation) introduced their "Hydrafrac" process for increasing well productivity by hydraulically fracturing the formation.[1] In 1949, the first commercial fracturing treatment was performed, thus initiating one of the most outstanding well stimulation procedures that the petroleum industry has ever known.[2] Since 1949, more than 400,000 fracturing treatments have been performed in the free world as well as an untold number behind the Iron Curtain.[2]

During the past 16 years, many advancements have been made in the concepts of hydraulic fracturing theory. The purpose of this paper is not to clarify the concepts of hydraulic fracturing theory, but to present a sound design method of effectively employing the concepts. Discussion of theory will be confined to only that necessary to justify the method of design. The design procedure presented in this paper is limited to vertical fractures and presents a method of optimizing fracture treatment sizes.

FRACTURE ORIENTATION

Since the initial development of the hydraulic fracturing process, one of the most controversial issues in hydraulic fracturing theory has been the orientation of induced fractures. Fracture orientation is very important since it dictates the procedure to be employed in designing fracture treatments.

Hubbert and Willis[3] concluded the following:

(1) If fluid pressure is applied locally within rocks and increased until fracturing of the rocks occurs, the plane along which fracturing will first occur is perpendicular to the least principal stress (Fig. 1).

(2) Horizontal fractures cannot be produced by hydraulic pressures less than the total pressure of the overburden.

(3) In sedimentary rocks, a close approximation of overburden is equal to 1.0 psi/ft of depth.

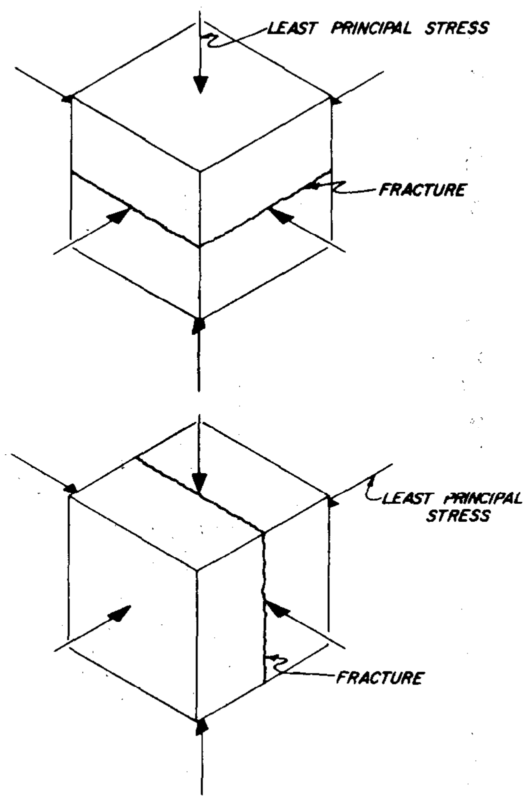


FIG. 1
STRESS ELEMENT AND PREFERRED PLANE OF FRACTURE

It is generally accepted throughout the West Texas area that the maximum depth to which horizontal fractures can be induced is approximately 4000 ft. As indicated in Fig. 2,[4] field data tend to confirm this limit if it is assumed that the values representing maximum treating gradients for depths less than 2000 ft are the result of horizontal fractures. As a general rule, the fracture plane is assumed to be horizontal when the fracture treating gradient is 1.0 psi/ft of depth or greater, and vertical when the fracture treating gradient is 0.7 psi/ft of depth or less.

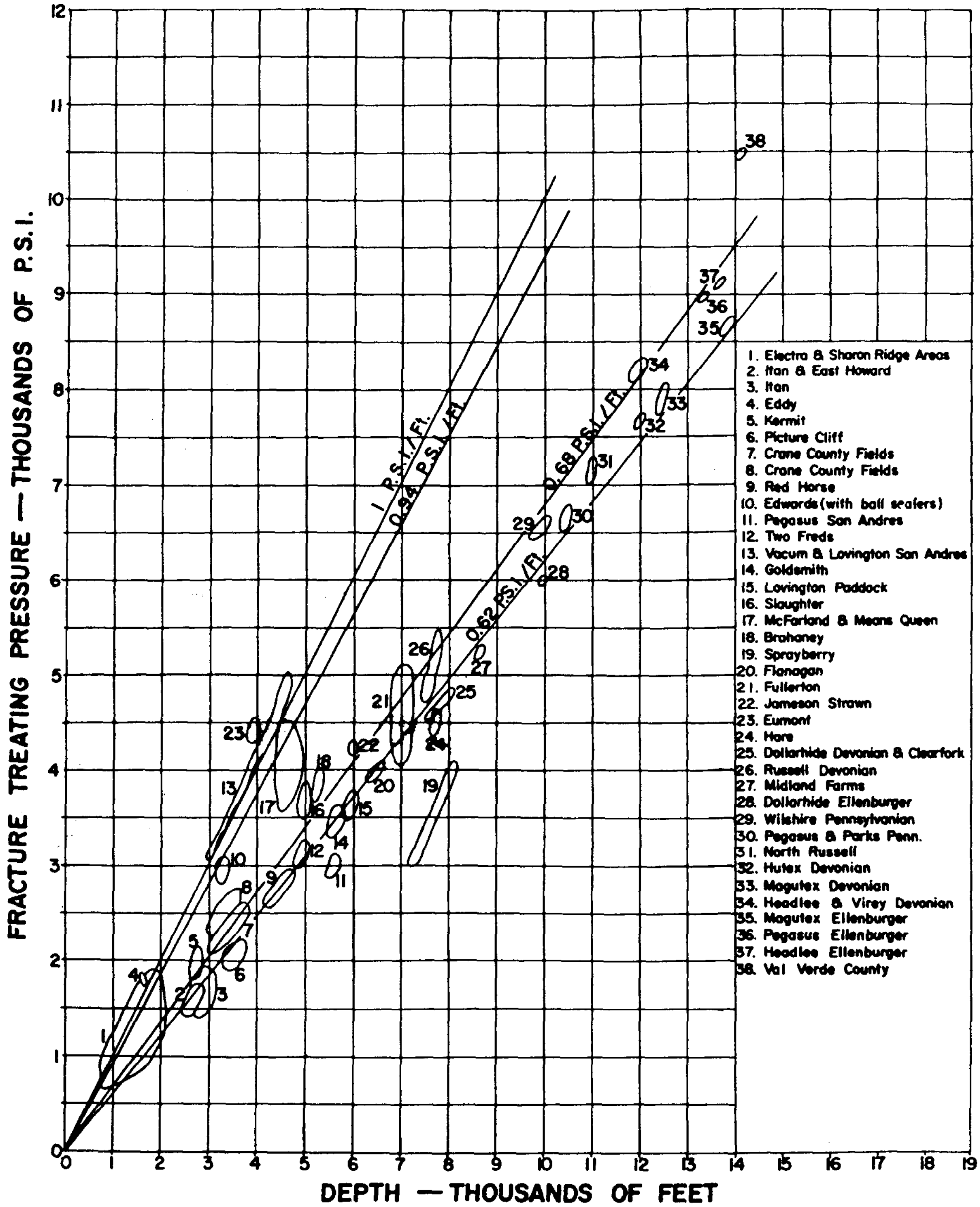


FIG. 2
FRACTURE TREATING PRESSURES FOR WEST TEXAS FIELDS

Crittendon[5] presented a formulation for equating fracture treating pressure with fracture orientation.

$$P_t = \frac{P_{ob}}{2}\left[\left(1+\frac{2\mu}{1-\mu}\right) + \left(1-\frac{2\mu}{1-\mu}\right)\cos 2\phi\right] \qquad [1]$$

Where: P_t = fracture treating pressure, psi.
P_{ob} = overburden pressure, psi.
μ = Poisson's ratio, dimensionless
ϕ = angle of fracture from horizontal degrees

Poisson's ratio is a measure of a material's elasticity and can be defined as the ratio of the strain perpendicular to an applied force to the strain parallel to the applied force. Poisson's ratios for rocks reported in the literature[6,7,8,9] range from approximately 0.05 to 0.45, with an average around 0.25. In general, the harder the rock, the lower the value of Poisson's ratio.

Fig. 3[4] illustrates the three types of fracture orientation that Eq. 1 indicates may be obtained. Fig. 4[5] presents a graphic solution to Eq. 1 in terms of fracture treating gradient, Poisson's ratio, and fracture orientation. If a Poisson's ratio of 0.25 is employed in Fig. 4, the fracture treating gradient required to obtain horizontal fractures is 0.94 psi/ft and the fracture treating gradient required to obtain vertical fractures is 0.62 psi/ft. These fracture treating gradients are in close agreement with the previously stated values of 1.0 psi/ft or greater for horizontal fractures and 0.7 psi/ft or less for vertical fractures.

Since the majority of the producing formations found in the West Texas area are below 4000 ft, the design method presented in this paper is restricted to vertical fractures. Horizontal fracture treatment design requires another approach, different data, and different calculations.

FRACTURING FLUIDS

In a hydraulic fracture treatment, the purpose of the fracturing fluid is to induce and extend the fracture and to transport and deposit the propping agent in the fracture. A large number of different types of fracturing fluids have evolved since the conception of the hydraulic fracturing process. Each type of fracturing fluid has different physical and chemical properties and, in many instances, the magnitude of production increase resulting from a fracturing treatment depends to a great degree on the choice of fracturing fluid.

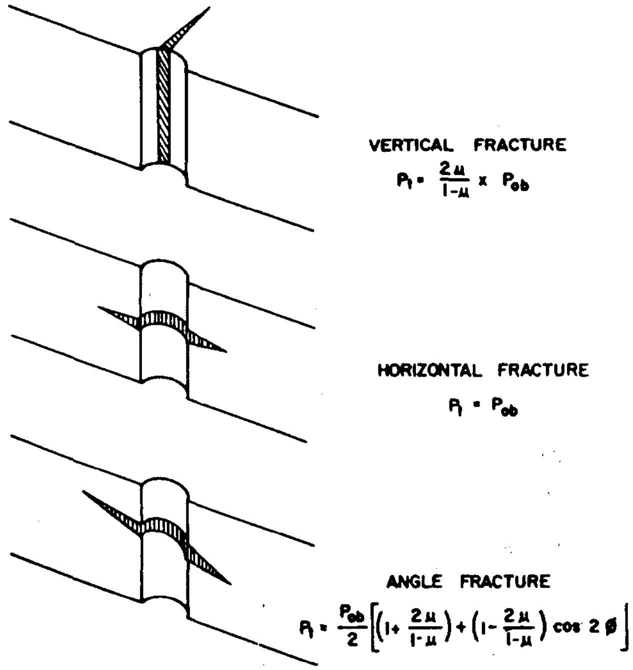


FIG. 3

TYPES OF FRACTURE ORIENTATION

Fracturing fluids may be classified into two groups:
(1) Newtonian
(2) Non-Newtonian

Newtonian fluids are defined as those fluids whose viscosities are constant at all rates of shear. Crude oil, refined oil, fresh water, salt water, oil field brine, and acid which contain no additives are Newtonian or true fluids.

Non-Newtonian fluids are defined as those fluids whose viscosities are not constant at all rates of shear. In general, a Newtonian type fluid is converted to a non-Newtonian type fluid by the addition of additives such as gelling agents, friction reducers, emulsifiers, and some fluid loss agents. Hydrocarbon gels, water base gels, and emulsions are non-Newtonian fluids.

Crude oil native to the formation in which it is being used as a fracturing fluid will not have a tendency to form an emulsion; however, crude oil foreign to the formation in which it is being used as a fracturing fluid may cause emulsion problems. Most crude oils have a high fluid loss, low viscosity, and poor sand suspending properties; however, with the proper fluid loss additive and injection rate, crude oil can be satisfactorily used as a fracturing fluid.

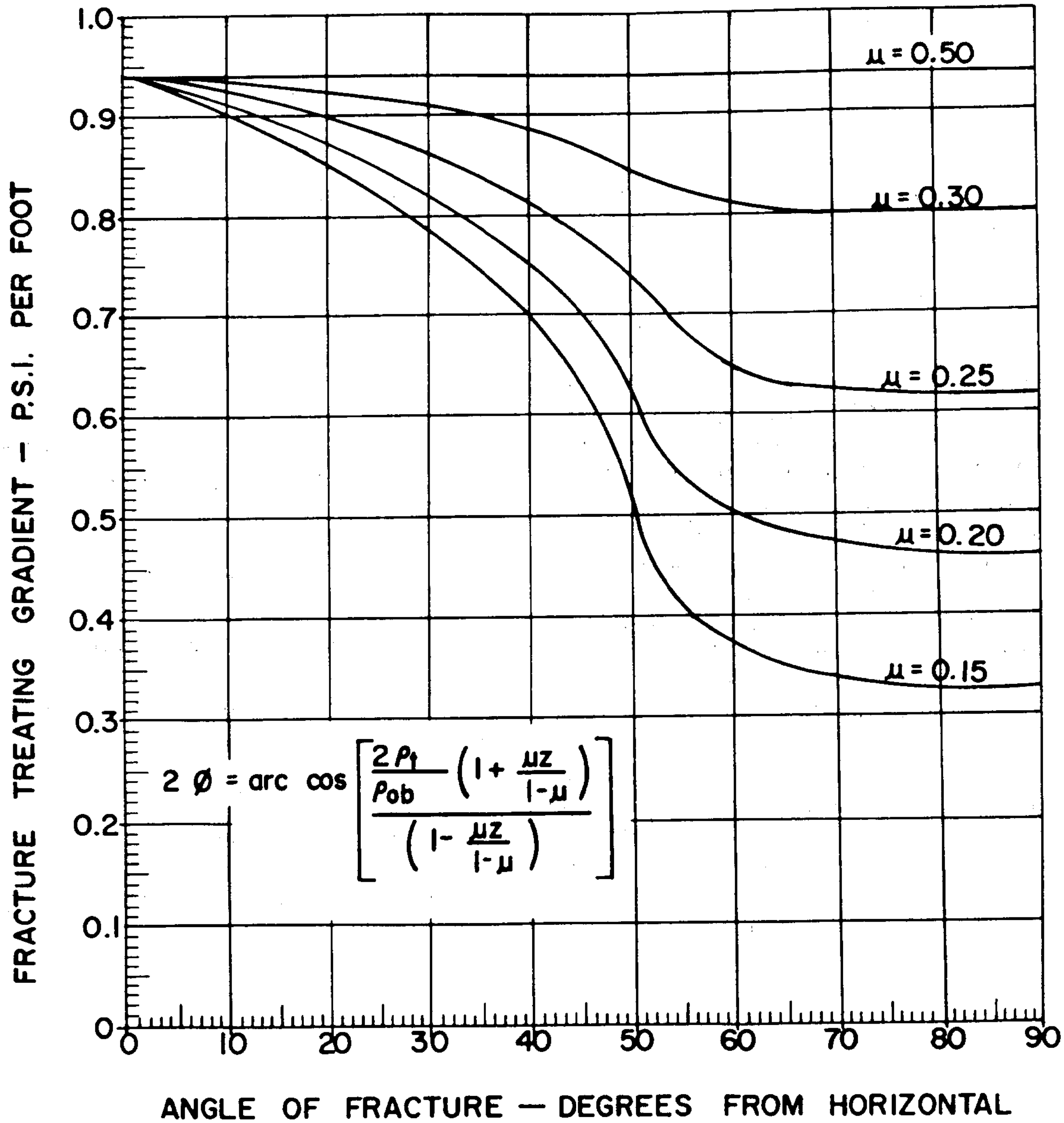


FIG. 4

CONFINING PRESSURE OF FRACTURES AT VARIOUS ANGLES

The refined oils that are currently being used as fracturing fluids are heavy fuel oils and have an API gravity ranging from 18° to 22°. Refined oils have a relatively low fluid loss as compared to lease oil, high viscosity, and good sand suspending properties, however, formation damage may result from the use of refined oil as a fracturing fluid since oils of different base composition will frequently precipitate asphaltic particles when mixed together. As a result of the removal of fine solid particles by the refining process, most refined oils have less tendency to form emulsions than does lease oil. When selecting a refined oil[10] as a fracturing fluid, care should be exercised to choose one having a pour point at least 20° below the formation temperature of the well to be fractured in order to avoid precipitation of wax crystals from the refined oil, a phenomenon taking place at 12° to 15° above the pour point of the refined oil.[10]

Promiscuous use of water as a fracturing fluid can completely block a productive reservoir. Analysis of the reservoir rock will usually indicate if water can be used. When clays, such as montmorillonite, are present in appreciable amounts in a reservoir, water normally cannot be used. If the water is treated, small amounts of this type of clay can be tolerated. Treatment of the water usually consists of the addition of small amounts of hydrochloric acid or calcium chloride. Treatment of the water is usually considered necessary only when it is less saline than the reservoir water.

Acid is possibly the most expensive fracturing fluid available. In practically all cases, the facturing cost in dollars per 1000 ft.^{2} of fracture area is higher with acid than other fracturing fluids. The main application of acid as a fracturing fluid is to increase fracture width by etching where propping agents can be successfully deposited in the fracture. If propping agents can be successfully placed in a fracture, the use of acid as a fracturing fluid appears superfluous.

When fracturing an oil productive reservoir, surface active agents should be used in conjunction with most water base fracturing fluids. The proper surfactant should be selected since it will serve as a non-emulsifier between the water base fracturing fluid and the reservoir crude oil. Surfactants are normally used in concentrations of from one to three gals. per 1000 gals. of fracturing fluid to be treated.

Water base fracturing fluids containing no additives have a very high fluid loss, very low viscosity, and very poor sand suspending properties; therefore, high injection rates are necessary for fracture extension and propping agent transportability. In cases where high injection rates are unobtainable, gelling the water base fracturing fluid will reduce the fluid loss, increase the viscosity, increase propping agent transportability and decrease friction loss.

Gels are composed of oil, water, or acid with the proper chemical or chemicals added to the base fluid to increase viscosity. Most hydrocarbons gels are produced by adding a thickening agent, usually a fatty acid salt or a metallic soap, to the hydrocarbon. Most water base gels are also produced by adding a thickening agent, usually a natural gum or a synthetic polymer, to the base fluid. Sand suspending characteristic of a gel is a function of the viscosity and density of the gel. Care should be exercised when selecting a gelling agent since most gelling agents produce some formation damage and several cause extensive damage. Several investigators[11, 12] have indicated that a high degree of formation damage can be tolerated; however, it certainly would be preferable to use one of the least damaging gelling agents, if possible.

Emulsions are comprised of a homogeneous mixture of two immiscible fluid phases, one of which is oil and the other is water or acid. Emulsions are formed by mixing the two fluid phases together in the proper proportions in the presence of an emulsifying agent to produce either water-in-oil or oil-in-water type emulsions. Most emulsions possess very good sand suspending properties; however, they usually have poorer fluid loss properties than gels.

Following a fracturing operation, gels and emulsions should break or lose sufficient viscosity for return to the well bore. The viscosity of gels and emulsions may be reduced in several different ways, depending upon the type of gelling or emulsifying agent employed. Gels may be broken by special additives referred to as gel breakers, electrolytes such as acid or salt water, bacterial growth within the gel, and dilution with formation fluid. Emulsions may be broken by special additives referred to as de-emulsifiers, electrolytes such as acid or salt water, absorption of the emulsifying agent into the formation, and dilution of the outer phase of the emulsion.

Because of the large number of fracturing fluids available, it is often quite difficult to select the best fracturing fluid for a particular fracturing treatment. The fracturing fluid having the most desirable and least damaging properties at the lowest cost is the logical one to use.

PROPPING AGENTS

The purpose of a propping agent in a hydraulic fracture treatment is to hold the fracture open and provide a permeable path for fluid flow into the well bore. The propping agent initially utilized in fracturing treatments was ordinary construction sand which has been screened to remove the larger particles and impurities.[2] Today, many types of propping agents are available; however, a round grained, carefully screened, high quality, clean, silica sand is usually considered superior to other propping agents. Since sand is currently the most popular propping agent, many sizes are available; however, 20-40 mesh is the most popular and consumed in the greatest quantity. Some of the most common fracturing sand mesh designations and grain sizes are given in Table I.

TABLE I

Mesh and Size of Fracturing Sand

Mesh Designation	Grain Size (In.)
4-8	0.1870 - 0.0937
8-12	0.0937 - 0.0661
10-20	0.0787 - 0.0331
20-40	0.0331 - 0.0165
40-60	0.0165 - 0.0098

In addition to sand, numerous other propping agents have gained recognition. Each type of propping agent has different physical properties which dictate such things as settling characteristics and ability to withstand deformation and crushing. As a means of comparison, the currently available propping agents together with some of their physical properties are listed in Table II.

TABLE II

Types and Properties of Propping Agents

Type	Specific Gravity	True Density Lbs/Gal	True Volume Gal/Lbs
Aluminum Pellets	2.71	22.59	0.044
Glass Beads	2.65	22.09	0.045
Iron Shot	7.14	59.52	0.017
Plastic Beads	1.05	8.75	0.114
Rounded Walnut Shells	1.28	10.67	0.094
Sand	2.65	22.09	0.045

The choice of a propping agent depends upon such factors as particle roundness, variation in particle size, deformation and crushing properties, cleanness, and cost.

Two approaches for propping fractures have gained recognition and are considered today — one is to pack the fracture, and the other is to place a monolayer or a partial monolayer in the fracture.

The concept of increased flow capacities with sparce propped fractures has been verified by laboratory measurements and is generally accepted throughout the oil industry; however, the actual placement of a propping agent as a monolayer or partial monolayer in a vertical fracture is very difficult since most propping agents have a much higher specific gravity than the carrying agents which are normally used. A packed fracture will usually have a lower flow capacity than a sparce propped fracture; however, the flow capacity of a packed fracture is sufficiently high to conduct all the fluid into a well bore that most of the wells in the West Texas area are capable of producing. Fig. 5 is a graph illustrating the flow capacity of various sand packs for different compaction pressures.

Packed fracture treatments are the most popular and most widely used today; therefore, only the packed fracture is considered in the design method presented in this paper.

FRACTURE AREA

The productivity of a fractured formation is determined to a large degree by the area of the fracture, and fracture area, in turn, is governed by treatment variables.

Howard and Fast presented the following equation, which was developed by R. D. Carter for determining the area of an induced fracture in terms of treating conditions.

$$A = \frac{QW}{4\pi C^2}\left[e^{x^2} \cdot \operatorname{erfc}(x) + \frac{2x}{\sqrt{\pi}} - 1\right] \qquad [2]$$

Where:
- A = total area of one face of the fracture at any time during injection, ft^2
- Q = constant injection rate during extension, bbl/min
- W = constant fracture clearance, ft
- C = a constant which is a measure of the flow resistance of the fluid leaking off into the formation during fracturing
- $x = \dfrac{2C\sqrt{\pi t}}{W}$
- t = total pumping time, min

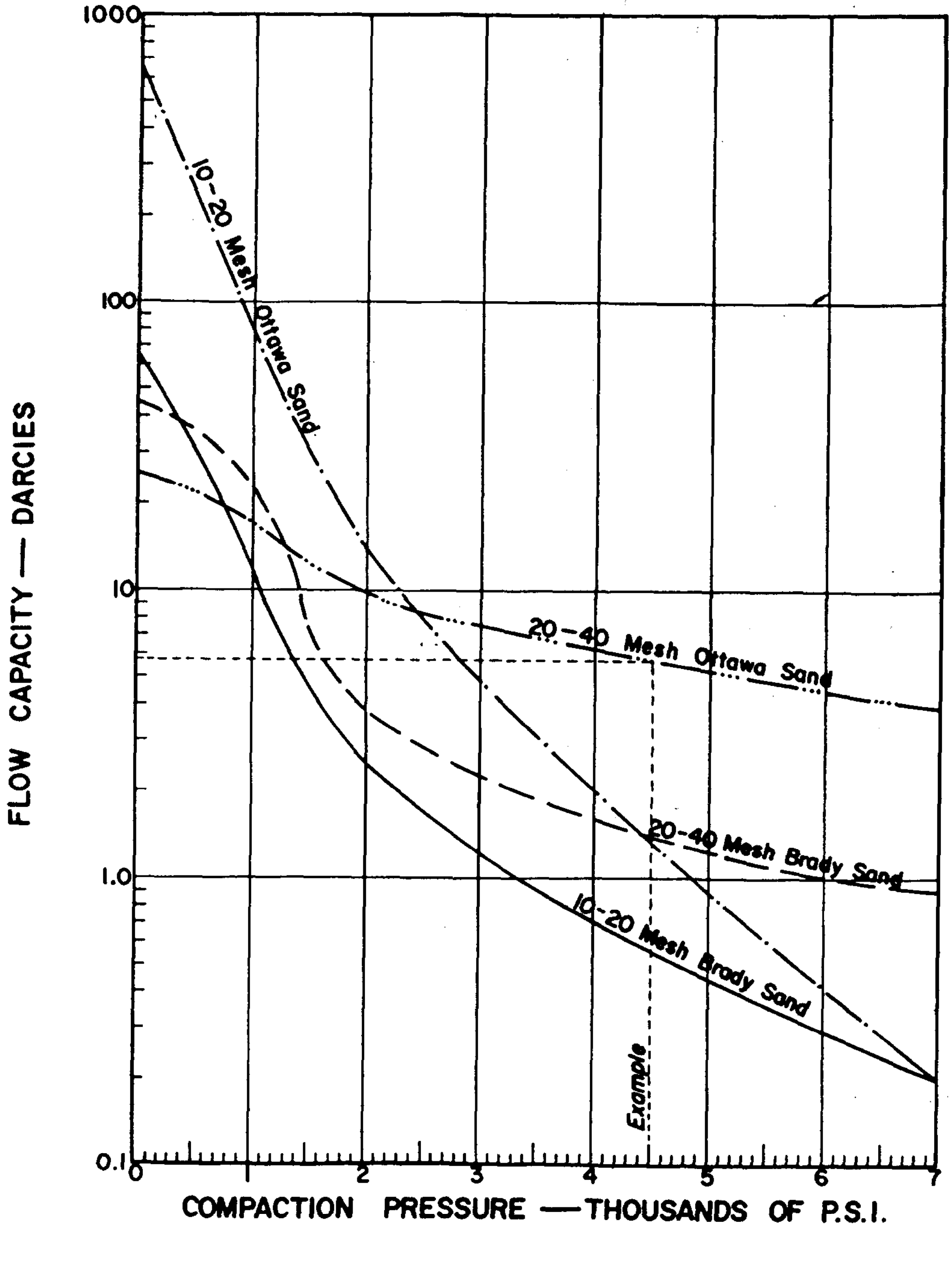


FIG. 5

VARIATION OF FLOW CAPACITIES OF SAND PACKS UNDER VARIOUS COMPACTION PRESSURES

Nomographs 1 and 2[14] provide a means of solving Eq 2 in terms of treatment volume. The prolem now resolves to one of determining the values of the treatment variables to be used in the nomographs.

Treatment Volume

The treatment volume to be used in Nomographs 1 and 2 is a combined volume of the fracturing fluid volume and propping agent quantity expressed in gallons. Fracturing fluid volumes are usually expressed in gallons; however, propping agent quantities are practically always expressed in pounds. The values of the true volume in Table II can be used to convert pounds to gallons for the commercially available propping agents.

Treating Fracture Width

Treating fracture width is a function of rock elasticity, injection rate, fracturing fluid properties, and fracture size. Treating fracture widths for restricted vertical fractures can be obtained from the following formulations:

Newtonian fluids in laminar flow in the fracture

$$W_t = 0.25\left[\frac{(Q)(u)(r_f)}{E}\right]^{1/4} \qquad [3]$$

Newtonian fluids in turbulent flow in the fracture

$$W_t = 0.4\left[\frac{(Q^2)(Sp.Gr.)(r_f)}{(E)(H_f)}\right]^{1/4} \qquad [4]$$

Conditions for laminar and turbulent flow

$$\frac{(Q)(Sp.Gr.)}{(H_f)(u)} < 0.32\text{, laminar, and}$$

$$\frac{(Q)(Sp.Gr.)}{(H_f)(u)} > 0.32\text{, turbulent}$$

Non-Newtonian fluids in laminar flow in the fracture

$$W_t = 8\left[\left(\frac{2^7}{3\pi}\right)(n'+1)\left(\frac{2n'+1}{n'}\right)^{n'}\left(\frac{0.9775}{144}\right)\left(\frac{5.61}{60}\right)^{n'}\right]\cdot\left[\frac{(Q^{n'})(K')(r_f)(H_f^{1-n'})}{E}\right]^{\frac{1}{2n'+2}} \qquad [5]$$

Where: W_t = treating fracture width, in.

Q = injection rate, bbls/min

u = viscosity, cps

r_f = fracture radius, ft

E = Young's modulus of elasticity, ps

$Sp.Gr.$ - specific gravity, dimensionless

H_f = fracture height, ft

n' = flow behavior index, dimensionless

K' = consistency index, lb-sec$^{n'}$/ft^2

The above equations are essentially the sam as those presented by Perkins and Kern[15] ex cept these expressions give the average width o an elliptical shaped fracture, whereas the formu lations presented by Perkins and Kern give th maximum width.

Fig. 6, 7 and 8 are graphic solutions of Eqs. 3 4 and 5, respectively. In order to solve Eqs. 3, and 5 and to use Fig. 8, values of Young's modul for various types of rocks must be known. Sinc a 10-to-20 fold variation in Young's moduli ma exist in rocks of the same type, the use of averag values of Young's moduli appears justified. Tabl III[15] presents values of Young's moduli that ca be used.

TABLE III

Values of Young's Moduli of Formation Rocks

Type of Rock	Range of Young's Moduli psi x 10^6	Averag Value o Young' Moduli psi x 10
Limestone and Dolomite	8.0 to 13.0	10.5
Hard, Dense Sandstone	5.0 to 7.5	6.2
Medium-Hardness Sandstone	2.0 to 4.0	3.0
Porous, Unconsolidated To Lightly Consolidated Sandstone	0.5 to 1.5	1.0

Injection Rate

The magnitude of injection rate is limited only by casing and well head strengths; however treatment variables will dictate the injection rate that is required for a particular treatment. An in jection rate should be selected that will result in a treating fracture width which is slightly larger than the healed fracture width or slightly larger than twice the maximum propping agent particle dimension, depending on which of the two is the

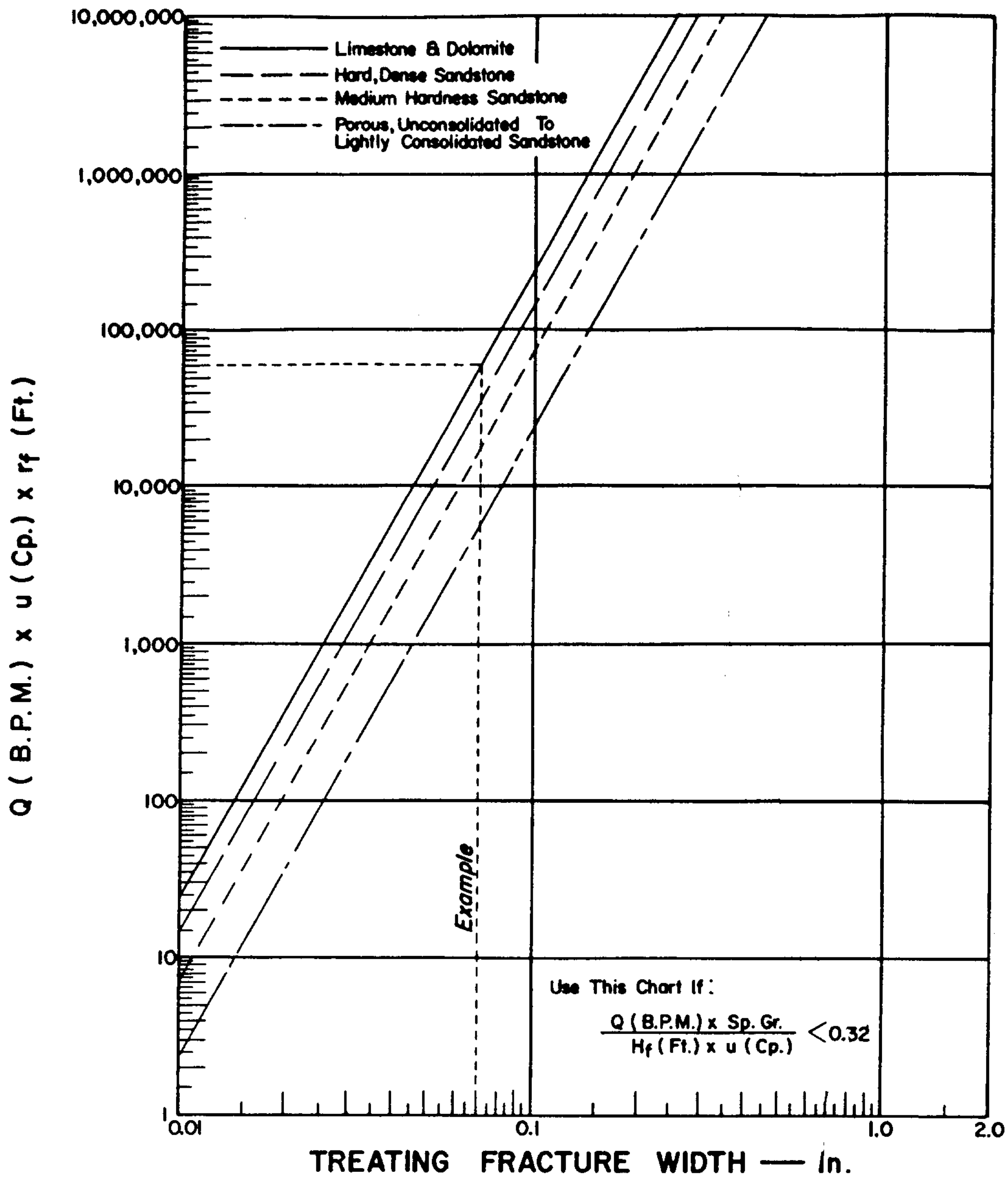


FIG. 6

CHART FOR DETERMINING PUMPING RATES AND TREATING FRACTURE WIDTHS FOR RESTRICTED VERTICAL FRACTURES RESULTING FROM NEWTONIAN FLUIDS IN LAMINAR FLOW IN THE FRACTURE

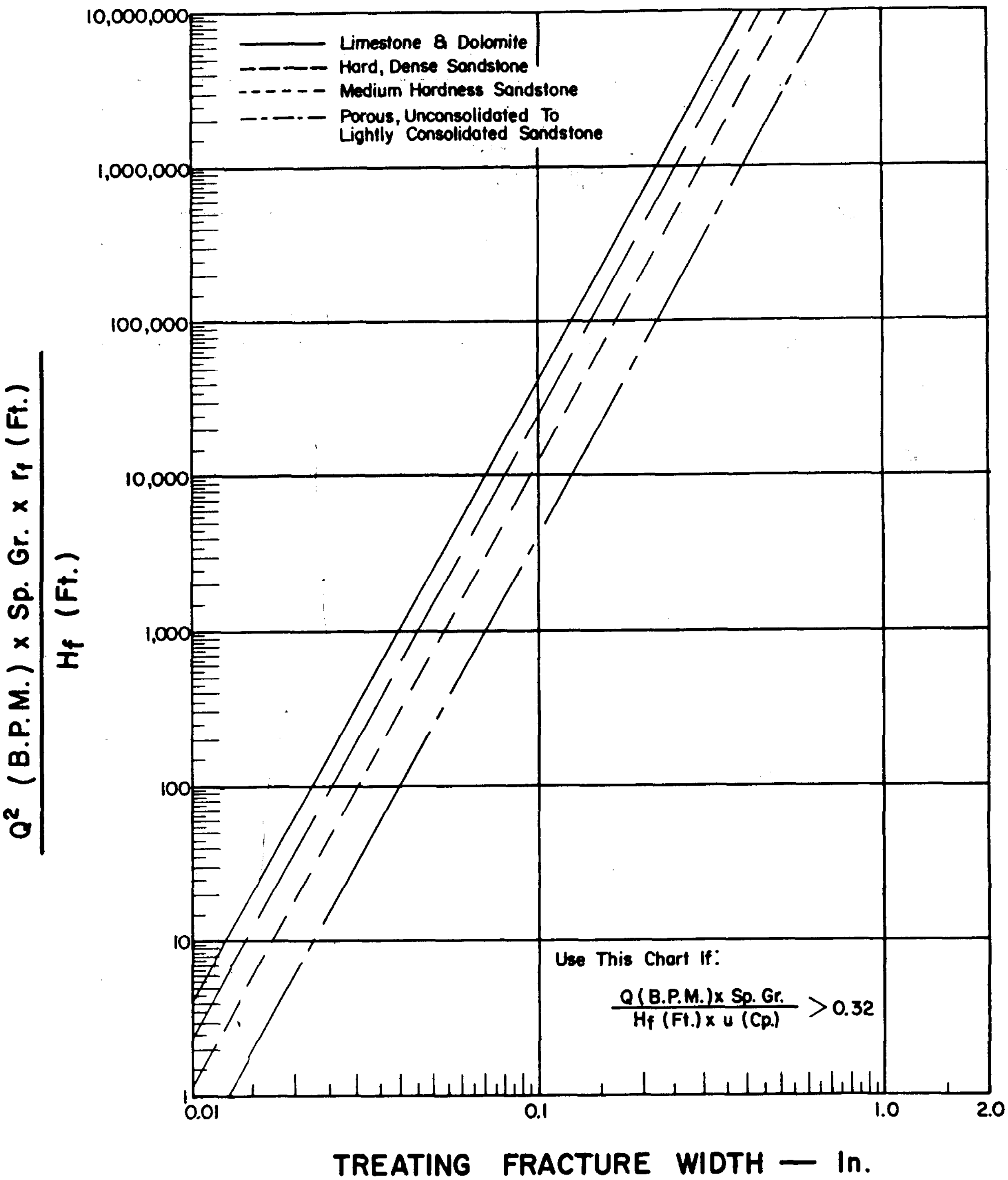


FIG. 7

CHART FOR DETERMINING PUMPING RATES AND TREATING FRACTURE WIDTHS FOR RESTRICTED VERTICAL FRACTURES RESULTING FROM NEWTONIAN FLUIDS IN TURBULENT FLOW IN THE FRACTURE

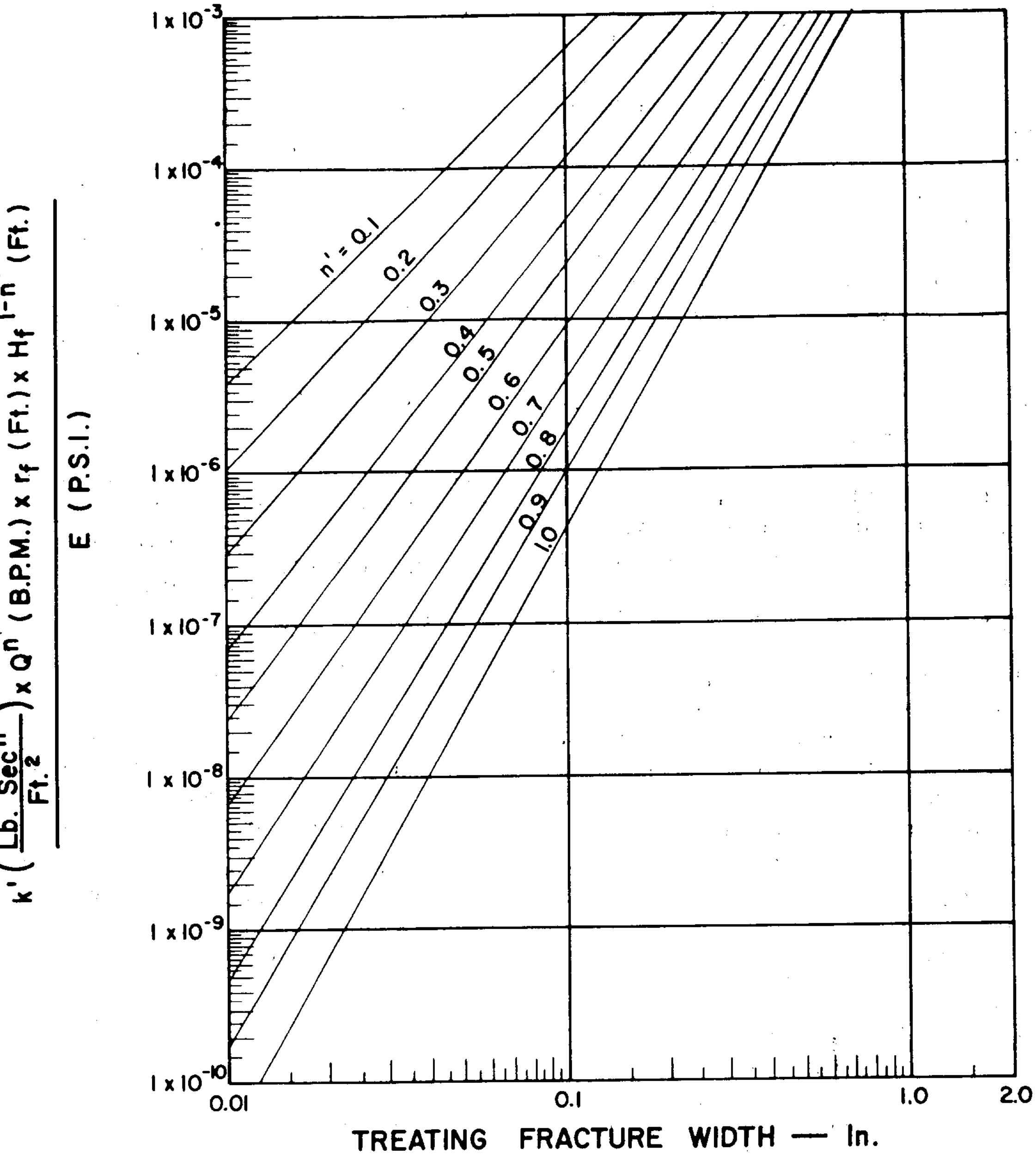


FIG. 8

CHART FOR DETERMINING PUMPING RATES AND TREATING FRACTURE WIDTHS FOR RESTRICTED VERTICAL FRACTURES RESULTING FROM NON-NEWTONIAN FLUIDS IN LAMINAR FLOW IN THE FRACTURE largest. Propping agent particle size can be obtained from Table I and healed fracture width can be determined from Eq. 18. Employing the appropriate treating fracture width, injection rate can be determined from Figs. 6, 7 or 8, depending on which of the figures satisfies treatment conditions.

Viscosity, flow behavior index, and consistency index that are used in Figs. 6, 7 and 8, should be for the fracturing slurry rather than for the fracturing fluid. These slurry properties can be determined by laboratory measurement; however, the following equation[16] can be used to correlate fracturing slurry with fracturing fluid viscosity.

$$u_{fs} = u_{ff}\left(1 + \frac{C_{pa}}{D_{pa}}\right) \qquad [6]$$

Where: u_{fs} = fracturing slurry viscosity, cps
u_{ff} = fracturing fluid viscosity, cps
C_{pa} = propping agent concentration lbs/gal.
D_{pa} = propping agent true density, lbs/gal.

The true density of the commercially available propping agents is given in Table II.

If sand is used as a propping agent, injection rate should be examined from the standpoint of the mechanics of sand movement in the fractures. The following equation[17] can be used for this purpose:

$$H_O = \frac{34 \cdot Q}{W_t \cdot v} \qquad [7]$$

Where: H_O = height of open section above settled sand, ft.
Q = injection rate, bbls/min.
W_t = treating fracture width, in.
v = equilibrium fluid velocity, ft/min

It is estimated that the equilibrium velocity for most Newtonian and non-Newtonian fluids is approximately 400 ft/min and 800 ft/min, respectively.[17,18]

If the height of the open section above the settled sand is very small, a sandout may occur and the injection rate should be increased; however, if the height of the open section above the settled sand is very large, an effective sand pack will not form and the injection rate should be decreased.

Composite Fracturing Coefficient

The composite fracturing coefficient is composed of the fracturing fluid coefficient, the reservoir fluid coefficient, and the fluid loss coefficient. In any fracturing treatment, each of thes coefficients acts simultaneously to varying ex tents and complements the other. It has been sug gested that these coefficients are analogous to series of electrical conductors and can be comb ned as such.

$$\frac{1}{C_{com}} = \frac{1}{C_I} + \frac{1}{C_{II}} + \frac{1}{C_{III}} \qquad [8]$$

C_{com} = composite fracturing coefficient, ft/$\sqrt{min}$
C_I = fracturing fluid coefficinet, ft/$\sqrt{m}$
C_{II} = reservoir fluid coefficent, ft/$\sqrt{min}$
C_{III} = fluid loss coefficient, ft/$\sqrt{min}$

The fracturing fluid coefficient can be obtaine from the following equation.

$$C_I = 0.0469\sqrt{\frac{K_{ff} \cdot \emptyset' \cdot \Delta P}{(1000) \cdot u_{ff}}} \qquad [9]$$

C_I = fracturing fluid coefficient, ft/$\sqrt{mi}$
K_{ff} = permeability to the fracturing fluid, md
$\emptyset'$ = effective porosity, fraction
ΔP = pressure differential across the fracture face, psi
u_{ff} = fracturing fluid viscosity, cps

Permeability to the fracturing fluid (oil or wa ter) may be obtained by correcting core analysi permeabilities to air by a factor of 0.6. Effectiv porosity should be obtained by correcting the fo mation porosity for irreductible oil and water sa turations. This correction is based on the assum tion that the irreducible oil and water will no move and acts as rock.

$$\emptyset' = \emptyset\left[1 - (S_{io} + S_{iw})\right] \qquad [10]$$

$\emptyset'$ = effective porosity, per cent
$\emptyset$ = formation porosity, per cent
S_{io} = irreductible oil saturation, fra
S_{iw} = irreductible water saturation, tion

Pressure differential across the fracture face the difference between bottom hole treating pre sure and reservoir pressure. Fracturing fluid vi cosity should be measured at reservoir tempera ture, and when a non-Newtonian fluid is use plastic viscosity should be employed.

The reservoir coefficent can be obtained from the following equation:[13]

$$C_{II} = \frac{0.0374\,\Delta P}{1}\sqrt{\frac{K_f \cdot \emptyset \cdot C_t}{1000 \cdot u_f}} \quad [11]$$

Where: C_{II} = reservoir fluid coefficent, ft/min

ΔP = pressure differential across fracture face psi

K_f = permeability to the mobile reservoir fluids, md

$\emptyset$ = formation porosity, fraction

C_t = total compressibility, vol/vol - psi x 10^{-6}

u_f = viscosity of the mobile reservoir fluids, cps

Permeability to the mobile reservoir fluids and viscosity of the mobile reservoir fluid should actually be determined from a complex relative permeability measurement and from a complex viscosity measurement; however, the following equations can be used to estimate the values.

$$K_f = \frac{\{S_o - S_{io}\}\{K_o\} + \{S_w - S_{iw}\}\{K_w\} + S_g K_g}{\{S_o - S_{io}\} + \{S_w - S_{iw}\} + S_g} \quad [12]$$

$$u_f = \frac{\{S_o - S_{io}\} \cdot u_o + \{S_w - S_{iw}\} \cdot u_w + S_g u_g}{\{S_o - S_{io}\} + \{S_w - S_{iw}\} + S_g} \quad [13]$$

Where:

K_f = permeability to the mobile reservoir, fluids, md

u_f = viscosity of the mobile reservoir fluids, cps

S_o = oil saturation, fraction

S_{io} = irreducible oil saturation, fraction

S_w = water saturation, fraction

S_{iw} = irreducible water saturation, fraction

S_g = gas saturation, fraction

K_o = permeability to oil, md

K_w = permeability to water, md

K_g = permeability to gas, md

u_o = oil viscosity, cps

u_w = water viscosity, cps

u_g = gas viscosity, cps

Irreducible oil and water saturations are each estimated to range from 15 to 20 per cent for most limestone and dolomite reservoirs. Permeability to oil and water may be obtained by correcting core analysis permeability to air by a factor of 0.6, and core analysis permeability to air should be used as permeability to gas. Oil, water and gas viscosities should be at reservoir conditions and can be obtained from most reservoir engineering texts and handbooks. Water saturation can be obtained from log analysis. If gas saturation can be determined, all of the saturations can be obtained since oil, water, and gas saturations constitute 100 per cent of the pore volume. The procedure to be followed in determining gas saturation can best be explained by means of an example. Assume the following:

limestone reservoir
producing gas-oil ratio = 750 SCF/STB
solution gas-oil ratio = 520 SCF/STB
reservoir temperature = 570°R

P_r = reservoir pressure = 2014.7 psi

Z = gas deviation factor = 0.59

β = formation volume factor = 1.28 res. bbl/STB

u_g = gas viscosity = 0.0227 cps

u_o = oil viscosity = 2.3 cps

The difference between the producing and solution gas-oil ratios is 230 SCF/STB and from the nonideal gas law, the reservoir volume of 230 SCF/STB is

$$\frac{P_1 V_1}{T_1 Z_1} = \frac{P_2 V_2}{T_2 Z_2} \qquad \frac{(14.7)(230)}{(520)(1.0)} = \frac{(2014.7)(V_2)}{(570)(0.59)}$$

$$V_2 = 1.09 \frac{ft^3 \text{ res. gas}}{STB}$$

The ratio of gas to oil flow rates in the reservoir is

$$\frac{q_g}{q_o} = \frac{V_2}{(5.61\ ft^3/bbl)(\beta)} = \frac{1.09}{(5.61)\ (1.28)}$$

$$= 0.15\ \frac{\text{bbl res gas}}{\text{bbl res oil}}$$

$\frac{q_g}{q_o}$ is equal to the mobility ratio

$$\frac{q_g}{q_o} = \text{Mobility Ratio} = \frac{K_g/u_g}{K_o/u_o}$$

$$\frac{K_g}{K_o} = \frac{(q_g/q_o)(u_g)}{u_o} = \frac{(0.15)(0.0227)}{2.3} = 0.0015$$

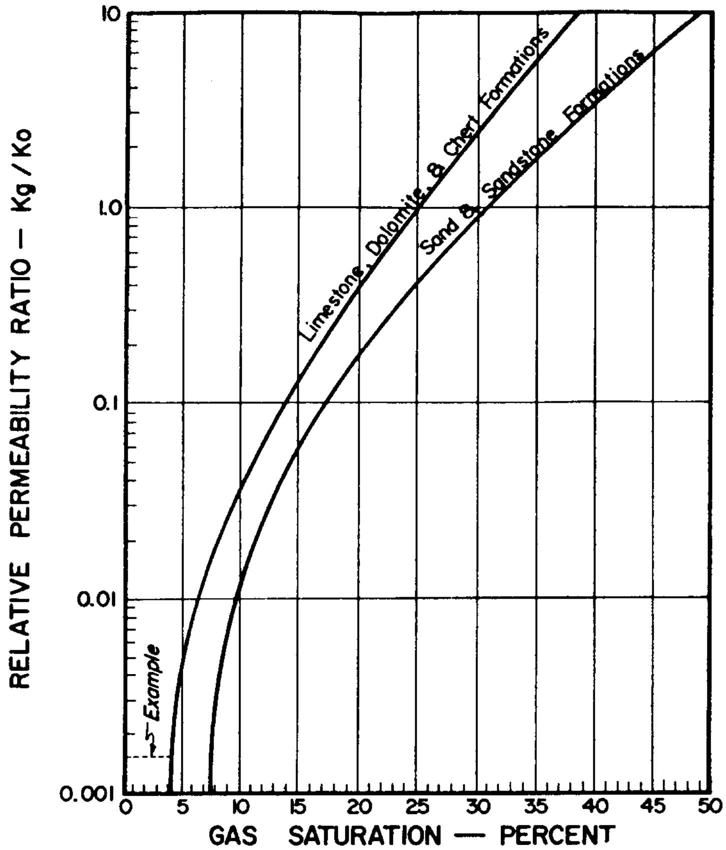


FIG. 9

RELATIVE PERMEABILITY RATIO vs GAS SATURATION

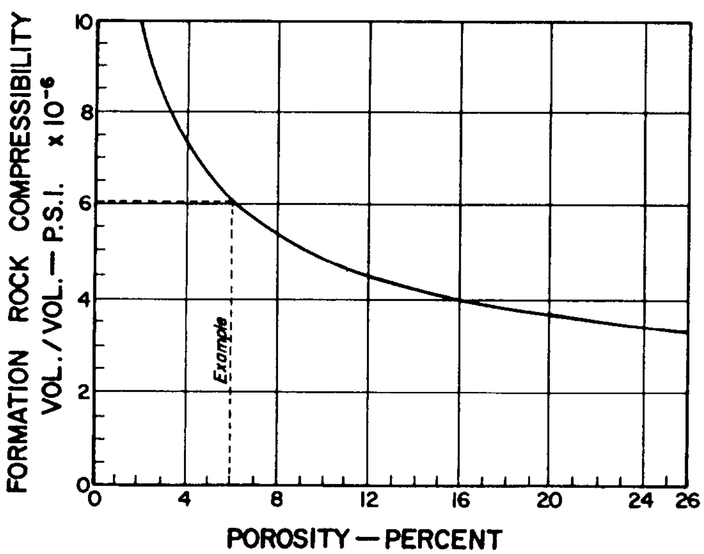


FIG. 10

FORMATION ROCK COMPRESSIBILITIES

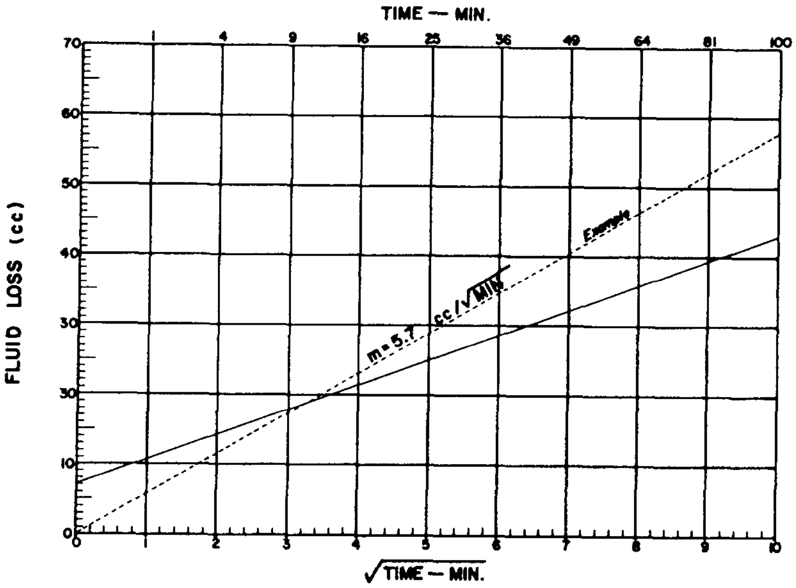


FIG. 11

FLUID LOSS PLOT FOR LEASE OIL (30° API) CONTAINING 25#/1000 GALS. OF ADOMITE MARK II. TEST CONDUCTED ON A BAROID FILTER PRESS AT 1000 P.S.I. AND 110°F

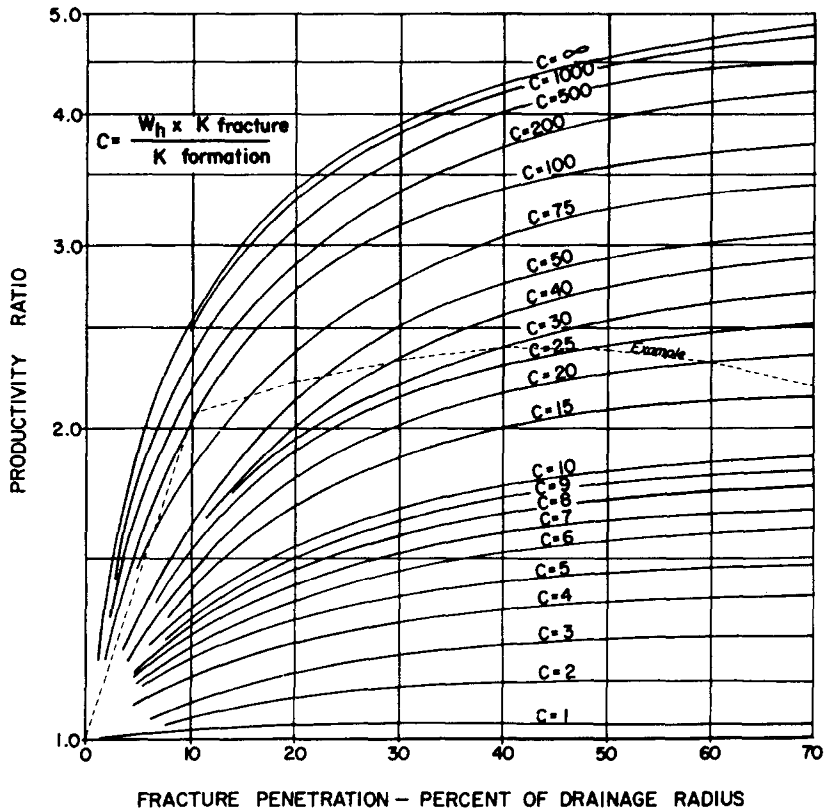


FIG. 12

EFFECT OF FRACTURE PENETRATION AND FRACTURE CONDUCTIVITY ON PRODUCTIVITY FOR VERTICAL FRACTURES

From Fig. 9,[19] gas saturation is four per cent. Total compressibility can be obtained from the following equation.

$$C_t = S_o C_o + S_w C_w + S_g C_g + C_f \quad [14]$$

Where:

C_t = total compressibility, vol/vol - psi

C_o = oil compressibility, vol/vol - psi

C_w = water compressibility, vol/vol - psi

C_g = gas compressibility, vol/vol - psi

C_f = formation rock compressibility, vol/vol - psi

S_o = oil saturation, fraction

S_w = water saturation, fraction

S_g = gas saturation, fraction

Oil, water, and gas saturations can be obtained as previously discussed, and oil, water, and gas compressibilities can be obtained from most reservoir engineering texts and handbooks. Formation rock compressibility can be obtained from Fig. 10.[20]

The fluid loss coefficient has to be determined from data obtained experimentally in a laboratory. A standard fluid loss test is conducted on a high pressure, high temperature Baroid filter press containing filter paper; however, fluid loss tests can be conducted where actual core samples are substituted for the filter paper. The fluid loss test is run with a pressure differential of 1000 psi and at a temperature of 125°F or at the actual reservoir temperature, if possible. The fluid loss in cubic centimeters is measured at time intervals of 1, 4, 9, 16, and 25 min, and these fluid loss values are then plotted on straight coordinate paper against the square root of time in minutes, as shown in Fig. 11. The experimental fluid loss coefficient is then calculated as follows:

$$C_{III\,exp} = \frac{(0.0164)(m)}{a} \quad [15]$$

Where: $C_{III\,exp}$ = experimental fluid loss coefficient, ft/$\sqrt{min}$

m = slope of the fluid loss curve, cc/$\sqrt{min}$

a = area of the filter medium, cm^2

The fluid loss of a fracture fluid is a combination of the fluid lost before a filter cake has begun to form (spurt loss) and the fluid loss through the filter cake during the treatment. The point where the fluid loss curve intersects the ordinate on a fluid loss plot is known as the spurt loss. Spurt loss is not accounted for in Eq. 15 and must be considered separately. Crittendon[4] presented the following method for including spurt loss in the fracturing fluid coefficient.

(1) Construct the fluid loss plot in the conventional manner.
(2) Draw a line running through the 0 intercept and the volume of fluid loss at a time equal to 1/2 of the total time that the fracturing fluid is being injected into the formation.
(3) Employ the slope of this line in Eq. 15.

The experimental fluid loss coefficient can be corrected from experimental conditions to actual conditions by the following equation.

$$C_{III} = C_{III\,exp}\sqrt{\frac{\Delta P_{act}}{\Delta P_{exp}} \cdot \frac{K_{act}}{K_{exp}} \cdot \frac{T_{act}}{T_{exp}}} \quad [16]$$

Where: C_{III} = fluid loss coefficient, ft/$\sqrt{min}$

$C_{III\,exp}$ = experimental fluid loss coefficient, ft/$\sqrt{min}$

ΔP_{act} = pressure differential across the fracture face, psi

ΔP_{exp} = experimental pressure differential, psi

K_{act} = permeability to the fracturing fluid, md

K_{exp} = permeability of the filter medium, (core sample or filter paper), md

T_{act} = reservoir temperature, °F

T_{exp} = experimental temperature, °F

The only value in Eq. 16 that has not been previously discussed is the permeability of filter medium. Permeability of filter paper decreases with increased pressure as a result of the compaction of the paper. It is estimated that the permeability of filter paper at 1000 psi is approximately 0.1 md.

PRODUCTIVITY RATIO

Productivity ratio is defined as the ratio of fractured productivity to that of unfractured productivity. Crittendon[4] correlated vertical fracture productivity ratios for various fracture penetrations with fracture conductivity as shown in Fig. 12. Fracture conductivity as used in Fig. 12 is expressed by the following equation:

$$C = \frac{W_h \cdot K_{fracture}}{K_{formation}} \quad [17]$$

Where:

C = fracture conductivity, ft

W_h = healed fracture width, ft

$K_{fracture}$ - fracture permeability, md

$K_{formation}$ = permeability to the mobile reservoir fluids, md

It requires 8.9 lbs of frac sand to produce a volume of 1 ft² cross section by 1 in. thick. If fraturing sand is used as a propping agent, the following equation can be used to calculate healed fracture width.

$$W_h = \frac{S\,Q}{A_f\,(8.9)} \quad [18]$$

Where:

W_h = healed fracture width, in.

$S\,Q$ = sand quantity, lbs

A_f = fracture area, ft²

8.9 = a constant, lbs/ft² - in.

Fracture permeability should be obtained from Fig. 5 using bottom hole treating pressure as compaction pressure, and the permeability to the mobile reservoir fluids should be obtained from Eq. 12.

The values of productivity ratio that are obtained from Fig. 12 are values which can be expected after a well has started on normal decline and are not the values immediately following a treatment.

BASIC STEPS TO FOLLOW IN FRACTURE TREATMENT DESIGN

There are eight basic steps to follow in the design method presented herein. These steps are as follows.

(1.) Determine the optimum fracture penetration and fracture area.
(2.) Determine the optimum sand quantity.
(3.) Determine the fracturing fluid coefficient and the reservoir fluid coefficient.
(4.) Assume a fracturing fluid volume and determine the treatment volume.
(5.) Determine the required injection rate.
(6.) Determine the fluid loss coefficient.
(7.) Determine the composite fracturing coefficient.
(8.) Employing the appropriate treatment variables, determine fracture area. If this fracture area does not agree with the fracture area that was obtained in Step 1, it will be necessary to repeat Steps 4 through 8 until the two fracture areas are in agreement. This is a trial and error solution.

EXAMPLE FRACTURE TREATMENT DESIGN

The concepts and design method previously discussed can best be explained by means of an example. Assume the following data are for a well that produces from 6500 ft. limestone reservoir and that is to be fractured with 20-40 mesh Ottawa sand and lease oil (30 degree API) containing 25 lbs. per 1,000 gals. of Adomite Mark II fluid loss additive.

r_e - drainage radius = 660 ft

H_f - fracture height = 150 ft

T = reservoir temperature = 110°F

P_r = reservoir pressure = 2000 psi

P_t = bottom hole treating pressure = 4500 psi

ΔP = pressure differential across the fracture face = 2500 psi

S_o = oil saturation = 0.71

S_w = water saturation = 0.25

S_g = gas saturation = 0.04

S_{io} = irreducible oil saturation = 0.15

S_{iw} = irreducible water saturation = 0.15

ϕ = formation porosity = 0.06

ϕ' - effective porosity = 0.042

K_o = permeability to oil = 1.5 md

K_w = permeability to water = 1.5 md

K_g = permeability to gas = 2.5 md.

K_f = permeability to the mobile reservoirs fluids = 1.6 md

K_{ff} - permeability to the fracturing fluid = 1.5 md

$K_{fracture}$ - fracture permeability = 5,700 md

u_o - oil viscosity = 2.3 cps

u_w - water viscosity = 0.6 cps

u_g — gas viscosity = 0.227 cps
u_f = viscosity of the mobile reservoir fluids = 1.9 cps
u_{ff} = fracturing fiuld viscosity = 9.0 cps
C_o = oil compressibility = 8.1 x 10^{-6} vol/vol - psi
C_w = water compressibility = 2.9 x 10^{-6} vol/vol - psi
C_g = gas compressibility = 258.4 x 10^{-6} vol/vol - psi
C_f = formation rock compressibility = 6.1 x 10^{-6} vol/vol - psi
C_t = total compressibility = 22.9 x 10^{-6} vol/vol - psi

Step 1. Determine the optimum fracture penetration and fracture area. For a fracture penetration of 10 per cent, calculate the fracture area as follows:

$$A_f = r_e\,(10\%)(2)\,H_f = (660)\ (0.10)\ (2)\ (150) = 19{,}800\ ft^2$$

Using the above fracture area, assume a sand quantity of 70,000 lbs. and calculate the healed fracture width from Eq. 18.

$$W_h = \frac{S\ Q}{A_f\ (8.9)} = \frac{70{,}000}{(19{,}800)\ (8.9)} = 0.397\ in. = 0.033\ ft$$

Using the above healed fracture width, calculate the fracture conductivity from Eq. 17.

$$C = \frac{W_h \cdot K_{fracture}}{K_{formation}} = \frac{(0.033)\ (5700)}{1.6} = 118\ ft$$

Productivity ratio=2.05 From Fig 12 at a fracture penetration of 10 per cent and C = 118 ft.

Repeat the above procedure for fracture penetrationsof 20, 30, 35, 40, 50, 60, and 70 per cent and tabulate the results as shown in the upper portion of Table IV. From this data, it is evident that the optimum fracture penetration is 40 per cent, and the optimum fracture area is 79,200 ft.[2] A plot of fracture conductivity vs fracture penetration as shown on Fig. 12 illustrates graphically the optimum fracture penetration.

Step 2. Determine the optimum sand quantity. Using the optimum fracture area of 79,200 ft,[2] **assume a sand quantity of 7,000 lbs. and calculate the healed fracture width from Eq. 18**

$$W_h = \frac{S\ Q}{A_f\ (8.9)} = \frac{7{,}000}{(79{,}200)\ (8.9)} = 0.010\ in = 0.0008\ ft$$

Using the above healed fracture width, calculate the fracture conductivity from Eq. 17.

$$C = \frac{W_h \cdot K_{fracture}}{K_{formation}} = \frac{(0.0008)\ (5{,}700)}{1.6}\ \ 3\ f$$

Productivity ratio = 1.25 -- From Fig. 12 at th optimum fracture penetration of 40 per cent an C = 3 ft.

Repeat the above procedure for various san quantities and tabulate the results as shown i the lower portion of Table IV. Since little benef is realized from the use of more than 35,000 lb of sand and since a reasonable healed fractur width results from the use of this 35,000 lbs o sand, this sand quantity is selected as the opt mum.

Step 3. Determine the fracturing fluid coeff cient and the reservoir fluid coefficient.

Calculate the fracturing fluid coefficient an the reservoir fluid coefficient from Eq. 9 and 1 respectively.

$$C_I = 0.0469\sqrt{\frac{K_{ff} \cdot \emptyset \cdot \Delta P}{(1000) \cdot u_{ff}}} = 0.0469$$

$$\cdot\sqrt{\frac{(1.5)\ (0.042)\ (2{,}500)}{(1{,}000)\ (9.0)}} = 6.2 \times 10^{-3}\ ft/\sqrt{mi}$$

$$C_{II} = \frac{0.0374\,\Delta P}{1}\sqrt{\frac{K_f \cdot \emptyset \cdot C_t}{1000 \cdot u_f}} = (0.0374)\ (25$$

$$\cdot\sqrt{\frac{(1.6)\ (0.06)\ (0.0000229)}{(1{,}000)\ (1.9)}} = 3.2 \times 10^{-3}\ ft\sqrt{min}$$

Step 4. Assume a fracturing fluid volume and determine the treatment volume.

Assumed fracturing fluid volume = 19,000 ga

Sand quantity = 35,000 lbs = (35,000) (0.045 = 1,575 gals.

Treatment volume = 19,000 + 1,575 = 20,575 gals.

Step 5. Determine the required injection rate Since the healed fracture width is 0.05 in. and since twice the largest particle dimension of 20 40 mesh sand is 0.066 in., a treating fracture wid th of 0.07 in. should be adequate. The fracture radius is 40 per cent of the drainage radius or 264 ft. The assumed fracturing fluid volume is 19,000 gals. and the optimum sand concentration is 35,000 lbs; therefore, the sand concentration is 1.84 lbs /gal. Using this sand concentration calculate the fracturing slurry viscosity from Eq 6.

$$u_{fs} = u_{ff}\left(1 + \frac{C_{pa}}{D_{pa}}\right) = 9.0\ \left(1 + \frac{1.84}{22.09}\right) = 9.7\ cps$$

Table IV

Fracture Penetration (%)	Fracture Area (Ft.^{2})	Sand Quantity (Lbs.)	Healed Fracture Width (In.)	Fracture Conductivity (Ft.)	Productivity Ratio (Dimensionless)
0	0	0	0	0	0
10	19,800	70,000	0.397	118	2.05
20	39,600	70,000	0.199	59	2.25
30	59,400	70,000	0.132	39	2.30
35	69,300	70,000	0.113	34	2.35
40	79,200	70,000	0.099	30	2.40
50	99,000	70,000	0.079	24	2.35
60	118,800	70,000	0.066	20	2.30
70	138,600	70,000	0.057	17	2.20
40	79,200	7,000	0.010	3	1.25
40	79,200	14,000	0.020	6	1.55
40	79,200	21,000	0.030	9	1.70
40	79,200	28,000	0.040	12	1.85
40	79,200	35,000	0.050	15	2.00
40	79,200	42,000	0.060	18	2.10
40	79,200	49,000	0.070	21	2.20
40	79,200	56,000	0.080	24	2.30
40	79,200	63,000	0.090	27	2.35
40	79,200	70,000	0.100	30	2.40

Check the injection rate for laminar flow.

$$\frac{(Q)(Sp.Gr.)}{(H_f)(u_{fs})} = \frac{(23)\ (1.00)}{(150)\ (9.7)} =$$

$$0.02 < 0.32 \quad \text{-- laminar}$$

Examine the injection rate from the standpoint of the mechanics of sand movement in the fracture using Eq. 7.

$$H_o = \frac{34 \cdot Q}{W_f \ v} = \frac{(34)\ (23)}{(0.07)\ (400)} = 28 \text{ ft}$$

Since the height of the fracture is 150 ft, and since the height of the open section above the settled sand is 28 ft, the injection rate of 23 bbls min is satisfactory.

Step 6. Determine the fluid loss coefficient.

Obtain the slope of the fluid loss curve in Fig. 11 and calculate the experimental fluid loss coefficient from Eq. 15.

$$C_{\mathrm{III}exp} = \frac{(0.0164)(m)}{a} = \frac{(0.0164)\ (5.7)}{22.9}$$

$$4.1 \times 10^{-3} \text{ ft } \sqrt{\text{min}}$$

Correct the experimental fluid loss coefficient from experimental to actual conditions by the use of Eq. 16.

$$C_{\mathrm{III}} = C_{\mathrm{III}exp}\sqrt{\frac{\Delta P_{act}}{\Delta P_{exp}} \cdot \frac{K_{act}}{K_{exp}} \cdot \frac{T_{act}}{T_{exp}}}$$

$$= 4.1\sqrt{\left(\frac{2{,}500}{1{,}000}\right)\left(\frac{1.5}{0.1}\right)\left(\frac{110}{110}\right)}$$

$$C_{\mathrm{III}} = 25.1 \times 10^{-3} \text{ ft}/\sqrt{\text{min}}$$

Step 7. Determine the composite fracturing coefficient.

Using the fracturing fluid coefficient of 6.2×10^{-3} ft $/\sqrt{\text{min}}$, the reservoir fluid coefficient of 3.2×10^{-3} ft $/ \sqrt{\text{min}}$, and the fluid loss coefficient of 25.1×10^{-3} ft $/\sqrt{\text{min}}$, calculate the composite fracturing fluid coefficient from Eq. 8:

$$\frac{1}{C_{com}} = \frac{1}{C_{\mathrm{I}}} + \frac{1}{C_{\mathrm{II}}} + \frac{1}{C_{\mathrm{III}}} =$$

$$\frac{1}{6.2 \times 10^{-3}} + \frac{1}{3.2 \times 10^{-3}} + \frac{1}{25.1 \times 10^{-3}} =$$

$$C_{com} = 1.9 \times 10^{-3} \text{ ft}/\sqrt{\text{min}}$$

Step 8. Employing the appropriate treatment variables, determine fracture area.

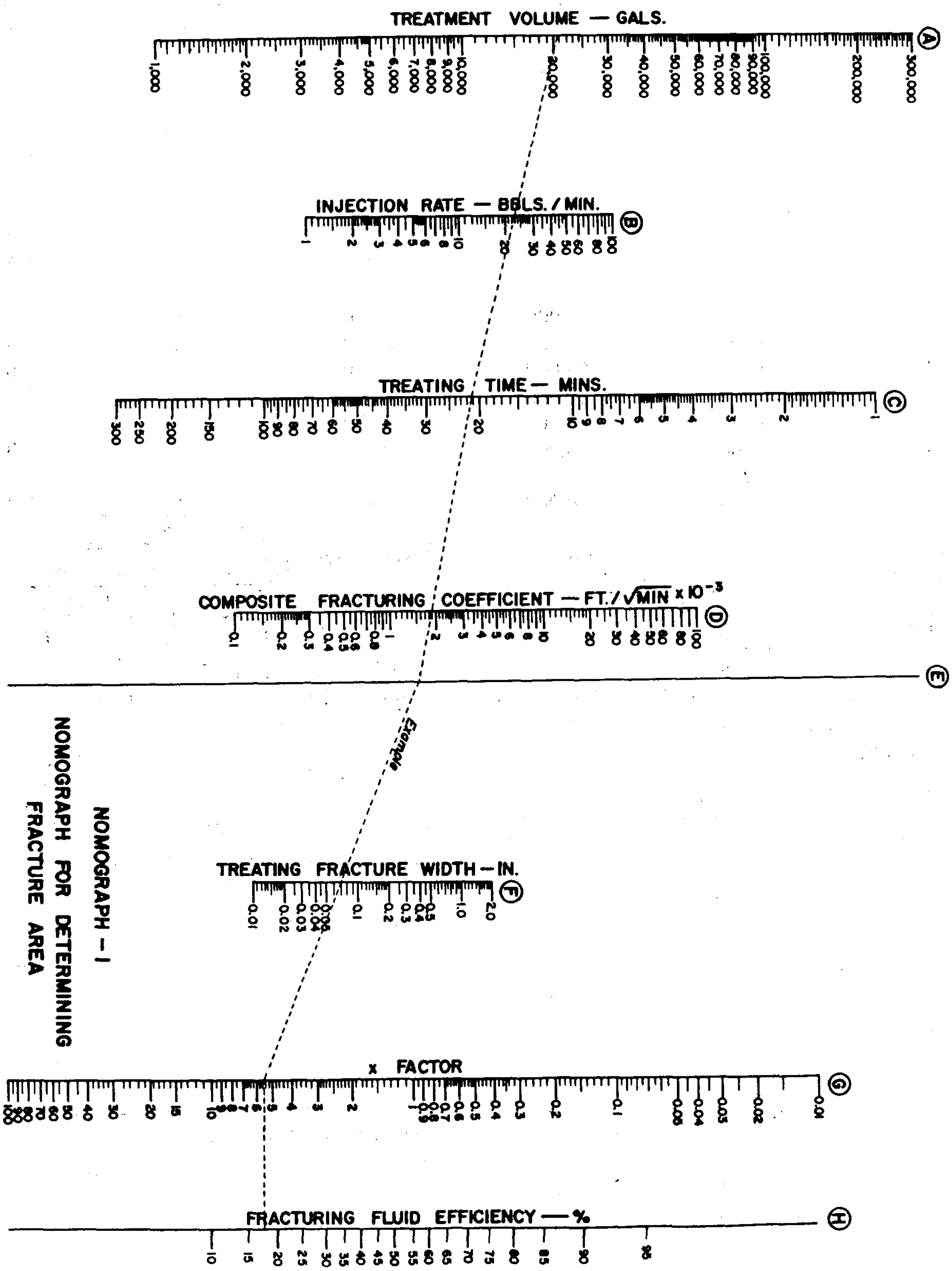
A_f = 79,200 f^2 From Nomographs 1 and 2 at treatment volume = 20,575 gals.

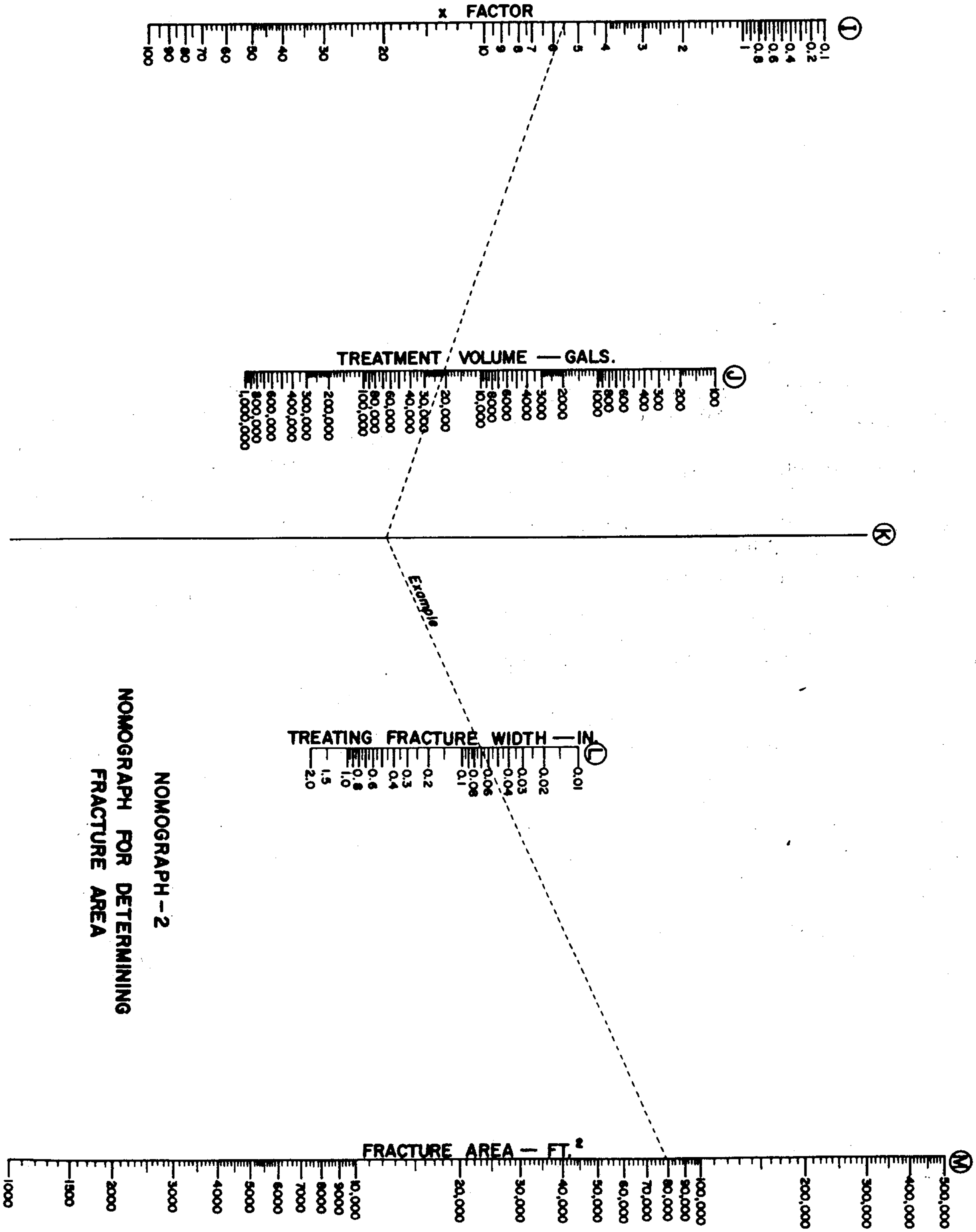
Q = 23 bbl/min $\quad$ C_{com} = 1.9×10^{-3}

Since the above fracture area agrees with the fracture area obtained in Step 1, the treatment design is complete. In summary, the treatment just designed consists of:

(1.) 19,000 gals. of lease oil (30° API) containing 25 lbs per 1000 gals. of Adomite Mark II fluid loss additive.

(2.) 35,000 lbs of 20-40 mesh Ottawa sand.
(3.) 23 bbls /min injection rate.





NOMOGRAPH-2
NOMOGRAPH FOR DETERMINING FRACTURE AREA

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ACKNOWLEDGMENTS

The author wishes to express his appreciati to the management of Mobil Oil Company f permission to publish this paper. The auth also wishes to extend his gratitude to the pe sonnel of Mobil Oil Company for their assistan in preparing this paper.