EULER LOADS AND MEASURED SUCKER ROD / SINKERBAR BUCKLING

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The Petroleum Industry is experiencing an increased awareness and understanding of sucker rod compression. This awareness has evolved during the past twelve years, from initial recognition of rod buckling, to a current desire to quantify (measure) the amount of compression required to initiate rod buckling.

Recent attention has focused on true or effective compressive loads in sucker rod strings. Measurement of these loads is being documented (1) and data collection is ongoing with improved technology.

This paper will provide a more accurate understanding of the amount of compression required to buckle sucker rods and sinkerbars of various diameters. This will be accomplished by presenting the following;

1. Predictive compressive loads that buckle various diameters and lengths of sucker rods and sinkerbars utilizing Euler loads.

2. Measured compressive loads that are required to buckle various diameter sucker rods and sinkerbars.

3. A comparison of predictive compressive loads to measured compressive loads.

Knowledge regarding the amount of rod string compression required to buckle various diameter sucker rods will provide the industry better rod string design guidelines.

Use of these guidelines will help identify dangerous compressive rod string loads which initiate rod-tubing contact, provide associated wear, and result in rod and/or tubing failure.

Euler Load (Critical Load, Pcr)

The famous mathematician Leonhard Euler (1707-1783), although hindered by loss of sight in one of his eyes in 1735 and the other in 1766, was the first to investigate buckling of slender columns. His investigations led to the determination of the critical load (Pcr) required to buckle an ideal, elastic column.

An ideal column is assumed to be perfectly straight and compressed by a centrally applied load (a load acting through the centroid of the cross section). An elastic column is assumed to reach stresses below the proportional limit before the critical load is reached.

The phenomenon of axial compression can be described by one of three (3) forms of equilibrium: stable, neutral or unstable buckling.

1. If the axial load (P) is less than the critical load (Pcr), the column remains straight and undergoes only axial compression. This straight form of equilibrium is called stable, which means that, if a lateral force is applied and a small deflection is produced, the deflection will disappear, and the column will return to its straight form when the lateral force is removed.

2. As the axial load (P) is gradually increased, a condition of neutral equilibrium is reached as the axial load (P) equals the critical load (Pcr). At this load the column theoretically may have any small value of lateral deflection. Under these conditions a small lateral force will produce a deflection which does not disappear when the lateral force is removed.

3. At higher load values, equal to or greater than the critical load, (Pcr), the column is unstable and will collapse. This phenomenon of instability is called buckling and we may consider that the column buckles, or becomes unstable, at the critical load (Pcr).

The critical load (Pcr), for an ideal, elastic column is often called the Euler Load. This load is defined as follows;

$$Pcr = \frac{(3.1416)^2 \times (E) \times (I)}{4(L)^2}$$

E = Modulus of Elasticity, for steel use 30.5×10^6 psi

I = Moment of Inertia, $(\text{inches})^4$

for a circle; $I = \frac{(3.1416) \times (D)^4}{64}$

L = Length of column, (inches)

The critical load (Pcr), of a slender column is directly proportional to the Modulus of Elasticity (E) and the Moment of Inertia (I), inversely proportional to the square of length of the column (L) and independent of the compressive strength of the material.

The critical load (Pcr), can be increased by increasing the Moment of Inertia (I) of the cross section or by decreasing the length of the slender column. For sucker rods or sinkerbars, the Moment of Inertia (I) can be increased by increasing the diameter of the rod or sinkerbar.

Other methods of calculation of the critical load (Pcr) or Euler Load can be developed based on the configuration of the ends of the slender column. Two (2) different ways of defining the end configuration are hinged ends or fixed ends. Critical loads for these two (2) configurations are as follows;

$$Pcr\ (hinged) = \frac{(3.1416)^2 \times (E) \times (I)}{(L)^2}$$

$$Pcr\ (fixed) = \frac{4 \times (3.1416)^2 \times (E) \times (I)}{(L)^2}$$

$$Pcr\ (fixed) = 4 \times (\ Pcr\ (hinged)\ )$$

Critical loads for hinged and fixed configurations are presented in figure 1.0 and 2.0, respectively. The critical load (Pcr) for a fixed end configuration is four (4) times greater than the critical load of a hinged end configuration.

Assuming these two (2) end configurations (fixed and/or hinged) in a slender column represent the extremes of end configurations in a sucker rod string, Table 1.0 presents calculated Euler Loads for various diameters of sucker rods and sinkerbars, to predict the fixed and hinged end critical loads.

Measured Buckling Loads

A test apparatus was constructed to measure the buckling load required to buckle various 25 foot long sections of sucker rods and sinkerbars. Refer to Figure 3.0

A 30' foot long joint of 7.0" O.D., 6.456" I.D., J-55, 20.0 lbs./ft., casing was modified by cutting a 5.0" wide windows the length of the casing. This joint of casing was secured in a vertical position. The top of the 7.0" O.D. casing was welded shut with a cap containing a centralizing cup on the inside of the casing cap to accept the pin end of a sucker rod or a sinkerbar.

The bottom of the 7.0" O.D. casing was modified to accept a 2.5" diameter, hydraulically actuated plunger. The top of the plunger, was fabricated with a centralizing cup to accept the pin end of a sucker rod or sinkerbar. Both centralizing cups were 1.77" I.D., and 2.0" deep.

Several buckling stresses (psi) were recorded from pressure gauge readings for each tested size sucker rod and sinkerbar. Extraneous gauge readings were eliminated, while the remaining gauge readings were averaged to arrive at an average stress (psi) reading and then converted to average Pcr (lbs.) depending on the diameter of sucker rod or sinkerbar. Measured plunger friction (lbs.) and weight of rod/sinkerbar in air (lbs.) was then subtracted to arrive at measured buckling load (lbs.).

The total weight of the sucker rods or sinkerbars in air, (lbs.) was removed to be consistent with the derivation of Euler Loads that does not consider the weight of the column.

Refer to table 2.0 for a comparison of calculated Euler Loads and Measured Buckling Loads vs. various sucker rod and sinkerbar diameters.

Measured Buckling Loads vs. Euler Loads (Critical Loads, Pcr)

The purpose of this paper is as follows;

1. To provide a more accurate understanding of the amount of compression required to buckle various diameter sucker rods and sinkerbars.

2. To investigate the possible use of the Euler Load Equation to estimate buckling loads and provide the Petroleum Industry with better guidelines to rod string designs.

Predicted Euler Loads and measured buckling loads for various diameter sucker rods and sinkerbars are presented in graph 1.0.

An interesting observation of the presented data is that as the pin diameter increased, the measured data curve tended to move away from the hinged end curve and closer to the fixed end curve.

This increase in Euler Load occurred in every case except that for the 1-3/8" sinkerbar. The measured data for the 1-3/8" sinkerbar more closely approximately that of the hinged end curve. It should be noted that pin on the 1-3/8" sinkerbar was a modified 3/4" diameter sucker rod pin, the same diameter as the 3/4" sucker rod.

Graph 2.0 incorporates all of the data of graph 1.0 with the addition of calculated Euler Load for 1/2", 5/8" diameter sucker rods and 1-1/2", 1-5/8", and 1-3/4" diameter sinkerbars. This graph provides a better perspective of the range of Euler Loads with various diameter sucker rods and sinkerbars.

Figures 9.0 through 12.0 provide Euler Loads for various lengths of sucker rods and sinkerbars. As in graphs 1.0 and 2.0, both hinged end and fixed end configurations are plotted to provide the reader a range of Euler Loads to consider in designing with sucker rods and sinkerbars.

Conclusions

1. Measured buckling loads, (except for the 3/4" rod) fall within the envelope of Euler Loads for fixed and hinged end configurations.

2. The size of the pins may have had an impact on the observed values of measured buckling loads.

3. Measured loads required which initiate buckling may be significantly less than those loads currently being documented by the Petroleum Industry (1).

4. The Euler Load (Critical Load, Pcr) Equations for fixed and hinged end configurations should be utilized as boundary conditions for estimating the amount of rod string compression required to buckle sucker rods or sinkerbars. To be conservative, use of only the hinged end configuration curve should be considered.

5. The relation between the pin diameter and the fixed or hinged end Euler Load (Pcr) calculation deserve further investigation.

6. The effects of greater than 25-foot long columns, off-center loading within the constraints of tubing and both full hole and slim hole rod couplings on Euler Loads should also be considered for future measured study.

References

(1) "Making the most of rod design - a comparison of measured and modeled rod string stresses emphasizing fiberglass rods" - R.E. Ott and G.L. Mendenhall

"Mechanics of Materials" - S.P. Timoshenko and James M. Gere, pp. 323-331

Table 1 - Euler Loads to Buckle Various Diameters

Euler Loads to Buckle Various Diameters

Rod and Sinkerbar Diameters	Calculated Euler Loads	
	Fixed	Hinged
1/2"	41 lbs	10 lbs
5/8"	100 lbs	25 lbs
3/4"	208 lbs	52 lbs.
7/8"	385 lbs	96 lbs.
1.0"	657 lbs	164 lbs.
1-3/8"	2,348 lbs.	587 lbs.
1-1/2"	3,325 lbs	831 lbs
1-5/8"	4,579 lbs.	1,145 lbs.
1-3/4"	6,160 lbs	1,540 lbs

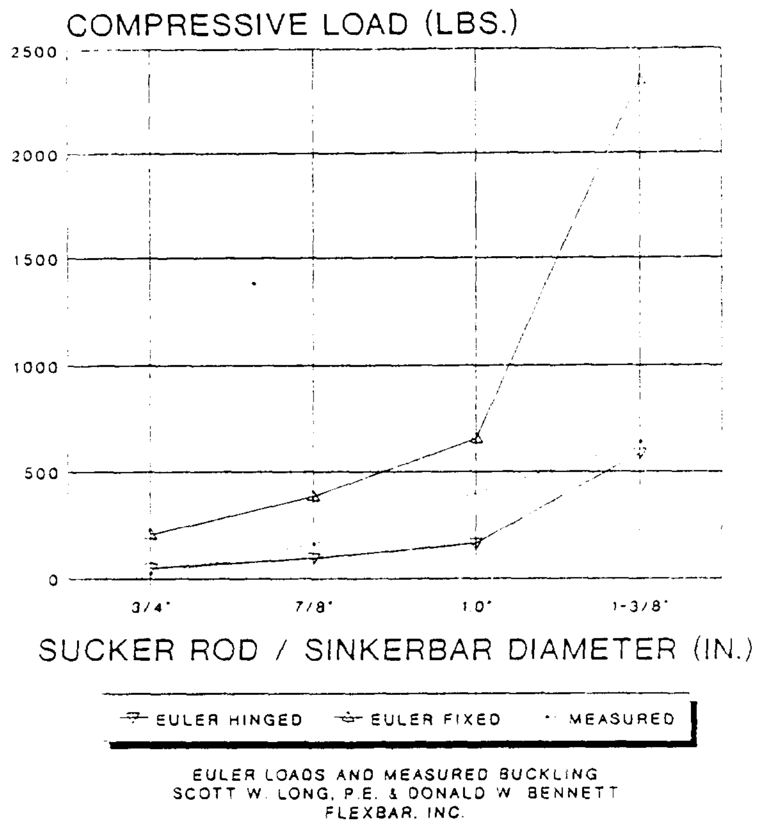
(All data based on 25-foot long sucker rods or sinkerbars with the total weight of sucker rods or sinkerbars in air, (lbs.) removed from calculation of measured loads to be consistent with the calculation of Euler Loads that does not consider the weight of the column)

Table 2 - Euler Loads and Measured Buckling Loads to Buckle Various Diameters

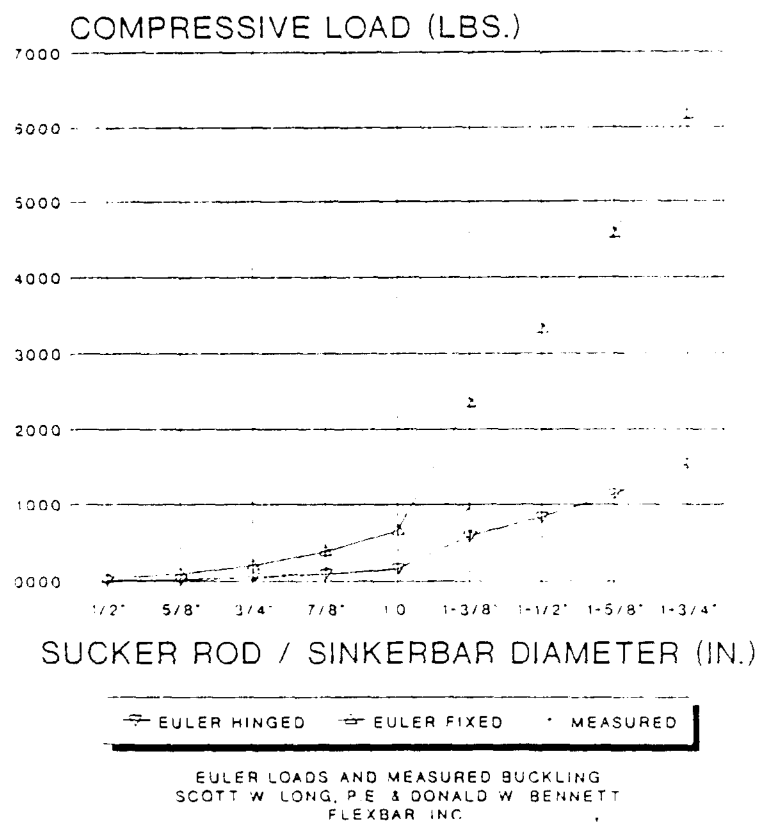
Euler Loads and Measured Buckling Loads to Buckle Various Diameters

Rod and Sinkerbar Diameters	Calculated Euler Loads		Measured Buckling Loads
	Fixed	Hinged	
1/2"	41 lbs.	10 lbs.	N.A.
5/8"	100 lbs.	25 lbs.	N.A.
3/4"	208 lbs.	52 lbs.	23 lbs.
7/8"	385 lbs.	96 lbs.	162 lbs
1.0"	657 lbs.	164 lbs.	N.A.
1-3/8"	2,348 lbs.	587 lbs.	641 lbs.
1-1/2"	3,325 lbs.	831 lbs.	N.A.
1-5/8"	4,579 lbs.	1,145 lbs.	N.A.
1-3/4"	6,160 lbs.	1,540 lbs.	N.A.

(All data based on 25-foot long sucker rods or sinkerbars with the total weight of sucker rods or sinkerbars in air, (lbs.) removed from calculation of measured loads to be consistent with the calculation of Euler Loads that does not consider the weight of the column)



Graph 1 - Euler & Measured Buckling
Loads for 25' Long Columns



Graph 2 - Euler & Measured Buckling
Loads for 25' Long Columns

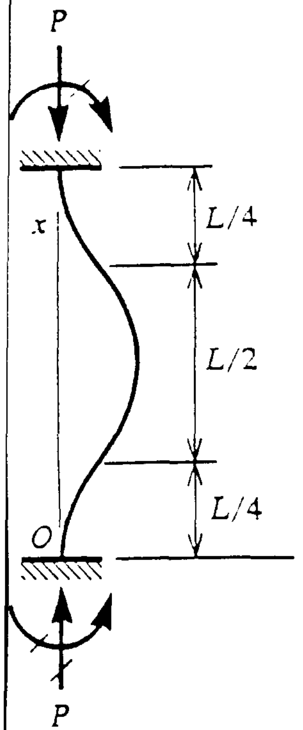


Figure 1

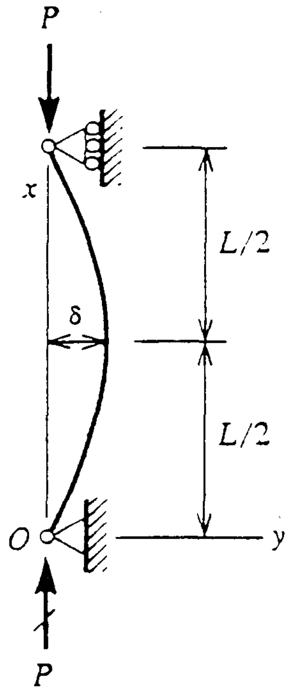


Figure 2

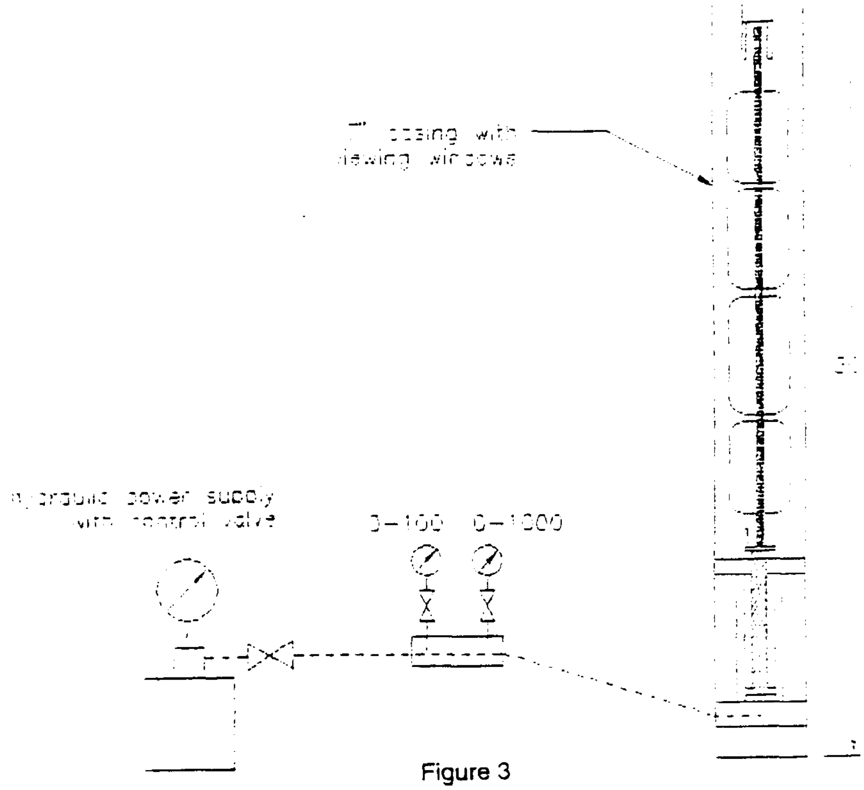


Figure 3

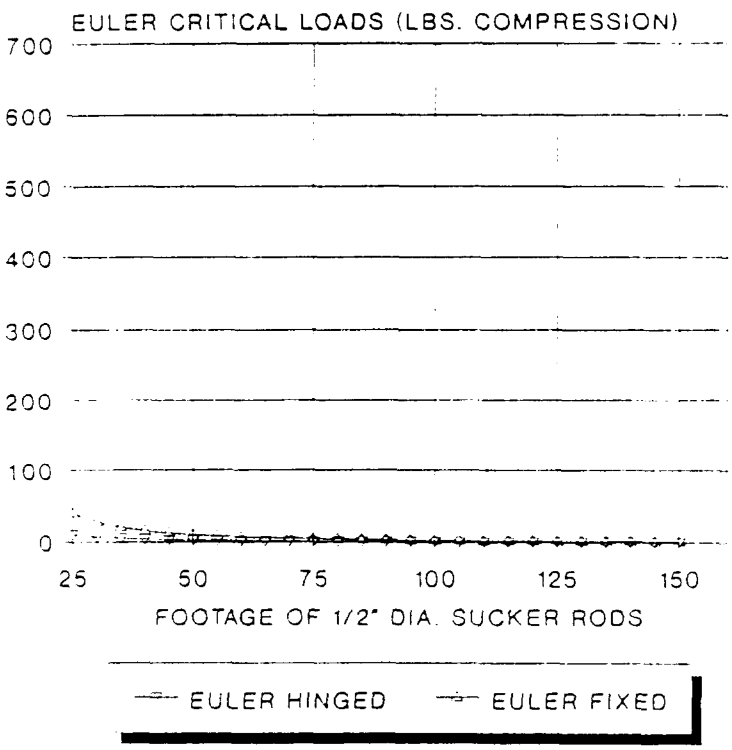


Figure 4 - Euler Critical Loads vs. Length of 1/2" Dia. Sucker Rods

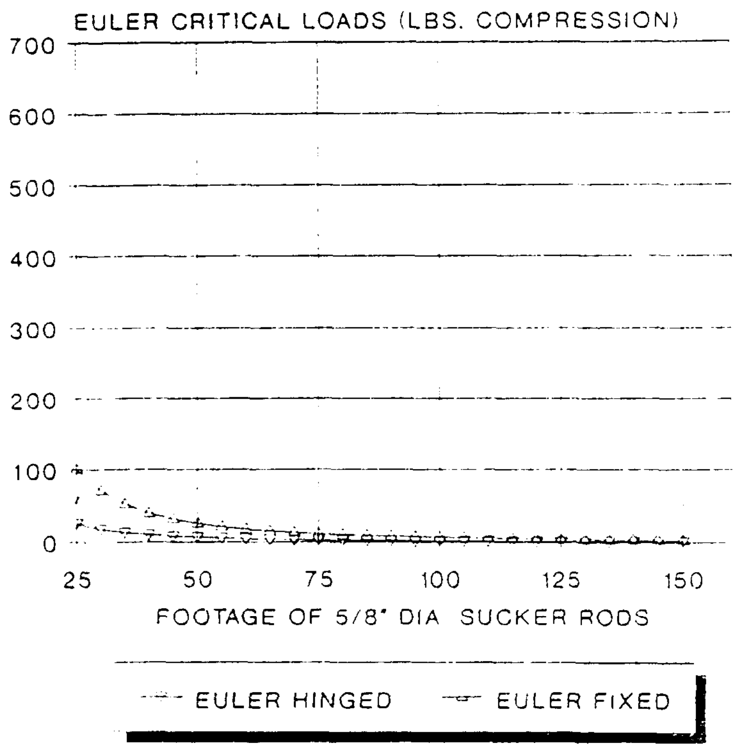


Figure 5 - Euler Critical Loads vs. Length of 5/8" Dia. Sucker Rods

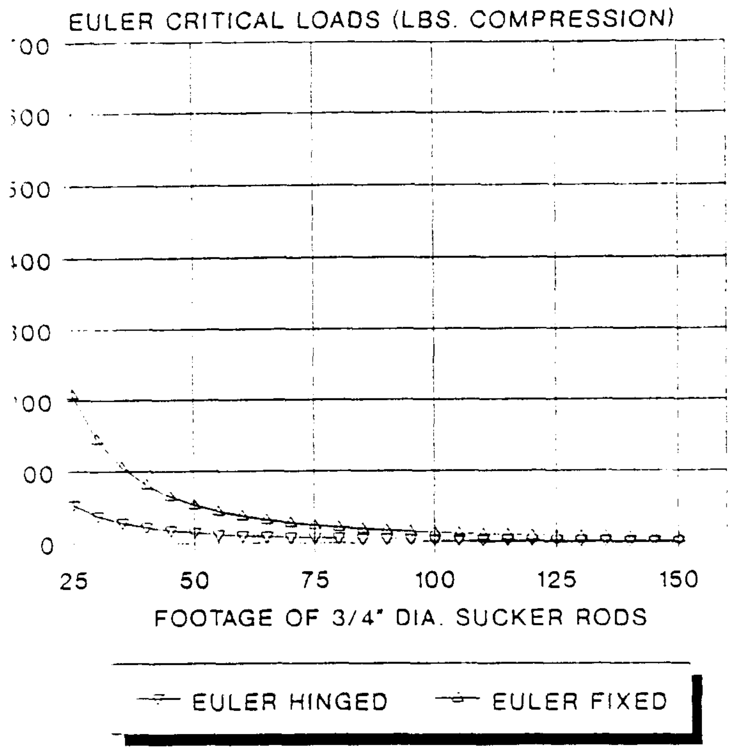


Figure 6 - Euler Critical Loads vs.
Length of 3/4" Dia. Sucker Rods

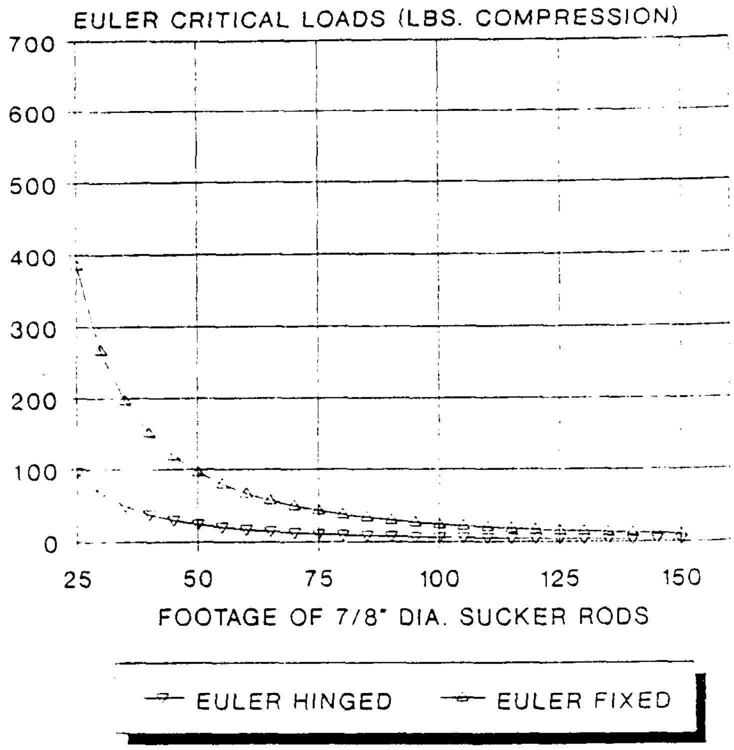


Figure 7 - Euler Critical Loads vs.
Length of 7/8" Dia. Sucker Rods

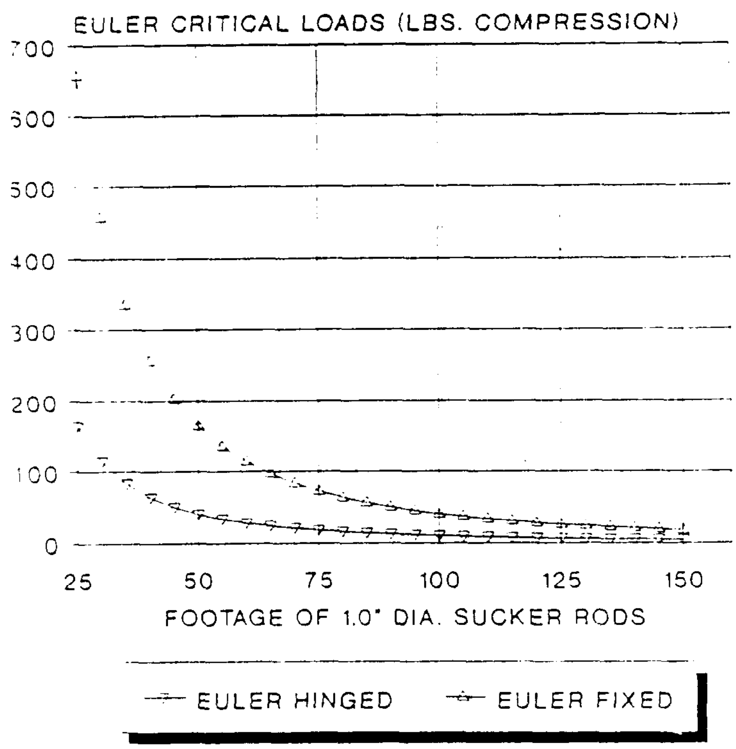


Figure 8 - Euler Critical Loads vs.
Length of 1.0" Dia. Sucker Rods

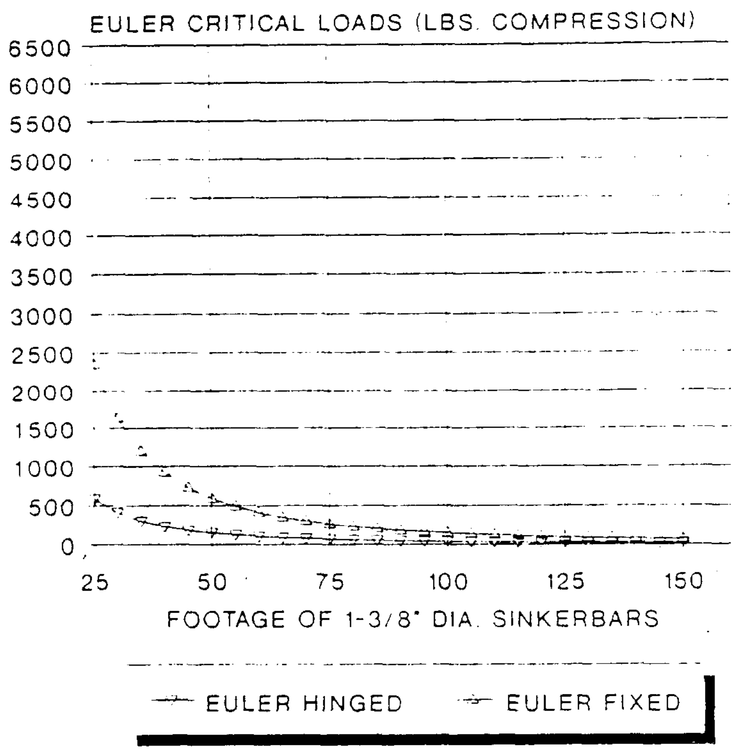


Figure 9 - Euler Critical Loads vs.
Length of 1-3/8" Dia. Sinkerbars

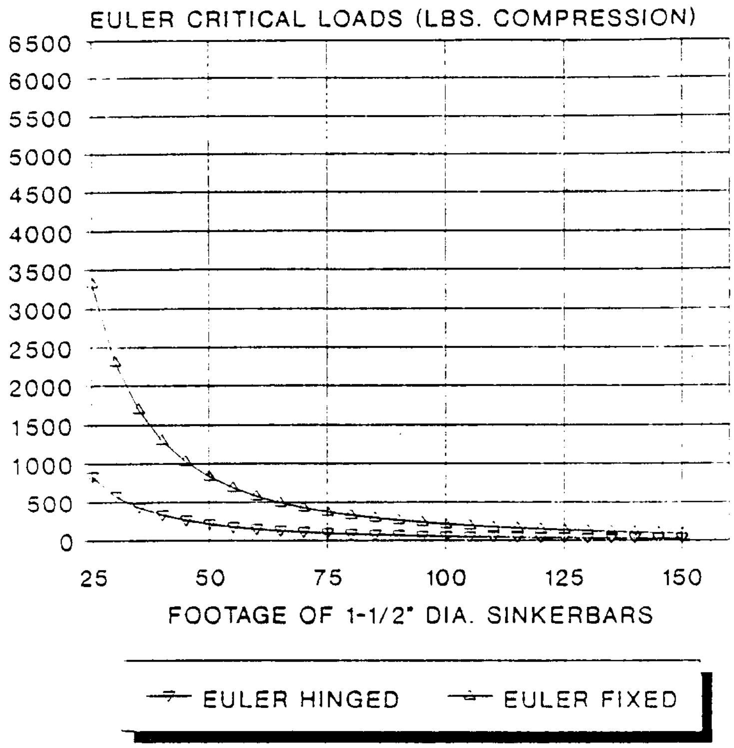


Figure 10 - Euler Critical Loads vs. Length of 1-1/2" Dia. Sinkerbars

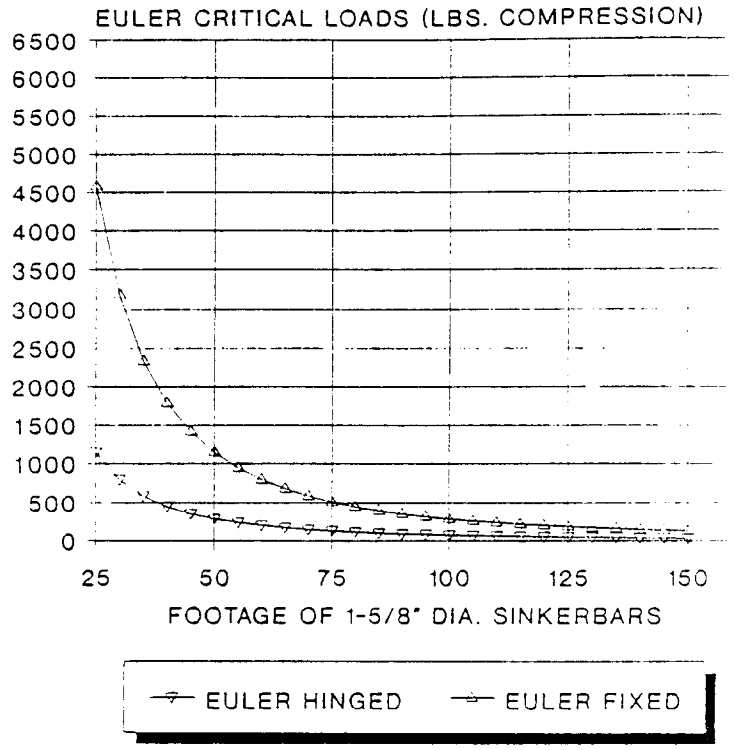


Figure 11 - Euler Critical Loads vs. Lenght of 1-5/8" Dia. Sinkerbars

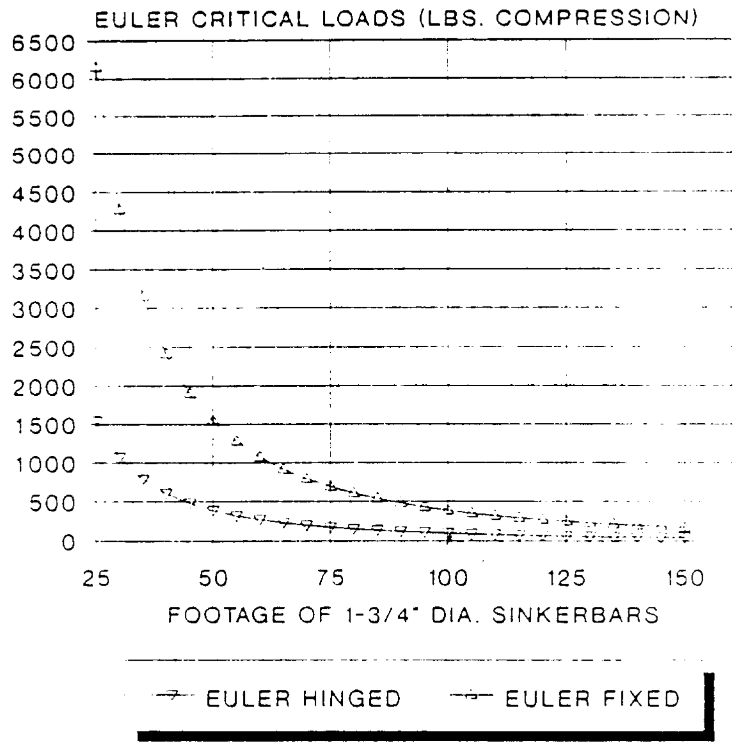


Figure 12 - Euler Critical Loads vs. Length of 1-3/4" Dia. Sinkerbars