WELL INFLOW ANALYSIS USING RESERVOIR DECONVOLUTION OF PRESSURE TRANSIENT TESTS

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ABSTRACT

Pressure transient tests are commonly used in production engineering to estimate reservoir pressure, well productivity, permeability and skin damage. In reservoir engineering, transient tests are also used to estimate the distribution of reservoir properties and presence of boundaries. During the last several decades deconvolution methods have been developed to remove wellbore effects and better estimate reservoir parameters using transient tests, but applications for production evaluation have been scarce.

In this paper deconvolution of pressure transient data is used to remove reservoir effects and determine inflow rates and wellbore fluid volumes, providing a better understanding of wellbore phenomena of interest in well maintenance, production engineering and artificial lift evaluation. The basic mathematical background is presented, along with real examples showing how the results add insight into understanding wellbore performance, including the evaluation of pump-off controller or timer settings and wellbore integrity.

INTRODUCTION

It has long been recognized that rate variations have an important effect on pressure transient behavior in wells and convolution techniques have been used to evaluate and represent the effects of variable rate on pressure transient tests. Examples of analytical and semi-analytical applications of convolution methods are presented in Van Everdingen and Hurst (1949), Agarwal, et.al. (1970) and W. Fair (1996) among others.

The use of deconvolution techniques in well testing is also well documented. Most references in the literature concentrate on removing variable flow rate effects from pressure transient data to better characterize reservoir response, in effect giving a longer time period for reservoir property analysis. On the other hand P. S. Fair and Simmons (1992) showed that deconvolution could also be used to remove reservoir effects from the early time response and thereby evaluate the rate in cases where it is not directly measured. In their paper they showed an example where a downhole flow meter was evaluated using deconvolution to generate the rates for comparison with the flow meter measurements and mentioned the possibility of evaluating variable storage effects.

In this paper the focus is not on reducing wellbore effects, but on evaluating wellbore effects. It is assumed that the reservoir acts infinitely at relatively short times and that the correct reservoir model is known after a pressure transient test has been evaluated conventionally. Under those conditions, it is possible to use the same deconvolution technique shown by P. S. Fair and Simmons (1992) to determine the wellbore performance from the measured transient data. Essentially deconvolution is used to remove the reservoir effects, giving an estimate of the bottomhole flow rate vs. time. Once the inflow rate is known, straightforward integration can be used to determine the change in wellbore fluid volume versus time, as well as versus the measured pressure. If anullar fluid level data is also available, the evaluation of fluid inflow volume versus fluid level can also provide insight into wellbore performance. As shown in the examples, this can provide a means to evaluate wellbore integrity and artificial lift efficiency independent of other methods.

It should be noted that this paper reports the status of an ongoing investigation, so the results are still preliminary. As discussed below, there are limitations in the application of the techniques in regards to wells with negative skin.

Further work needs to be done to remove that limitation, quantitatively determine the sensitivity to reservoir models and data precision and also implement higher order numerical schemes.

DECONVOLUTION METHOD

Numerical deconvolution techniques are well documented in the literature. Cinar, et al (2006) present a summary of the most important deconvolution methods currently in use, while Von Schroeter, et al. (2001) presents a comprehensive review of numerical deconvolution algorithms. Specific algorithms are presented by Roumboutsos and Stewart (1988), Mendes (1989), P. S. Fair and Simmons (1992) and others. As shown, there is a tendency to use numerical Laplace transforms to convert well test pressure and rate data to Laplace space where a simple division is equivalent to deconvolution. Afterwards the resulting transformed data is numerically inverted to determine the real deconvolved pressure or rate. As discussed in the references, this technique has both advantages and disadvantages.

The convolution integral normally used in pressure transient analysis is shown in Equation 1 in dimensionless variables, where p_{wD} is the wellbore pressure variation, p_D represents the reservoir pressure response to a unit change in flow rate, q_{wD} is the rate relative to a constant rate, and S represents the pressure loss due to near wellbore skin effect. Note that the derivative of rate or of pressure can equivalently be used in the convolution.

$$p_{wD}(t_D) = \int_0^{t_D} q_{wD}(\tau)\frac{dp_D}{dt_D}(t_D-\tau)d\tau + Sq_{wD} = \int_0^{t_D}\frac{dq_{wD}}{dt_D}(\tau)p_D(t_D-\tau)d\tau + Sq_{wD} \qquad (1)$$

Deconvolution, on the other hand, is the inverse procedure. In most reservoir applications, we are given the measured pressure data and the rate variations as a function of time and we need to determine the ideal constant rate reservoir response. In other references the Laplace transform of the convolution integral is taken yielding Equation 2. As can be seen, if p_{wD} is measured and p_D and S are known, then it is possible to solve for q_{wD} as in Equation 3, but the procedure involves taking numerical Laplace transforms and then the inverse Laplace transform of the numerical data.

$$L\{p_{wD}\} = sL\{q_{wD}\}L\{p_D\} + SL\{q_{wD}\} \qquad (2)$$

$$L\{p_D\} = \frac{L\{p_{wD}\} - SL\{q_{wD}\}}{sL\{q_{wD}\}} \qquad (3)$$

In this present work we state the same deconvolution problem slightly differently. We have the measured pressure response and assume we know the ideal constant rate reservoir response as a result of the pressure transient analysis and we want to determine the rate variations required to yield the observed pressures. While it would be possible to solve (2) for the rate, in this case it appears that a simpler procedure is possible by directly evaluating the convolution integral and solving for the unknown rate at each measured time.

Using the second form of convolution in (1) at dimensionless time t_{Dn} as shown in (4) and applying the trapezoidal rule for numerical integration we can write Equation 5.

$$p_{wDn} = p_{wD}(t_{Dn}) = \int_0^{t_{Dn}}\frac{dq_{wD}}{dt_D}(\tau)p_D(t_{Dn}-\tau)d\tau + Sq_{wD}(t_{Dn}) \qquad (4)$$

$$p_{wDn} - Sq_{wDn} = \sum_{i=1}^{n}\left\{\left.\frac{dq_{wD}}{dt_D}\right|_{t_{Di-1/2}}\left(\frac{1}{2}\right)\left[p_D(t_{Dn}-t_{Di-1}) + p_D(t_{Dn}-t_{Di})\right](t_{Di}-t_{Di-1})\right\} \qquad (5)$$

The rate derivative can be approximated as a difference quotient as shown in Equation 6.

$$\left.\frac{dq_{wD}}{dt_D}\right|_{t_{Di-1/2}} \approx \frac{q_{wDi} - q_{wDi-1}}{t_{Di} - t_{Di-1}} \tag{6}$$

Substituting (6) into (5) and rearranging yields Equation 7.

$$\begin{aligned} p_{wDn} - Sq_{wDn} = \; & \frac{1}{2} q_{wDn} \left[p_D (t_{Dn} - t_{Dn}) + p_D (t_{Dn} - t_{Dn-1}) \right] \\ & - \frac{1}{2} q_{wDn-1} \left[p_D (t_{Dn} - t_{Dn}) + p_D (t_{Dn} - t_{Dn-1}) \right] \\ & + \frac{1}{2} \sum_{i=1}^{n-1} \left\{ (q_{wDi} - q_{wDi-1}) \left[p_D (t_{Dn} - t_{Di}) + p_D (t_{Dn} - t_{Di-1}) \right] \right\} + p_{wDn} \end{aligned} \tag{7}$$

Noting that $p_D(t_{Dn} - t_{Dn}) = p_D(0) = 0$ and solving for q_{wDn} yields the desired formula shown in Equation 8.

$$q_{wDn} = \frac{2p_{wDn} + q_{wDn-1} p_D (t_{Dn} - t_{Dn-1}) - \sum_{i=1}^{n-1} \left\{ (q_{wDi} - q_{wDi-1}) \left[p_D (t_{Dn} - t_{Di}) + p_D (t_{Dn} - t_{Di-1}) \right] \right\}}{p_D (t_{Dn} - t_{Dn-1}) + 2S} \tag{8}$$

Since in (8) q_{wDn} depends only on previous time values of q_{wD}, the rate at time t_n can be solved explicitly in a straightforward manner. Once the rates have been determined, the cumulative volume can also be estimated by numerically integrating the rate versus time. Using the trapezoidal rule yields the desired formula in dimensional variables shown in Equation 9.

$$V_{wn} = \sum_{i=1}^{n} \frac{1}{2} \left[(q_{wn} + q_{wn-1})(t_n - t_{n-1}) \right] \tag{9}$$

LIMITATIONS

Although the principle of deconvolution and the equations and procedures presented here are general, there are some theoretical and practical limitations. It is apparent that the data must be of high enough precision to allow meaningful estimates to be computed. Failure to measure enough data points, especially at early times, will make the numerical integration procedures inaccurate.

Since an inherent assumption in the method is that the reservoir model is known, it seems obvious that the use of an incorrect model (dual porosity, fractured, radial, etc.) would yield meaningless results. Fortunately, however, nearly all of the common radial flow models yield the same early time behavior, so the errors may not be significant. It is important that the correct well inflow model is used, for example a radial flow (pressure changes with the logarithm of time or exponential integral) versus a hydraulically fractured well (pressure changes proportional to the square root or fourth root of time).

There is another subtle problem in applying the technique in wells with a negative skin. In the case of a negative skin, a singularity in the pressure may occur due to subtracting a pressure drop near the wellbore. The normal means to evaluate wellbore effects in the presence of a negative skin is to use a wellbore radius larger than real to compute less pressure drop than expected. However it is also well known that wellbore storage and skin are highly correlated, so the use of a negative skin implies no wellbore storage, so valid inflow rates are not obtained. A solution to this problem is currently under investigation.

ALGORITHM VALIDATION

To validate the algorithm presented above, the analysis of a synthetic drawdown test is presented in Figure 1. Since the input parameters were all known, the deconvolution algorithm was applied to the synthetic data set and the inflow rate was computed as shown in the figure. Since the data is synthetic, the actual inflow rate was also computed from the synthetic data as shown. Note that the apparent rate oscillation is a figment of the integration algorithm, since an over-estimated rate in one point is compensated by an under-estimated rate in the next point, because the convolution integral is forced to match the measured pressure points. Although not shown here, more closer spaced pressure measurements would help to minimize this effect.

Note that the overall trend and long term behavior is adequately represented by the deconvolution algorithm, but the tendency appears to somewhat over-estimate the short time rates.

ANALYSIS PROCEDURE

Using the deconvolution formula in (8) the application of the technique to evaluate buildup and drawdown well tests is straightforward. The first step is to perform a normal interpretation to determine the reservoir and well model as well as permeability and wellbore skin. Once the interpretation has been completed, it is then assumed that the reservoir model and reservoir and skin parameters are correct and the inferred model is used to calculate p_D and p_{wD} for all of the measured pressure points. Equation 8 is then applied to all of the measured pressured versus time points to compute the dimensionless rates, which are then multiplied by the given constant rate to determine actual well bottomhole flow rates. Finally, Equation 9 can be applied to determine the changes in wellbore fluid volumes during the well test. A summary of the recommended analysis procedure is as follows. Note that the first 3 steps are the same as for any well test interpretation.

1. Quality control check all pressure and well data.
2. Attempt a standard type curve match to determine the most appropriate well and reservoir model.
3. Use standard nonlinear regression techniques to determine the applicable reservoir and skin parameters.
4. Assume that the reservoir and skin parameters are correct and calculate the inflow rate for each measured pressure data point.
5. Using the computed rates, compute the wellbore fluid volume change for each point.
6. Evaluate the rate and volume versus time, pressure and fluid level to infer wellbore performance.

EXAMPLE WELL TESTS

EXAMPLE 1

Figure 2 shows the interpreted buildup test for an example well. As can be seen, the buildup test was fit with a model representing radial flow, storage and skin. Note that the semilog line does not appear to have been reached at the end of the test, but the regression curve fit appears to be quite good. This test was recorded using acoustic equipment, so the recorded fluid level is also available for comparison with the wellbore volume estimates.

Using the estimated rates as shown, the wellbore volume was calculated. Figure 3 shows the relationships of wellbore storage volume with respect to time, pressure and fluid level. Note that the volume versus pressure trend appears nearly perfectly linear, with a slight change in slope at around 100 psi pressure change. The fluid level versus wellbore volume also appears to be linear, indicating no anomalous behavior. From the wellbore volume versus time, the initial trend is linear bending over to a lower slope. Since the higher slope indicates a higher inflow rate, the volume versus time graph could be used to estimate an optimum shut-in time for a pump timer. When the rate starts to slow down, consideration could be given to start the pump, but before that time the reservoir inflow rate has not substantially changed.

EXAMPLE 2

The second example is a well test taken from Rosa and Horne (1900) and is presented in Figure 4. As can be seen, the test indicates a small amount of phase segregation causing a slight dip in the derivative curve. The computed rate is shown to decline steady, then abruptly level off. The wellbore volume also shows the same behavior, with an increasing wellbore volume that abruptly stabilizes. Since no further information is available, the cause of this behavior cannot be determined.

This example shows the application to a well test measured with a downhole gauge.

EXAMPLE 3

Figure 5 shows the analysis of a well test on a high rate well affected by momentum of the wellbore fluids. When the well was shut-in, the fluids continued to move due to inertia and the pressure rose slightly above the reservoir pressure level, then fell back to reach equilibrium. Note that the wellbore volume shows a sharp increase followed by a slower decrease as the fluids approached equilibrium. The volume versus pressure graph illustrates this and the storage value is impossible to estimate due to the momentum effects.

EXAMPLE 4

Figure 6 presents a pressure buildup test measured acoustically, so that fluid level data was available. As can be seen, type curve fit appears normal with storage and skin indicated. The wellbore volume variation, however, shows an anomalous decrease in volume after a sharp increase, so it is apparent from the deconvolution that something else is also happening. The volume versus pressure shows that the wellbore storage appears to be constant initially, as shown by the linear trend, but at longer times the wellbore fluid volume decreases almost independently of pressure. The fluid level also appears to reach a stable value and remain constant. For true wellbore storage the volume should remain linear with pressure while the fluid volume change should be accompanied by a continuing rise in the fluid level.

It appears that in this well there is a thief zone or a mechanical leak, so that fluids entering the wellbore from the reservoir are somehow being lost. It is also apparent that something affected the test at around 15 to 20 hours of shut-in time. Further investigation of the mechanical integrity of the well is warranted. In the event that the mechanical integrity is intact, then it appears that the optimum shut-in time for a pump-off timer would be several hours, since the maximum volume from the reservoir has accumulated in the wellbore when the pressure rise reaches just over 300 psi or the fluid level stabilizes at about 600 ft. For longer shut-in times, the volume in the wellbore does not increase and fluids may be lost to either a leak or a thief zone.

CONCLUSIONS AND RECOMMENDATIONS

As a result of the deconvolution techniques presented in this paper, the following conclusions can be made:

- Deconvolution to determine the transient rates is a viable technique to add insight into well performance.
- The indirect determination of inflow rates and volumes may aid in setting pump off controllers and timers for beam pumped wells.
- Visualization of the pressure versus wellbore volume performance can assist in evaluating wellbore integrity.

As with any technical investigation, the additional insight gained leads to recommendations concerning application of the procedures and also leads to additional questions and ideas for further investigations. As a result, the following recommendations are made.

- Determine the detailed effects of inaccurate reservoir models.
- Investigate the possibility of using higher order numerical integration methods to improve the rate and volume estimates.
- Determine methods to compute flow rates for wells with negative skins.
- Develop a catalog of rate and volume responses associated with typical well problems.

REFERENCES

Agarwal, R. G., Al-Hussainy, Rafi, and Ramey, H. J., 1970. "An Investigation of Well Bore Storage and Skin Effect in Unsteady Liquid Flow: I. Analytical Treatment," **SPEJ**, Sept 1970, Trans AIME 249.

Cinar, M., Ilk, D., Onur, M., Valko, P. P., and Blasingame, T. A., (2006). "A Comparative Study of Recent Robust Deconvolution Algorithms for Well-Test and Production Analysis," *SPE Paper 102575, SPE Annual Technical Conference and Exhibition*, San Antonio, TX.

Van Everdingen, A. F. and Hurst, W., 1949. "The Application of the Laplace Transformation to Flow Problems in Reservoirs," **Trans. AIME** 186, 305-324.

Fair, P. S. and Simmons, J. F., 1992. "Novel Well Testing Applications of Laplace Transform Deconvolution," *SPE 24716, SPE Annual Technical Conference and Exhibition* Washington, D. C.

Fair, Walter Jr., 1996. "Generalization of Wellbore Effects in Pressure-Transient Analysis," **SPE Formation Evaluation**, June 1996.

Roumboutsos, A. and Stewart, G., 1988. "A Direct Deconvolution or Convolution Algorithm for Well Test Analysis," *SPE Paper 18157, SPE Annual Technical Conference and Exhibition*, Houston, TX.

Von Schroeter, Thomas, Hollaender, Florian and Gringarten, Alain C., 2001. "Deconvolution of Well Test Data as a Nonlinear Total Least Squares Problem," *SPE Paper 71574, SPE Annual Technical Conference and Exhibition*, New Orleans, LA.

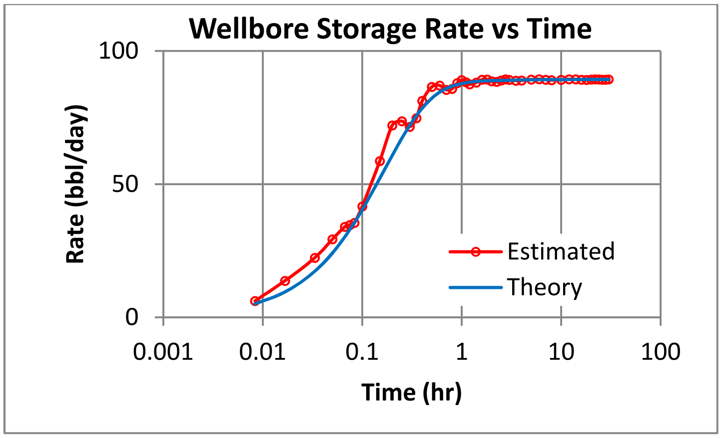

Figure 1 - Comparison of Computed and Exact Wellbore Storage Rates for a Synthetic Storage Curve

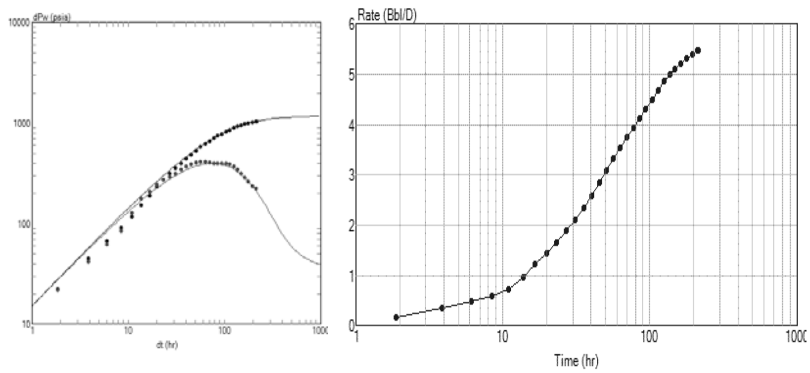

Figure 2 - Example Well 1: Ideal Storage

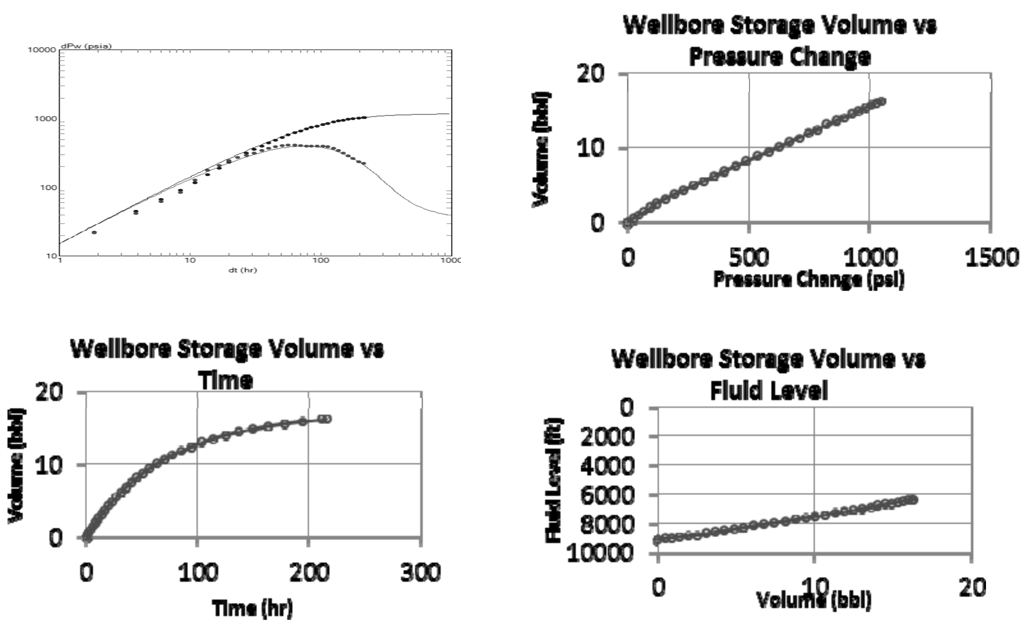

Figure 3 - Wellbore Storage Behavior of Example 1

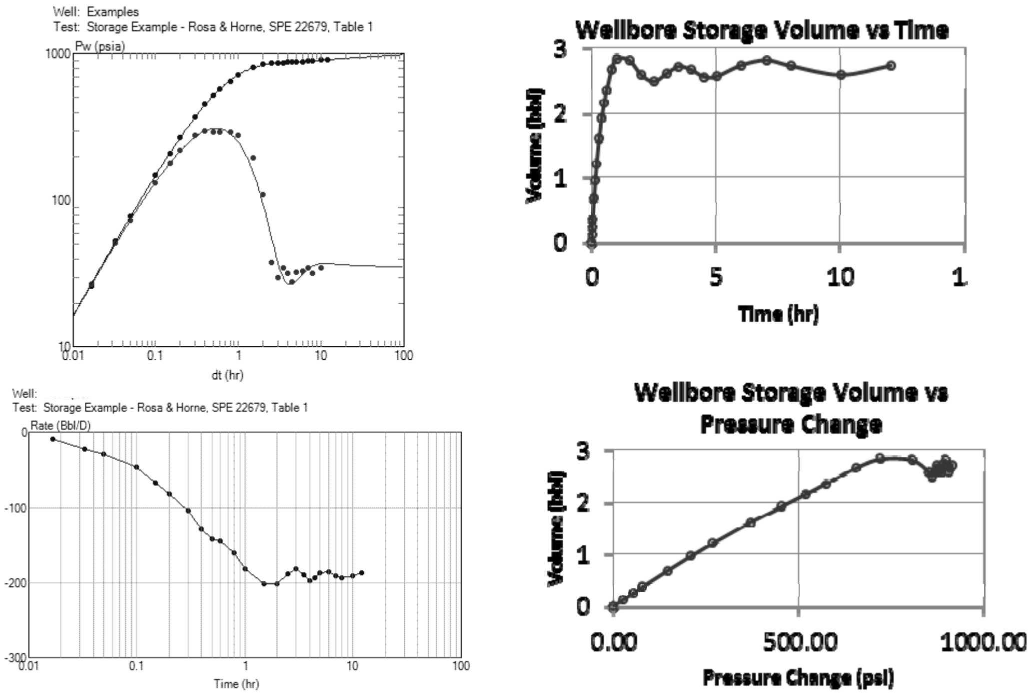

Figure 4 - Wellbore Storage Behavior of Example 2

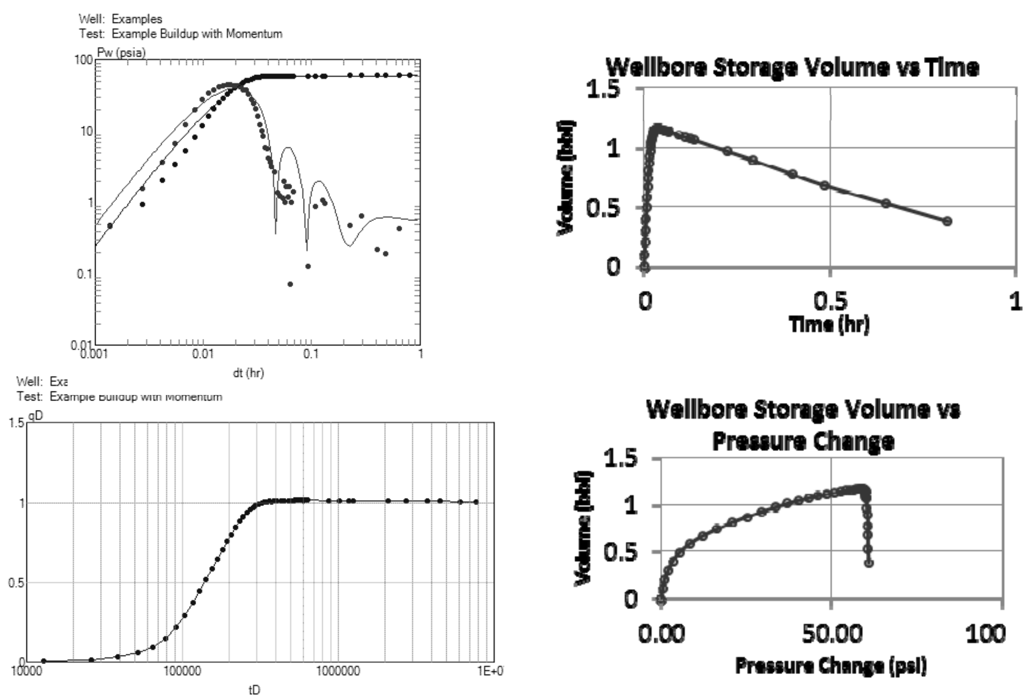

Figure 5 - Wellbore Storage Behavior of Example 3 (Wellbore Momentum)

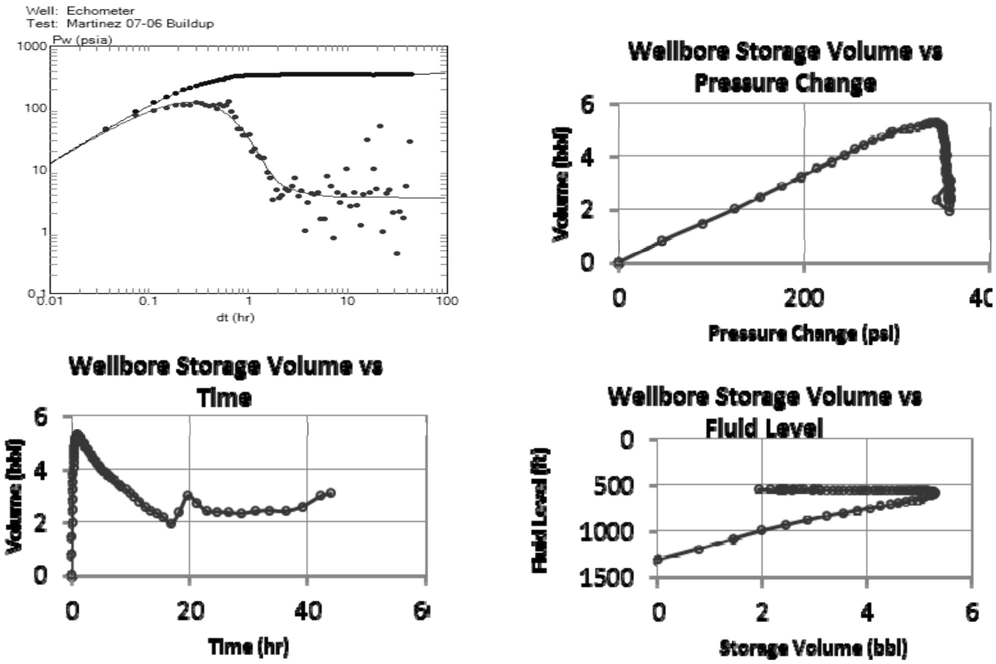

Figure 6 - Wellbore Storage Behavior of Example 4 – Possible Thief Zone