# Down-Hole Thermal Stress Analysis

By KURT LEUTWYLER

Baker Oil Tools, Inc.

# INTRODUCTION

Completions for thermal recovery installations are closely related to the many new and sophisticated approaches to well design which have come of age during the last decade. This writer likes to compare them with completions in the 15,000 -20,000 ft range, to high pressure gas completions in corrosive environments, perhaps to wells on platforms in deep water or completed by TFL methods. This comparison is made not because the magnitude of engineering difficulty in each instance is equal, but because the total problem content of each of these completion techniques represents a distinct engineering challenge and required unique solutions. Finding these solutions in turn required the use of classical methods and theories mixed with original thinking. Let us then examine why thermal recovery wells qualify for a position amongst difficult and sophisticated technological accomplishments.

## **Environmental Considerations**

Very few completions are engineered for temperatures exceding 300°F. As a matter of fact, this temperature level has been accepted as a design objective for most retrievable down hole equipment, although drillable tools and special equipment will perform at temperatures up to and sometimes exceeding 375°F. It is then safe to make the statement that conventional completions are generally made in an environmental range from 100°F to 400°F. Since 400°F is just about the starting point of the thermal recovery range, and specifically the steam injection temperature range, we find that little problem overlap exists. Thus the new technology is of the nature of a second or third generation of methods and techniques. What, may we ask, is the upper temperature limit of this new technology? We do know that several distinct levels of technique exist:

- (1) 400°F 700°F: Hot water or saturated steam.
- (2) 700°F 1050°F: Superheated steam.
- (3) Above 1050°F: Exotic recovery methods.

Most thermal recovery work has been done in the area of the first level of technique in the hot water or saturated steam injection temperature range. The third temperature range must be considered in in situ combustion processes for parts of the ignition well, but we shall consider this a special case. Designing completions for potential exotic recovery processes is certainly going to be a challenge, since even conventional steam power plant technology stops between 1000°F and 1100° F. However is appears that, going from first level techniques to second level techniques, exceeding the critical point of water, would be possible based on available technology. It must be said, however, that one will find a considerable economic barrier at 700°F and therefore this analysis will be confined to the first level of technique, 400°F to 700°F.

## Thermal Stress

Temperature changes, such as the one from geothermal to injection equilibrium, result in thermal expansion of the completion components. Since these components, such as casing and tubing, are often restrained, and thus prevented from expanding, they are subjected to an increase in stress level. This increase in stress level is the result of thermal stresses, which under pure compressive conditions are the same as would be required to shorten the members to their original length had they been free to expand under the influence of the temperature change. Mathematically this condition can be analyzed as follows:

$$\Delta L = \sigma_c \frac{L}{E}$$

Where  $\Delta L$  - Expansion or Contraction

L = Length of Member

E - Modulus of Elasticity

♂<sub>C</sub> - Average Stress Level in Cross-Section

Thermal Expansion:

 $\Delta L = L \beta \Delta t$ 

Where:  $\Delta L = -Expansion$ 

- L = Length of Member
- $\Delta t$  Temperature Change of Member

Based on the equivalence of trermal and elastic displacements, we can equate the two terms

 $\sigma_{\rm C} \frac{\rm L}{\rm E} = {\rm L} \beta \Delta t$ 

and hence

240

$$\sigma_{\text{thermal}} - E \beta \Delta t$$
 [1]

Analyzing Eq. 1 dimensionally we find that

therefore:

$$\sigma_{\text{thermal}} = 200 \, \Delta t \, \text{(psi)}$$
 [2]



This condition is illustrated in Fig. 1. Since long tubular members, such as tubing and casing, do not behave in an elastically stable manner, Eq. 2 not adequately describe the thermal stress conditions in steam injection environments. When they expand thermally under restrained conditions, they buckle helically against the surrounding structure. Mathematically this condition can be analyized as follows:

not adequately describe the thermal stress conditions in steam injection environments. When they expand thermally under restrained conditions, they buckle helically against the surrounding supporting structure. Mathematically this condition can be analyized as follows:

It can be proven that the strain in a helix at any depth of a string is \_\_\_\_\_?

Where 
$$\mathcal{E}_z$$
 = Strain at Depth  $\mathcal{E}_z = \frac{r^2}{4E1} F_z$ 

- r = Radial Clearance
- E = Modulus of Elasticity
- I = Moment of Inertia
- F = Force on Bottom Unit

Helix Under Construction



o<sub>c</sub> =

Fig. 2 illustrates such a system. It is evident that the total  $\triangle L$  is the summation of all unit strains as a function of  $F_7 = \frac{z}{r}$  F

$$\Delta L - \int_{n-L}^{L} \mathcal{E}_z d_z = \frac{r^2}{4EI} \frac{F}{n} \int_{n-L}^{L} z dz$$

From this results the following approximation

 $\Delta L = \frac{L}{E} \left( \frac{1}{A} + \frac{r^2}{4I} \right) F$ (3)
Where  $\Delta L = \text{Contraction}$ 

r = Radial Clearance

E - Modulus of Elasticity

I = Monment of Inertia

- A = Cross-Section of Member
- L = Length of Member
- F Force on Bottom of String

 $\frac{F}{A} \xrightarrow{\text{However, the compressive stress in the tubular}}_{\text{ross-section is}} [4]$ 

and the bending stress at the outer fibre of the member is

$$\sigma_{\rm b} = \frac{\rm D r}{4 \rm I} \rm F$$
 [5]

Where D = OD of Member

For other nomenclature, see Eqs. 3 and 5. Combining Eqs. 3.4 and 5, we find an expression for the maximum combined stress at the bottom of the string as a function of the temperature change  $\triangle t$ .

$$\sigma_{\text{thermal}} = \frac{\left(\frac{1}{A} + \frac{Dr}{4I}\right)}{\left(\frac{1}{A} + \frac{r^2}{4I}\right)} E \beta \Delta t \qquad [6]$$

Eq. 6 was used to analyze strings of various geometrical configurations and the plot illustrated in Fig. 3 resulted. As a further approximation, it was found that the combined thermal stress can be described by Eq. 7 for elastically instable conditions:  $\sigma_{thermal} = 260 \text{ }\Delta t \text{ (psi)}$  [7] A radial clearance of one in, was used for this aproximation. Increase of this clearance due to wash-outs or change in pipe or hole size will lead to higher stresses, as predicted by Ref. 14. Stress conditions along the length of the string can be evaluated by the obvious means of evaluating F at any depth and proceeding by means of Eqs. 3, 4, and 5.



#### CONCLUSIONS

This summary of down hole thermal stress analysis is based on the reference material listed. The theoretical concept presented has been verified by some field experience, although much correlative work remains to be done. Many approximations had to be used, some in this paper, more in the original work used for reference purposes. Engineering concepts are often a compromise between physical phenomena and the mathematical techniques describing them. As long as the accuracy of the final result is not unduly diluted such methods are valid, and it is felt that the method of thermal stress analysis described here should result at least in good knowledge of the order of magnitude of the problem. Good quantitative approximations or measurements of the average change of temperature of the members considered are required and the reader is again referred to the list of references.

#### REFERENCES

- Moss, J. T. and White, R. D.: "How to Calculate Temperature Profiles in a Water Injection Well", <u>Oil and Gas Journal</u> (March 9,1959) No. 11.
- Squier, D. P., Smith, D. D. and Dougherty. E. L.: "Calculated Temperature Behavior of Hot-Water Injection Wells", <u>Jour. Pet</u> <u>Tech.</u> (April, 1962) 436.
- 3. Ramey, H. J., Jr.: "Wellbore Heat Transmission", Jour. Pet. Tech. (April, 1962) 427.
- 4. McAdams, H. J., <u>Heat Transmission</u>, Third Edition, McGraw-Hill Book Co., Inc., New York (1964) 55.
- 6. Leutwyler, Kurt and Johnson, B. K.: "Heat Transmission by Radiation During Steam Injection", <u>Unpublished Research Report</u>, Baker Oil Tools, Inc.
- 7. Lubinski, A., Althouse, W. S. and Logan,
- J. L.: "Helical Buckling of Tubing Sealed in Packers", <u>Jour. Pet. Tech.</u> (June, 1962) 655.
- 8. Timoshenko, S. and Goodier, J. N.: <u>Theory</u> of <u>Elasticity</u>, Second Edition McGraw-Hill Book Co., Inc., New York (1951) 33.

 $\{ (1,2) \}$ 

- Timoshenko, S. and MacCullough, G. H.: <u>Elements of Strengths of Materials</u>, third Edition, D. Van Nostrand Co., Inc. (1959) 374.
- Humphrey, H. C.: "Casing Failures Caused by Thermal Expansion". <u>World Oil</u> (Nov., 1960).
   Injection Systems". Baker Oil Tools. Inc., European Technical Seminar Brochure (1964).
- Leutwyler, Kurt and Bigelow, H. L.: "Steam Injection Systems", Baker Oil Tools, Inc., <u>European Technical Seminar Brochure</u> (1964)).
- 12. Sokolunkoff, I. S. and E. S.: <u>Higher Mathematics for Engineers and Physicists</u>. Second Edition. McGraw-Hill Book Co., Inc. A New York (1964) 33.
- 13. Faires, V. M.: <u>Applied Thermodynamics</u>. Revised Edition, the MacMilland Co. (1948).
- Luetwyler, Kurt and Bigelow, H. L.: "Temperature Effects on Subsurface Equipment in Steam Injection Systems", Jour. Pet. Tech., (January, 1965).