Fundamental Orifice Flow Caclulations

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The theory of the measurement of the flow of a liquid through an orifice is based on the two fundamental laws of hydraulics. Gas flow involves some further modifications which will be mentioned after the simpler case of liquid flow has been developed.

The first law of hydraulics is that known as the Law of Continuity. In its general form it states that the quantity of fluid flowing into any closed portion of space in a given time is equal to that flowing out in the same time provided no change in storage is taking place in the region. In the case of a pipe filled with liquid, storage changes do not occur and the law applies. Since the discharge, Q, is the product of cross-sectional area, A, and mean velocity, V, the law is often written

$$Q = AV \text{ or } A_1V_1 = A_2V_2 \qquad (1)$$

where the customary units in engineering practice are

Q: cubic feet per second (cfs),
A: square feet, and
V: feet per second (fps).

It is also convenient when dealing with circular pipe to replace the area by its equivalent, ''D2/4, where D is the diameter in feet. This gives

$$D_1^2 V_1 = D_2^2 V_2 \qquad (2)$$

Since an orifice is essentially a segment of smaller diameter it may be seen from this equation that a liquid flows faster through an orifice than in the rest of the pipe.

Bernoulli's Equation, the second law of hydraulics, is a modification of the general energy equation of thermodynamics. For liquids, only three forms of energy may be transformed into useful work. These are the kinetic energy and the mechanical potential energies of pressure and of elevation. For greater convenience the energies per pound of fluid flowing are used. These quantities, measured in foot-pounds per pound, or feet, are called heads. Thus the kinetic energy becomes the velocity head and the other two energies become the pressure and elevation heads. Some energy also becomes unavailable by a deterioration into kinetic energy of small rotations in regions of turbulence and then, ultimately, into heat. This energy, or the corresponding head, the hydraulic engineer refers to as lost head. Such degradation is caused by pipe roughness, by changes in direction, or by sudden changes in cross-section.

In words, Bernoulli's equation states that if a liquid flows from point 1 to point 2, the sum of the three types of head present at point 1 will be greater than the corresponding sum at point 2 by the amount of the lost head.

In symbols

$$\frac{V_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{w} + z_2 + h_L, \qquad (3)$$

Where

v: velocity in feet per second.

g: acceleration of gravity (= 32.2 fps^2)

p: pressure in pounds per square foot.

w: specific weight (= 62.4 specific gravity).

z: elevation above some arbitrary datum in feet.

h_L: head lost in foot-pounds per pound.

A crude orifice meter could be made by using standard pipe fittings as illustrated in Figure 1. The liquid

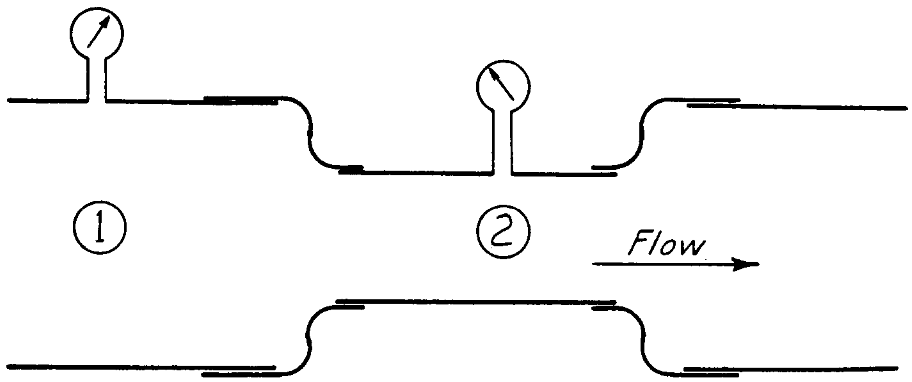

Figure 1 — The Essentials of an Orifice Meter

velocities in the line pipe, V_1, and in the throat, V_2, would be related by the law of continuity, (2). By giving the meter a horizontal setting, $z_1 = z_2$. Thus equation (3) may be written

$$\frac{V_1^2}{2g} + \frac{p_1}{w} = \left(\frac{D_1}{D_2}\right)^4 \frac{V_1^2}{2g} + \frac{p_2}{w} + h_L. \qquad (4)$$

Standard algebraic manipulations on equation (4) lead to

$$V_1 = \sqrt{\frac{2g\left[\frac{p_1 - p_2}{w} - h_L\right]}{\left(\frac{D_1}{D_2}\right)^4 - 1}} \qquad (5)$$

Since 2g = 64.4, a constant, and since D_1 and D_2 are fixed for a given meter it is convenient to calculate them once and for all and then rewrite equation (5) as

$$V_1 = K_1 \sqrt{\frac{p_1 - p_2}{w} - h_L} \qquad (6)$$

The discharge may be found by use of equation (1):

$$Q = A_1 V_1 = \frac{\pi}{4} D_1^2 K_1 \times \sqrt{\frac{p_1 - p_2}{w} - h_L}. \qquad (7)$$

This, in turn, may be simplified to

$$Q = K_2 \sqrt{\frac{p_1 - p_2}{w} - h_L}. \qquad (8)$$

If, now, we could find h_L a simple calculation involving the pressure difference would enable us to compute the quantity flowing. The advantage of the commercial meters over the meter just described is that they involve a predictable loss in head; one may thus prepare a table, chart, or equation giving the information desired.

Actually, it is customary to modify equation (8) by observing that $h_L = K_3 (V^2/2g)$, where K_3 is not strictly a constant but is nearly so in most cases. When this is substituted in equation (4) one obtains

$$V_1 = \sqrt{\frac{2g\left(\frac{P_1 - P_2}{w}\right)}{\left(\frac{D_1}{D_2}\right)^4 + K_3 - 1}}, \qquad (9)$$

or, finally,

$$Q = K_4 \sqrt{\frac{P_1 - P_2}{w}} \qquad (10)$$

The coefficient, K_4, in this expression is not strictly a constant, but must be determined over the range of flows which it is desired to measure by actual calibration.

In Figure 1 we showed two Bourdon gauges. Usually it is more convenient and gives better accuracy to replace these with a manometer.

If the manometer fluid (shown shaded in Figure 2) be heavier than the liquid flowing it will tend to remain in the bottom of the U-tube. Sometimes a lighter liquid or a gas may be used; in such a case the tube must be inverted and placed above the pipe line.

For the case shown, if S_L be the specific gravity of the liquid flowing and S_M be the specific gravity of the manometer liquid, the law of hydrostatics applied at the level A — A' in the two legs gives

$$\frac{P_1}{w} + (y+h)\,S_L = \frac{P_2}{w} + y\,S_L + h\,S_M. \qquad (11)$$

Thus

$$\frac{P_1 - P_2}{w} = (S_M - S_L)\,h. \qquad (12)$$

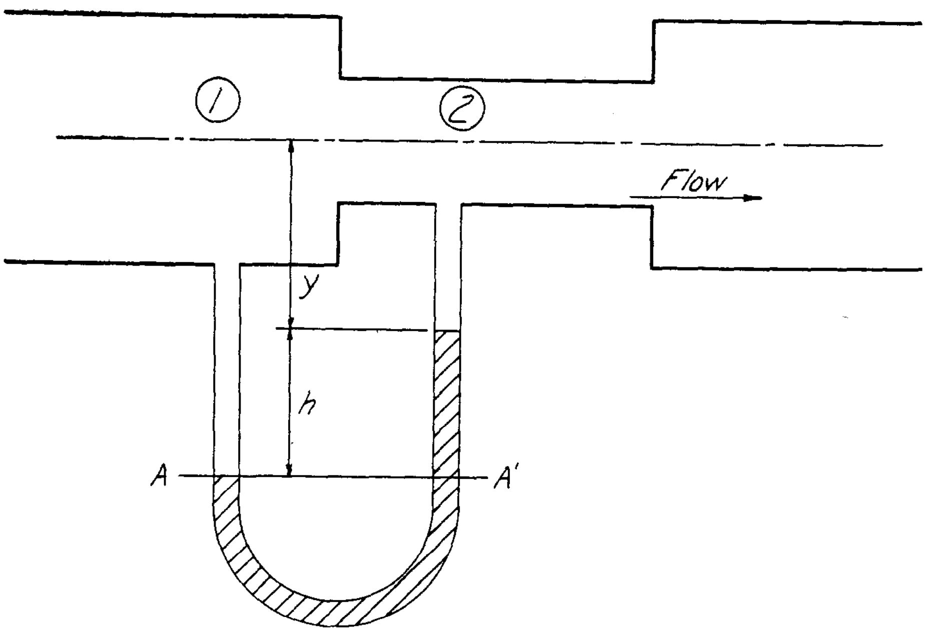

Figure 2 — Orifice Meter with Manometer (Schematic)

By combining $(S_M - S_L)K_4$ into a new coefficient, K_5, we have finally

$$Q = K_5 \sqrt{h}. \qquad (13)$$

There are several styles of commercial meters. The one which looks most similar to that of our example is the Venturi meter. Here the irregular rough pipe fittings are replaced by carefully machined cones and the pressure taps are arranged to assure correct measurements. The loss through such a meter is very smal and hence the accuracy is high. But so is the cost.

An effective meter can be made by inserting a plate with a hole of smaller diameter than that of the pipe drilled in its center. Here the jet resulting from flow through the hole, or orifice, tends to maintain itself for a short distance beyond the plate; this permits the measurement of pressure and velocity differences and thus leads to determination of rate of flow.

No general agreement as to the proper location of the pressure taps exists. Any of the standard placements, however, will lead to reliable results if the user has the calibration curve or table for the meter at hand.

The flow air or gases may also be measured by means of pipe orifices. Here, however, the theoretical formulas become much more complicated in order to take into account the compressibility. For example, equation (5) would need to be replaced by Eq. 14:

$$V_1 = \left(\frac{P_2}{P_1}\right)^{\frac{1}{K}} \frac{Ca}{A} \sqrt{\frac{2g\left(\frac{K}{K-1}\right)\left(\frac{P_1}{w_1}\right)\left[\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}} - 1\right]}{\left(\frac{P_2}{P_1}\right)^{2/K}\left(\frac{Ca}{A}\right)^2 - 1}}, \qquad (14)$$

where K is the adiabatic constant for the gas,

a is the area of the orifice,

A is the area of the pipe,

C is the coefficient of jet contraction, and the other letters have the meanings previously assigned to them.

In practice, a compressibility factor, K, may be introduced as a correction to the theory based on incompressible flow. Since this factor is found tabulated in the handbooks little additional difficulty results in applications to flows at the usual rates. High-velocity flows may introduce further complications but they fall outside the scope of a paper on fundamental orifice flow calculations.