Fracture Analysis with the Square Root of Time Plot.

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ABSTRACT

Literature examples of the calculation of permeability and fracture length from the square root of time plot usually involve single phase flow. This paper discusses (1) fracture analysis based on multiple phase flow, (2) use of the slope of the linearity and the end of linearity for permeability and fracture length calculation, (3) what is the end of linearity and (4) problems in drawing the proper linearity in a square root of time plot.

INTRODUCTION

In Clark's[1] work on fracture analysis he used the permeability from a radial flow analysis (semi-log plot) to calculate the fracture length. Gringarten, Ramey and Raghavan [2,3] pointed out that the linearity on the square root of time plot for a uniform flux fracture in an infinite reservoir ended at a dimensionless time of $t_D = .16$, permitting the fracture length and the permeability to be calculated from the square root of time plot alone. When there is enough data for both a radial flow and a fracture analysis, this provides two independent calculations of the permeability.

The diffusivity equation basic to transient flow analysis balances the total mobility of the reservoir fluids against the storage capacity of the porosity. Therefore, in any equation involving diffusivity, $.00026368\ k/(\phi\mu C)$, such as those used in the calculation of the skin effect, the radius of investigation or the dimensionless time (as is used in type curve matching), it is essential that the k/μ used be $(k/\mu)_t$, the total mobility. Miller-Dyes-Hutchinson [4] pointed out the need for using total mobility in calculating dimensionless time. Earlougher [6], page 18, clearly states the need and the procedure for getting both $(k/\mu)_t$ and C_t when there is multiple phase flow. However, since it is common practice even with multiphase flow to use k_o calculated from $q_o\mu_oB_o$ as "the" permeability, k, (see Ref. 6, p 137), and to define t_D as $\frac{.00026368kt}{\phi\mu C_t r^2}$ (Ref. 5, p 24) where the need for the total compressibility, C_t, is clearly indicated, but the need for using total mobility $(k/\mu)_t$ is not, there is apt to be confusion. We have run buildup, drawdown or falloff curves on thousands of wells, mainly pumping oil wells but also water supply wells, water injection wells, gas wells and flowing wells. About 85% of our tests have involved multiple phase flow in the reservoir, Table 1. This problem does not arise in many published papers because the examples involve single phase flow[1,2,3,7].

It is the purpose of this paper to derive the permeability and the fracture length from the end of linearity and slope of the square root of time plot keeping in mind multiple phase flow, and to discuss some of the factors involved.

DERIVATION OF PERMEABILITY AND FRACTURE LENGTH

We have:

The definition of dimensionless time

$$t_D = \frac{.00026368\ (k/\mu)_t t}{\phi\ C_t X^2{}_F} \tag{1}$$

The definition of dimensionless pressure

$$P_D = \frac{kh\ \Delta P}{141.2\ q\mu B} = \frac{(k/\mu)_t\ h\Delta P}{141.2\ q_t B_t} \tag{2}$$

$$q_t B_t = q_o B_o + q_w B_w + \left(q_g - \frac{q_o R_{so} - q_w R_{sw}}{1000}\right) B_g \tag{3}$$

The basic equation for linear flow

$$P_D = \sqrt{\pi t_D} \tag{4}$$

We will take as the end of linearity

$$t_{D_{Le}} = .16 \text{(Ref. 3)} \tag{5}$$

with corresponding values P_{DLe}, t_{Le} and ΔP_{Le} for this time. We will define m_L as the slope of the square root of time linearity, psi/$\sqrt{\text{hours}}$ and t_{Le} as the hours at the end of linearity as shown in Fig. 1.

Rather than taking the pressure rise as $P_e - P_o$ [3,7] we will use

$$\Delta P = m_L \sqrt{t_{Le}} \tag{6}$$

This avoids using the early time pressure rise data which are frequently distorted by afterflow, phase readjustment, leaky valves in pumping wells [8] and perhaps other effects and utilizes the slope of the square root of t plot linearity, which should be a more reliable datum.

Substituting P_D and ΔP values in (2)

$$\sqrt{.16\pi} = \frac{(k/\mu)_t\ h m_L \sqrt{t_{Le}}}{141.2 q_t B_t}$$

$$(k/\mu)_t = \frac{100.11 q_t B_t}{h m_L \sqrt{t_{Le}}} \tag{7}$$

Substituting (6) in (1) at the end of linearity

$$X_F = .40618 \sqrt{\frac{q_t B_t \sqrt{t_{Le}}}{h\phi C_t m_L}} \tag{8}$$

$$k_o = \frac{100.11\ q_o \mu_o B_o}{hm_L\ \sqrt{t_{Le}}} \tag{9}$$

The k_o as derived above is the effective permeability to oil and in line with common usage in radial flow analysis would be regarded as k for use in kh, etc. This k is a function not only of rock properties but also of saturation conditions at the time it is measured. It would be expected to change rapidly and considerably during fillup of an element of a reservoir in a water flood, for example. To get the permeability of the rock - which is the permeability to a single phase (often referred to as the 'air permeability' in the laboratory) one requires estimates of the relative permeabilities. The single phase permeability is not a function of saturation and should remain the same in the reservoir from year to year except for factors such as compaction or clogging by particle movement.

Incidentally, calculation of C_t also requires that relative permeabilities be estimated since the saturation of each phase must be known. In our company's reports the relative permeabilities are estimated from published correlations, and C_t and single phase rock permeability are estimated. The calculation of relative permeabilities is beyond the scope of this paper.

THE 'END OF LINEARITY' ON THE SQUARE ROOT OF TIME PLOT.

Consider the flow lines for a uniform flux fracture, X_F, in an infinite reservoir. Next to the fracture they are equally spaced, perpendicular to the fracture - we have linear flow. At a distance large compared to X_F, the flow lines will have curved to become equally spaced and radial - we have radial flow. The change from linear flow to radial is gradual - there is, of course, really no point that corresponds to the 'end of linearity'. The deviation between the pressure build up for linear flow, $P_D = \sqrt{\pi t_D}$, and for buildup with a uniform flux fracture (given in Ref. 2, Table 1), is given in Table 2.

We have found that generally by the time the deviation of the curve from the linearity is 1.3% of ΔP (Eq(6)), it is a recognizable break from the linearity. The value 1.3% corresponds to t_D = .16 as given by Gringarten et al [3] and used in the derivation above. If, following Raghavan [7], a value of t_D = .19 is taken for the end of linearity, then a point on the curve which is 2% below linearity should be used as the end. Using a 2% deviation and t_D = .19 should give the same X_F as 1.3% with .16. This is in effect a 'linearity plus single point type curve' fit. From a practical viewpoint, since the gradual change from linear to radial flow and the irregularities of field data make the whole process give only an approximation of X_F, picking the 'end of linearity' does not usually cause a problem. The value of X_F is not sensitive to differences in deciding where the 'end of linearity' is.

This analysis assumed the fracture was short compared to the drainage radius. Gringarten et al [2], in their Fig. 3, show curves for the effect of boundaries on buildup with a uniform flux fracture. The effect is to cause the curve to rise substantially above the semilog tangent. We almost never see this, and they say (Ref. 3, p 888) "Virtually all field-test data have followed an infinite reservoir curve, $X_e/X_f = \infty$ ". We do not believe that the use of these infinite reservoir curves in the development significantly detracts from the usefulness of this method.

In our early work we always calculated and reported fracture lengths and permeabilities two ways: first for a uniform flux fracture and second for an infinite conductivity fracture. We soon stopped the infinite conductivity calculations because the results were not consistent with radial flow permeability and other information. No doubt the uniform flux concept is seldom met very exactly in the field, but there seems to be a rough balance between the tendency of flow lines to concentrate near the end of the fracture because of the higher pressure there, and the tendency of the limited flow capacity of the fracture to cause flow lines to concentrate near the well. In view of the approximate nature of the analysis, the uniform flux concept appears to be adequate for general application.

PICKING THE LINEARITY IN THE SQUARE ROOT OF TIME PLOT.

Because afterflow controls early time build up and radial flow dominates later times it is not surprising that many buildup curves on fractured wells do not show a well defined $\sqrt{t}$ linearity. Further, there are many $\sqrt{t}$ plots which start out with a low slope, rise rapidly and then flatten again. There is a straight line segment at the inflection point and this may or may not be the proper straight section for linear flow. Also, it should be noted that the $\sqrt{t}$ plot always straightens out at long times because the curvature of $P = P_o + m \log(t)$ on a $\sqrt{t}$ plot is approximately $-\frac{2\,m}{2.3t}$. If the linearity at long times or at the inflection point does not pass close to P_o at $t = o$, it is probably not the correct linearity.

It is often said that the type curve slope will be one half during linear flow. This is only true if the linearity passes through P_o at $t = o$. Errors in P_o caused by failure of the well to be at steady state when shut-in or other reasons, may cause the type curve half slope not to coincide with linear flow.

A very useful guide to locating the $\sqrt{t}$ and the log t linearity is given by Wattenbarger and Ramey [9] who found that in an infinite conductivity system the pressure at the start of log t linearity is twice that at the end of the $\sqrt{t}$ linearity. Looking at Figs. 3 and 8 in Ref. 2, the relationship appears to hold much better for a uniform flux fracture than for an infinite conductivity fracture. In practice, we sometimes run across cases for which the pressure separation of well defined $\sqrt{t}$ and log t linearities is much less than a factor of two. This is likely because the reservoir does not meet the criteria on which the equations are based: a single, uniform rock matrix filled with a single uniform fluid of low compressibility. Old pumping wells with a low producing bottom hole pressure are apt to have a highly compressible region near the well which will affect our analyses.

EXAMPLE OF A BUILDUP CURVE ON A PUMPING WELL

Figs. 1, 2 and 3 show the $\sqrt{t}$, semi-log and type curve plots; Table 3 gives the reservoir data and Table 4 the buildup for a pumping well in Lea County, New Mexico. The end of the $\sqrt{t}$ linearity was at 5.19 hrs, the slope of the $\sqrt{t}$ linearity was 31.1 psi/$\sqrt{\text{hour}}$, and the semilog slope was 144 psi/log cycle. Using values from Table 3 we have (neglecting the small amount of gas dissolved in the water):

From Eq 3

$$Q_t B_t = 15 * 1.025 + 5 * 1.003 + \left(22 - \frac{15 * 45.1}{1000}\right) * 17.76$$

$$= 15.4 + 5.0 + 379 = 399 \text{ res bbl/day}$$

From Eq 7

$$(k/\mu)_t = \frac{100.11 * 399}{25 * 31.1 * 2.3} = 22.3 \text{ md/cp}$$

From Eq 8

$$X_F = .40618 \sqrt{\frac{399 * 2.3}{25 * .19 * 2.22E\text{-}3 * 31.1}} = 21.5 \text{ feet}$$

Fracture length = $2X_F$ = 43 feet

From Eq 9

$$k_o = \frac{100.11 * 15 * 2.17 * 1.025}{2.5 * 31.1 * 2.3} = 1.87 \text{ md}$$

$$k_o/\mu_o = 1.87/2.17 = .86 \text{ md/cp}$$

Radial flow k_o from Fig. 2

$$k_o = \frac{162.6 * 15 * 2.17 * 1.025}{25 * 144} = 1.51,\ k_o/\mu_o = .69 \text{ md}$$

Fracture half length calculated from k_o/μ_o instead of k_t/μ_t(Eq 1):

$$X_F = \sqrt{\frac{.00026368 * .86 * 5.29}{.19 * 2.22E\text{-}3 * .16}} = 4.22 \text{ feet}$$

The pressure at the start of log t linearity is 164 psi and at the end of linearity on the $\sqrt{t}$ plot 86 psi giving a close check of the 'double ΔP rule' as Raghavan calls it, (Eq 11.11, Ref. 5).

Another check of the validity of our linearities is to compare the k_o's calculated from the fracture analysis ($\sqrt{t}$) and **from the** radial flow analysis (log t). The fracture analysis $k_o = 1.9$ is close check of the radial flow analysis $k_o = 1.5$, giving further confidence in the linearity picks.

CONCLUSIONS

1. Care must be taken to use $(k/\mu)_t$ rather than k_o/μ_o in equations involving diffusivity, with multiphase flow.

2. The slope and the end of linearity on a $\sqrt{t}$ plot, together with the relation $t_D = .16$ at the end, permit an estimate of X_F and k_o.

3. Of the pumping oil wells we have run about 30% have oil plus water flowing in the reservoir and about 60% oil plus water plus gas. Almost none have single phase flow.

NOMENCLATURE

B_g; formation volume factor, gas; res bbl/Mcf

B_o, B_w; formation volume factor, oil, water; res bbl/STB

C_t; total compressibility in the reservoir; 1/psi

h; formation thickness; feet

k; permeability; md

k_o, k_w, k_g; effective permeability to oil, water or gas; md

k_o/μ_o, k_w/μ_w, k_g/μ_g; mobility of oil, water, gas; md/cp

$(k/\mu)_t = k_o/\mu_o + k_w/\mu_w + k_g/\mu_g$; total mobility; md/cp

m; semi-log slope; psi/log cycle

m_L; $\sqrt{t}$ slope; psi/$\sqrt{\text{hours}}$

P; pressure; psia

P_D; dimensionless pressure

P_o; pressure at t = o; psia

ΔP; pressure change; psi

q_g; gas production rate, Mcf/D

q_o, q_w; oil, water production rate, STB/D

r; radial distance into reservoir; feet

R_{so}, R_{sw}; dissolved gas in oil, water; scf/STB

t; time; hours

t_D; dimensionless time

t_{Le}; time at end of $\sqrt{t}$ linearity; hours

t_{DLe}; dimensionless time at end of $\sqrt{t_D}$ linearity

X_F; half fracture length; feet

ϕ; porosity; fraction

μ; viscosity; cp

μ_o, μ_w, μ_g; viscosity of reservoir oil, water or gas, cp

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TABLE 1.

Frequency of occurrence of multiple phase flow in 500 successive well tests, including pumping oil wells, flowing oil wells, gas wells and water injection wells.

Phases Flowing in the Reservoir.	% of tests
o	6
w	7
g	2
o + w	22
o + g	10
w + g	0
o + w + g	53

TABLE 2.

Deviation of P_D from linear flow for a uniform flux fracture in an infinite reservoir.

$$t_D = \frac{.00026368\ (k/\mu)_t\ t}{\phi\ C_t\ X_F^{\ 2}}\ ; \qquad P_D = \frac{kh\ \Delta P}{141.2\ q\mu B}$$

t_D	$\sqrt{\pi t_D}$	P_D	$\frac{\sqrt{\pi t_D} - P_D}{P_D}$
0	0	0	0
.05	.3963	.3963	<.02%
.10	.5605	.5587	.32%
.15	.6865	.6790	1.10%
.16			1.30%
.19			1.95%
.2	.7927	.7756	2.20%
.3	.9708	.9261	4.83%
.5	1.2533	1.1355	10.37%

TABLE 3

Reservoir Data on a Pumping Well (see Table 4 and Fig. 1, 2 and 3)

Location: Lea Co., N.M.

Production Rates		
Oil, q_o	15	STB/D
Water, q_w	5	STB/D
Gas, q_g	22	Mcf/D
Cumulative oil production	136496	STB
Reservoir temperature	95	°F
Net pay thickness	25	Feet
Gas gravity	1.0806	Relative to air

Properties at avg. pressure from t = 0 to end of buildup, P = 147 psia

Formation Volume Factors		
Oil, B_o	1.025	Res bbl/STB
Water, B_w	1.003	Res bbl/STB
Gas, B_g	17.76	Res bbl/Mcf
Viscosity in the Reservoir		
Oil, μ_o	2.17	cp
Water, μ_w	.735	cp
Gas, μ_g	.0104	cp
Solubility of gas in reservoir oil	45.1	Scf/STB
Total compressibility, C_t	2.22E-3	1/psi
Slope from Fig. 1, m_L	31.1	psi/$\sqrt{\text{hour}}$
Slope from Fig. 2, m	144	psi/log cycle
End of linearity Fig. 1, T_{Le}	5.29	hours ($\sqrt{t_{Le}}$=2.3$\sqrt{\text{hours}}$)

TABLE 4

Buildup Curve on Pumping Well (see Table 3 and Figs. 1, 2 and 3)

t (hrs.)	$P-P_O = \Delta P$
0	$P_O = 42.33$
.1	4.97
.13	6.6
.17	8.24
.22	10.7
.25	12.4
.28	14.8
.32	17.0
.92	40.2
1.83	53.0
3.07	64.8
4.6	76.7
6.47	87.5
8.63	96.1
11.12	103.9
13.92	111.5
17.02	119.0
20.45	126.2
24.18	132.9
28.23	139.4
32.6	146
37.28	152.5
42.27	159.0
47.57	165.3
53.18	171.5
59.12	177.6
65.37	183.7
71.93	189.7
74.02	191.6
76.18	193.5
80.70	197.3
84.25	200.1
87.97	203.1
91.85	206.0
95.90	208.7

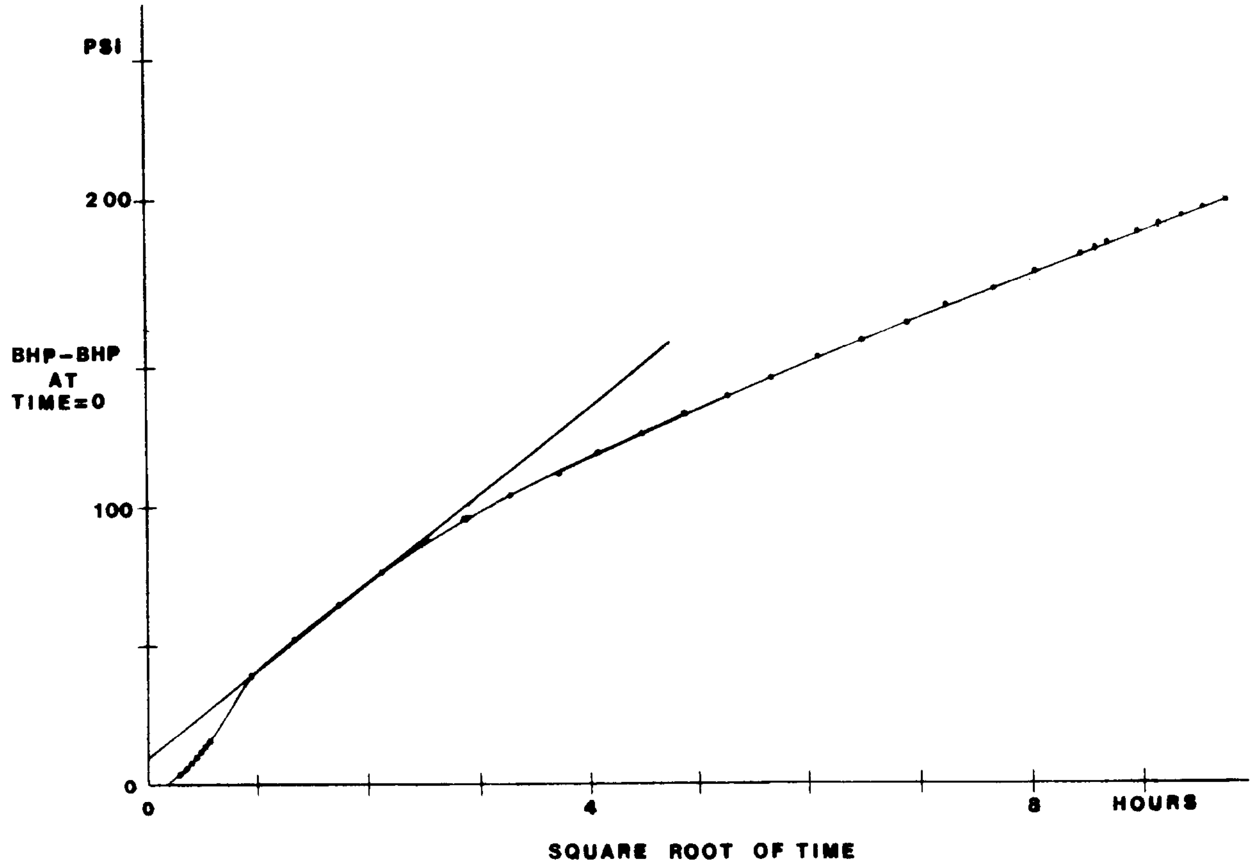

Fig. 1 Plot of $P-P_O$ vs. square root of time

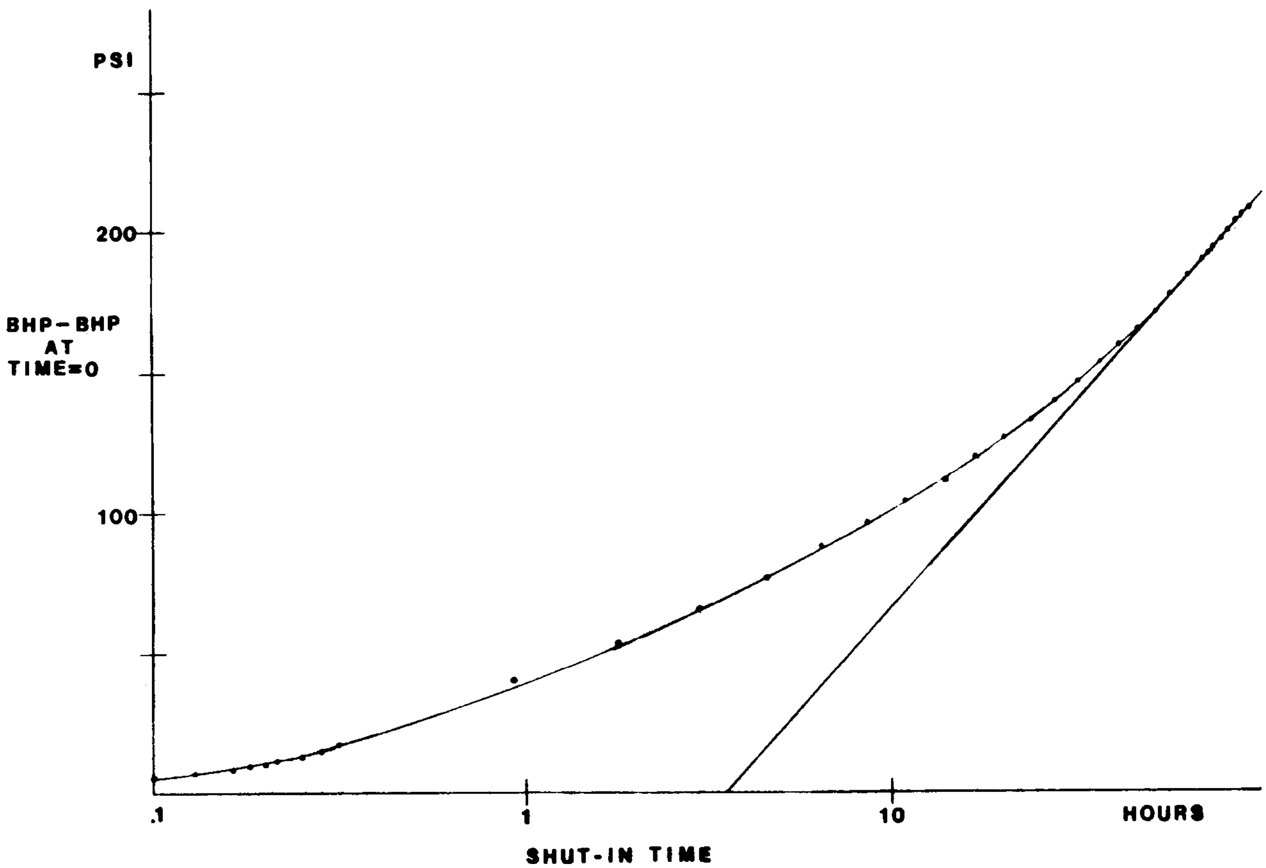

Fig. 2. Plot of $P-P_O$ vs. log (time)

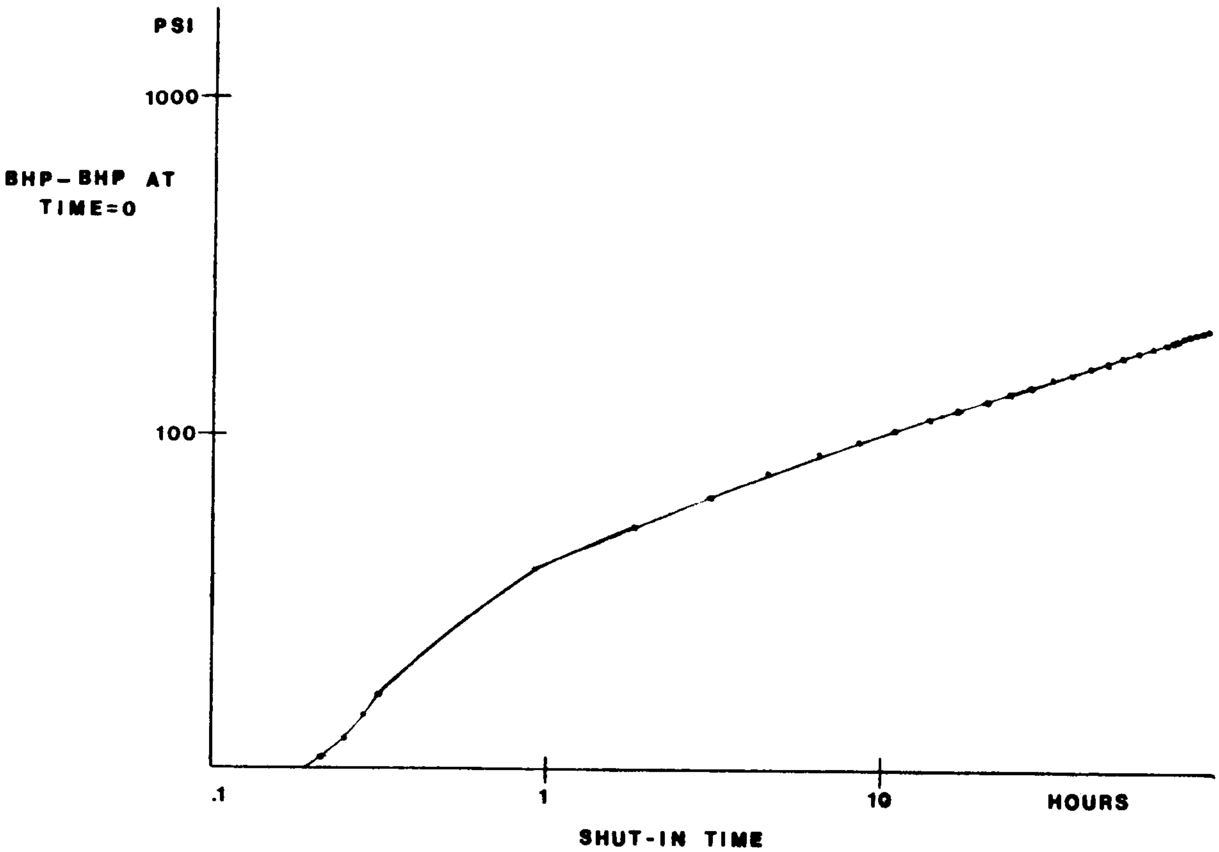

Fig. 3. Plot of log $(P-P_o)$ vs. log (time)