CRITICAL EVALUATION
OF SUCKER ROD STRING DESIGN PROCEDURES

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ABSTRACT
A properly designed sucker-rod string should provide failure-free pumping operations for an extended period. Improper design of rod tapers can lead to early mechanical failures (rod breaks) with a complete termination of pumping action and an inevitable loss of production. According to its prime importance in sucker-rod pumping technology several design procedures based on different assumptions were developed in the past. Correct description of rod loading conditions during the pumping cycle led to the inclusion of fatigue endurance limits into rod string design methods; all modern designs incorporate the modified Goodman diagram that describes fatigue failures. The paper details the main features of available rod string designs and discusses their main characteristics; design results are compared for example cases. The paper also provides a more thorough comparison of designs involving the calculation of loads and stresses predicted from the solution of the damped wave equation. Using a predictive analysis program rod stresses are calculated that, plotted on the modified Goodman diagram, provide a proper comparison of the merits of the different rod string design methods.

INTRODUCTION
The sucker-rod string is a very peculiar piece of man-made structures because its maximum diameter (about one inch) is absolutely negligible as compared to its length of several thousand feet making it an absolute “slender” bar. The weight of the string is distributed along the length and any section has to carry at least the weight of all the rods below it. This fact suggests that the ideal shape would be an inverted cone, continuously tapering from top to bottom. Since such a rod string is impossible to manufacture one tries to approach the ideal shape by designing tapered strings with sections of increasing diameters toward the surface. For shallow wells, straight rod strings made up from one rod size only are also used but deeper wells inevitably require the application of tapered strings.

In order to find an ideal solution of the design goals detailed mechanical calculations should be performed with the actual well conditions properly taken into account. The designer faces two basic problems when designing a sucker-rod string:

- how to calculate rod loads during the pumping cycle, and
- what principle to use for the determination of taper lengths.

Rod Loads

The different kinds of possible rod string loads that occur during the pumping cycle can be classified into the following groups:

- Weight of rods; this load is distributed along the string. At any section, it is equal to the weight of the rods below the given section.
- Buoyancy force; always opposes the rod weight and is equal to the hydraulic lift caused by immersing the rods into the produced liquid.
- Fluid load is a concentrated force acting at the bottom of the string during upstroke only and equals the force resulting from the net hydrostatic pressure acting on the area of the pump plunger.
- Dynamic loads are the results of changes in acceleration of the moving masses (rods, fluid column).
- Frictional forces are of two kinds: (1) fluid friction between the rods and the produced liquid and (2) mechanical friction between the rods and the tubing string.

Investigation of the effects of these forces shows that the rod string is exposed to a cyclic loading. Although the upper rods are always in tension, the tension level considerably increases during the upstroke due to the load of the fluid lifted. The downstroke load, however, consists of only the buoyant weight of the rods minus dynamic and friction forces. Thus, the loading of the sucker rod string is pulsating tension; this fact must be accounted for in the mechanical design of the string.

When calculating the loads in the rod string dynamic and frictional loads are almost impossible to estimate; this is the reason they are not included in most of the traditional design procedures. The remaining loads: buoyant rod string weight and fluid load are easy to determine but depend on the taper lengths that are about to be determined. Rod string design, therefore, is an iterative process.

In order to ensure the reliability of rod string design it is customary to use some important assumptions that increase the calculated rod loads and thus improve the safety of the design: (a) water is pumped, and (b) the fluid level is at the pump setting depth, i.e. the well is pumped off.

Overview of Design Principles

Early rod string design methods utilized the simplifying assumption that the string was exposed to a static tension loading. Their goal was to keep the maximum rod stresses below a percentage of the tensile strength or the yield point of the rod material. Several different procedures were developed to design tapered rod strings; the one proposed in the Bethlehem Handbook [1] and later adopted by the API in the earlier editions of RP 11L [2] gained wide acceptance. This is called a “balanced” design based on static loading conditions and sets the maximum stress at the top of each section equal.

The simple design procedures gave reasonable rod life in shallower wells, but as rod stresses became higher in deeper wells it became inadequate and overloading of the lower rod sections was experienced, as observed by Eickmeier [3] as early as in 1967. Rod breaks were occurring at operating stresses well below the tensile strength or the yield strength of the rod material. Such breaks are typical fatigue failures; this finding caused designers to realize that a proper rod string design must account for the cyclic nature of rod loading. This is why the string has to be designed for fatigue endurance, as done in most of the present-day procedures.

The fatigue endurance limit of API steel rod materials, called the allowable stress is calculated from the modified Goodman formula [4]:

$$S_a = SF\left(\frac{T_a}{4} + 0.5625\ S_{min}\right) \qquad \mathbf{1}$$

where:
- S_a = fatigue endurance limit (allowable stress), psi
- SF = service factor, -
- T_a = minimum tensile strength of the rod material, psi
- S_{min} = minimum rod stress, psi.

The first tapered rod string design procedure specifically developed for fatigue loading was proposed by West [5, 6]. West developed a taper design that maintains the same amount of safety for every taper section. The objective of the design is to have the same ratio of maximum stress to allowable stress (represented by the Service Factor used) in each taper. Rod strings designed this way will have the same safety factor included in every taper and will not have any weak points.

The next rod string design proposed by Neely [7] introduced the concept of modified stress and aimed at reaching the same modified stress values at the top of each taper section. His design method was adopted by the American Petroleum Institute in 1976 and taper percentages calculated using his procedure were published in the later editions of RP 11L [2]. The tables published in API RP 11L gained wide acceptance mainly because they eliminated the time necessary for detailed design calculations.

The latest design procedure was proposed by Gault-Takacs [8] which overcomes the drawbacks of previous procedures and gives a more theoretically sound design method while requiring only moderate computational time. The goal of the design is to have the same degree of safety in every taper section. Service factors for all tapers will be the same, and rod strings will be subjected to a uniform level of fatigue loading all along the string.

QUICK COMPARISON OF DESIGN PROCEDURES

Main Features

The main features of available rod string design procedures are collected in **Table 1** which contains information on the different ways authors estimate rod loads and presents each model’s basic design goal.

The early design proposed in the Bethlehem Handbook disregards minimum and dynamic loads and does not consider buoyancy effects. West, too, ignores buoyancy forces to compensate for friction forces that are usually unknown but tend to act opposite the buoyant force. Although he considers dynamic loads but uses the obsolete Mills acceleration factor method which gives reasonable predicted loads for small pumps and medium pumping depths only. Neely's approach to load calculations includes buoyancy as well as dynamic loads for which he recommends an empirical correlation. In this design model, dynamic loads are assumed to linearly decrease downward. The Gault-Takacs model is the only one that tries to include the effect of the force wave reflections in the rod string when calculating rod loads and finds the surface dynamic loads from RP 11L (now called API TL 11L) calculations. These dynamic loads are distributed along the string in proportion to the mass being moved and have different magnitudes during the up-, and downstroke.

Investigation of the design goals of the different models reveals huge differences in basic principles. The Bethlehem model designs strings with the same maximum stresses at the top of each taper. This means that the S_{min} - S_{max} points plotted on the Modified Goodman diagram [4] for the different taper sections will fall on a horizontal line which must inevitably cross several ***SF*** = constant lines. Lower tapers (with lower minimum stresses) have a higher service factor and a consequently reduced safety than the tapers higher up the string. The fatigue loading on lower tapers, therefore, is higher and these sections are more likely to experience premature failure.

Setting the service factors (***SF***s) equal in each taper, as done by West and Gault-Takacs, ensures the same amount of safety for every taper section. Rod strings designed this way have the same safety factor included in every taper and do not have any weak points.

Finally, Neely defines a "modified stress" and forces those at the top of each taper equal:

$$S_{mod} = S_{max} - 0.5625\ S_{min} \qquad \mathbf{2}$$

where: S_{mod} = "modified" stress, psi,
S_{min}, S_{max} = maximum and minimum rod stresses, psi.

Setting the modified stresses equal in each taper means that S_{min} - S_{max} points belonging to the different tapers, when plotted on the modified Goodman diagram (MGD), will lie on a parallel to the ***SF*** = 1 line. This line, however, inevitably crosses the lines corresponding to any service factor other than 1.0. Therefore, the design generates different safety factors for each taper; upper tapers are relatively more loaded than lower ones. This situation is just the opposite of early design methods where usually the lower tapers were under-designed.

Sample Case

In order to illustrate the main features of rod string design models an example case of a three-taper rod string is presented. Well parameters are given here; calculated taper lengths found from each design are displayed in **Table 2**.

Pump Setting Depth	6,000 ft	Plunger Size	2 in
PR Stroke Length	120 in	Pumping Speed	6 SPM
Rod Grade	API D	Service Factor	0.9
API Rod Code	86	Liquid Sp. Gr.	1.0

Comparison of the four rod string designs is done on the modified Goodman diagram in **Fig. 1**. The stresses plotted are calculated according to each model's basic assumptions, as detailed in **Table 1**. **Fig. 1** presents a non-dimensional form of the modified Goodman diagram (MGD) and contains S_{min} - S_{max} points for each taper and each design procedure. The two dashed lines represent the fatigue endurance limits of the given rod material for ***SF*** = 0.8 and 0.9.

As seen, all four design methods produced strings that can safely handle the estimated well loads indicated by the fact that all maximum stresses are below the fatigue endurance limits of the rod material for ***SF*** = 0.9. Their behavior in relation to the fatigue loading of the individual tapers, however, is different. Two design models (West and Gault-Takacs) have their three points belonging to the three tapers located on ***SF*** = constant lines; this is in accordance with their design principles. These strings are, therefore, uniformly loaded and have tapers designed with the same safety against fatigue failure.

The string designed by Neely's procedure is not uniformly loaded because the line connecting the three points corresponding to the three tapers is clearly seen to intersect a ***SF*** = constant line. The reason for this is that according to Neely's design principle the three points must fall on a constant modified stress line. This line, however, has a slope identical to that of the ***SF*** = 1 line, i.e. 0.5625; but the slope of any ***SF*** = const. line is less than that and equals 0.5625 ***SF***, see **Eq. 1**. In conclusion, this method results in having different service factors in the design of each taper as seen in **Fig. 1**. The top taper (represented by the rightmost point) is relatively more loaded and is bound to fail first.

If the Bethlehem model is followed then it is easy to see that even greater differences in the fatigue loading of the different tapers take place. As shown in **Fig. 1** maximum rod stresses are about the same and the three points belonging to the tapers lie nearly on a horizontal. This means that the bottom taper (corresponding to the leftmost point) is much more loaded than the upper ones. This fact was proved in the oil fields when in the late 1960's rod strings designed according to this model failed almost invariably at the bottom taper.

Predicted Loads

Up to this point all loads and stresses were calculated according to each design procedure's basic assumptions, as detailed in **Table 1**. These loads, however, can only be considered as estimates for the given pumping conditions. In order to exactly compare the features of the different rod string designs, one would have to actually measure the loads occurring in the different strings just designed. Since this would be close to impossible to do, this is, of course, not a viable approach. The best possible solution of the problem involves the calculation of rod loads from the solution of the damped wave equation written on the rod string. This approach is justified by experience gained since the introduction of the wave equation in the late 1960s [9] proving that the predictive solution of the damped wave equation gives loads that very closely match measurements. Since mechanical stresses in the rods must be calculated from "true" loads the solution must include the effects of buoyant forces that occur on the different tapers. [10] Therefore, the rod strings designed by different procedures will be compared in this paper on the basis of loads and stresses calculated using predictive analysis techniques.

The rod strings designed for the sample case were used to find the distribution of rod loads during the up-, and the downstroke using the predictive solution of the damped wave equation. For the calculations we assumed a conventional pumping unit, a vertical well, average damping factors, Grade D rods, and pumping water in a pump-off condition. Minimum and maximum pumping loads were determined at the top of each rod taper that allowed the calculation of mechanical stresses at the top sections of the tapers. These stresses, plotted on the modified Goodman diagram (MGD) define the fatigue loading of the strings and allow one to derive important conclusions on the merits of the different procedures.

Fig. 2 contains a comparison of the four design procedures for the example case based on the predicted rod stresses and plotted on the modified Goodman diagram. Rod stresses, compared to those shown in **Fig. 1**, have considerably changed; the general trend is that minimum stresses became lower while maximum stresses became higher. This is a clear indication that predicted dynamic loads are higher than those assumed by the design procedures because dynamic loads tend to alter rod loads that way. This effect is most pronounced for the Bethlehem model where the bottom taper's fatigue loading has extremely increased.

Comparison of **Fig. 1** and **Fig. 2** makes it clear that a meaningful evaluation of the rod string design procedures must not be based on each model's calculation procedure because design loads are very different from predicted ones. Using predicted loads received from the solution of the damped wave equation, on the other hand, ensures the highest possible approximation of measured loads and can thus give a reliable foundation to further investigations.

SYSTEMATIC EVALUATION OF ROD STRING DESIGNS

Background

It must be clear that an ideally designed sucker-rod string should have the same level of safety in each taper so that none of the tapers is a weak link in the system. The safety of a given taper section against fatigue failure is defined as the ratio of the actual maximum stress and the allowable stress. Using **Eq. 1** that describes the modified Goodman diagram (MGD) and solving it for the service factor we get:

$$SF = \frac{S_{max}}{\left(\frac{T_a}{4} + 0.5625\, S_{min} \right)} \qquad \textbf{3}$$

where: $\boldsymbol{SF}$ = service factor, **-,**
$\boldsymbol{T_a}$ = minimum tensile strength of the rod material, psi,
$\boldsymbol{S_{min}}$, $\boldsymbol{S_{max}}$ = minimum and maximum rod stresses, psi.

This formula indicates that the safety of any taper section is proportional to the service factor (***SF***) calculated from the actual rod stresses. Therefore, to attain the same amount of safety in every taper section one has to ensure that calculated ***SF***s for each taper are the same; this is the basic requirement for an ideal design. Procedures following this idea like West [5, 6] and Gault-Takacs [8 [, therefore, have a sound background and can be expected to properly design the rod string.

Since all design procedures use rod loads that are only estimated during the design process, investigations using predicted loads and stresses can reveal features not relevant in the original calculation model. This analysis involves the calculation of ***SF*** values for each taper based on the predicted stresses in the top section of the tapers using **Eq. 3**. If calculated ***SF***s are equal then $\boldsymbol{S_{min}} - \boldsymbol{S_{max}}$ points plotted on the MGD chart must fall on the same ***SF*** = const. line. In case the points fall on different ***SF*** = const. lines, the design procedure does not meet the criteria for an ideal design.

The merits of the design procedures are evaluated based on their ability to force predicted stresses on a ***SF*** = const. line. In order to check this feature a ***SF*** = const. line on the modified Goodman diagram is fitted between the $\boldsymbol{S_{min}} - \boldsymbol{S_{max}}$ points belonging to the different tapers. Using the method of least squares the service factor belonging to the best fitting line is found from the following formula:

$$SF = \frac{\frac{T_a}{4} \sum_{i=1}^{n} S_{max(i)} + 0.5625 \sum_{i=1}^{n} \left(S_{min(i)}\, S_{max(i)} \right)}{\frac{n\, T_a^2}{16} + \frac{T_a\, 0.5625}{2} \sum_{i=1}^{n} S_{min(i)} + 0.5625^2 \sum_{i=1}^{n} S_{min(i)}^2} \qquad \textbf{4}$$

where: $\boldsymbol{SF}$ = service factor, **-,**
$\boldsymbol{T_a}$ = minimum tensile strength of the rod material, psi,
$\boldsymbol{S_{min\,(i)}}$, $\boldsymbol{S_{max\,(i)}}$ = minimum and maximum rod stresses in the i^{th} taper , psi,
$\boldsymbol{n}$ = number of tapers in the string, -.

The deviation of the $\boldsymbol{S_{min}} - \boldsymbol{S_{max}}$ points from the line belonging to this ***SF*** value on the MGD chart indicates how efficiently the given procedure can approximate the ideal rod string design. Its measure is the goodness of fit defined in the next formula. Perfect fit is indicated by $\boldsymbol{R^2}$ = 1; lower values are received if points scatter more from the best fitting line.

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \left(S_{max(i)} - S_{max,calc(i)} \right)^2}{\sum_{i=1}^{n} \left(S_{max(i)} - S_{max,avg} \right)} \qquad \textbf{5}$$

where: $\boldsymbol{R^2}$ = goodness of fit, **-,**
$\boldsymbol{S_{max\,(i)}}$ = maximum rod stress in the i^{th} taper, psi,
$\boldsymbol{S_{max,\,calc\,(i)}}$ = maximum rod stress found from **Eq. 1** in the i^{th} taper, psi,
$\boldsymbol{S_{max\,avg}}$ = average of the maximum rod stresses in the string, psi,
$\boldsymbol{n}$ = number of tapers in the string, -.

Results for the Sample Case

The best-fitting lines' ***SF***, as well as $\boldsymbol{R^2}$ values for the sample case are given in **Table 2** for the four design procedures. As seen, the closes approach to an ideal design (where $\boldsymbol{R^2}$ = 1) is provided by the Gault-Takacs procedure and the Bethlehem method is the worst performer. Good results were also obtained with the West and the Neely procedures and $\boldsymbol{R^2}$ values above 0.9 were found; this indicates that they perform properly even with predicted rod stresses i.e. under actual conditions. The best and the worst designs are compared in **Figs. 3** and **4** where stresses belonging to each taper are plotted for two conditions: (a) as calculated in the design procedure, and (b) as found from the predictive solution of the wave equation. The best-fitting line using the ***SF*** value found from **Eq. 4** is also plotted. As seen in **Fig. 3**, the rod string designed by the Gault-Takacs procedure has a low deviation between the best-fitting line and the three points belonging to the predicted stresses. The early method proposed by Bethlehem, on the other hand, performs very poorly as indicated in **Fig. 4** because predicted stresses widely scatter around the best fitting line.

Generalized Evaluation

In order to evaluate the merits of the rod string design procedures in a more general way the following project was set up. Investigations assumed a conventional pumping unit, a vertical well, average damping factors, Grade D rods, and pumping water in a pump-off condition as general constraints. A framework of pump setting depths and pump sizes was laid down and for each combination of those a matrix of pumping speeds and polished rod stroke lengths was selected. After selecting different possible API taper combinations every element of the matrix contained all parameters required for the design and strings were designed using the different procedures. The next step of the project involved the calculation of rod stresses in each taper of the strings using the predictive solution of the damped wave equation. Predicted stresses were used to find the best fitting $\boldsymbol{SF}$ = const. line on the modified Goodman diagram using **Eq. 4**. Finally **Eq. 5** was used to evaluate the goodness of fit.

The Bethlehem procedure was excluded from further study because it gave taper lengths with widely deviating predicted stresses from the best fitting line. Results for the other design methods are contained in **Tables 3 – 5** where $\boldsymbol{R^2}$ values are displayed; these give a good indication of the merits of the designs. Designs with a goodness of fit of at least $\boldsymbol{R^2}$ = 0.9 are shaded and are considered as being close to ideal designs. Cells containing "-" indicate that the design is not valid because the calculated service factor is greater than unity. The tables enable one to check the behavior of the different rod string design procedures in the investigated ranges of operating conditions. One general observation is that the reliability of the designs deteriorates at pumping speeds above $\boldsymbol{SPM}$ = 10.

Evaluation of the RODSTAR Model

The commercial computer program package RODSTAR [11] includes the design of the rod string while performing a predictive analysis of the rod pumping system. The design procedure is based on predicted rod loads that are calculated from the solution of the damped wave equation so the design does not rely on approximate calculations like the four procedures discussed so far. Taper lengths are selected so that their loading is identical at the top of each taper; loading is defined as follows:

$$Loading = \frac{S_{\max 1} - S_{\min 1}}{S_{a1} - S_{\min 1}} = \frac{S_{\max 2} - S_{\min 2}}{S_{a2} - S_{\min 2}} = \frac{S_{\max 3} - S_{\min 3}}{S_{a3} - S_{\min 3}} = \ldots = C \qquad \mathbf{6}$$

where: $\boldsymbol{S_{max\,(i)}}$, $\boldsymbol{S_{min\,(i)}}$ = maximum and minimum rod stresses in the ith taper, psi,
$\boldsymbol{S_{a\,(i)}}$ = allowed rod stress in the ith taper, found from **Eq. 1**, psi.

Substituting the formula for $\boldsymbol{S_{a\,(i)}}$ from **Eq. 1** and expressing the maximum stress in the ith taper section we get:

$$S_{\max(i)} = C\ SF\ \frac{T_a}{4} + S_{\min(i)}\left(C\ SF\ 0.5625 - C + 1\right) \qquad \mathbf{7}$$

In this formula $\boldsymbol{C}$ denotes the identical loading of the tapers and can attain values between 1 (fully loaded) and 0 (no load). In case of $\boldsymbol{C}$ = 1 (fully loaded tapers) **Eq. 7** is identical to **Eq. 1** and the string design is ideal because the safety against fatigue failure is the same in each rod taper. This situation, however, changes if the tapers are not fully loaded when the results deviate from the ideal case. This is clearly seen on the MGD presented in **Fig. 5** where $\boldsymbol{C}$ =const. lines calculated from **Eq. 7** (shown in dashed lines) are plotted along with $\boldsymbol{SF}$ = const. lines for a case when $\boldsymbol{SF}$ = 0.9 is used during the design process. As shown, the RODSTAR model gives a perfect design only if the string is fully (100%) loaded, but at any other loading the points belonging to the stresses in the different tapers will fall on lines intersecting the $\boldsymbol{SF}$ = const. lines. In conclusion, the string design based on equal loadings does not meet the criteria for an ideal design because the safety against fatigue failure is different for each taper section.

The RODSTAR design applied to the sample case is included in **Table 2** which indicates the parameters of the best-fitting $\boldsymbol{SF}$ = const. line as well. Calculated stresses for the three tapers are plotted on the MGD in **Fig. 6** along with the best-fitting line; the points are seen to scatter and do not fall on the line belonging to the best-fitting line. This indicates the systematic flaw of the design principle.

GENERAL CONCLUSIONS

The comprehensive evaluation of the available sucker-rod string design procedures presented in the paper resulted in the following general conclusions.

- The proper assessment of rod string designs, in lieu of measurements, must rely on predicted stresses calculated from the solution of the damped wave equation because loads and stresses estimated by the different procedures are not a true measure of the actual conditions.
- Optimum rod string design must provide the same safety against fatigue failures in each rod taper section; this is ensured by having the same Service Factor (***SF***) at the top of each taper.
- The early (Bethlehem) design principle of setting equal the maximum stresses at the top of each taper produces rod tapers with different safety; bottom tapers are usually more loaded than upper ones.
- The West, Neely, and Gault-Takacs design methods behave properly and provide equal safety in each taper in most of the ranges investigated in the paper.
- The design principle utilized in the RODSTAR program package does not meet the criteria of an ideal string design.

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Table 1

Model	Year	Min. Load	Max. Load	Dyn. Loads	Design Goal
Bethlehem	1953	-	Fluid load plus rod weight in air	-	Equal max. stresses
West	1973	Rod weight in air	Fluid load plus rod weight in air plus dynamic loads	Mills acceleration factor	SF = const.
Neely	1976	Buoyant rod weight	Fluid load plus buoyant rod weight plus dynamic loads	Special formula	Equal modified stresses
Gault - Takacs	1990	Buoyant rod weight	Fluid load plus buoyant rod weight plus dynamic loads	From RP 11L	SF = const.

Table 2

Model	¾" taper	7/8" taper	1" taper	SF	R^2
Bethlehem	2,364 ft	1,853 ft	1,783 ft	0.842	0.142
West	1,469 ft	2,400 ft	2,131 ft	0.817	0.933
Neely	1,543 ft	2,254 ft	2,203 ft	0.819	0.947
Gault - Takacs	1,774 ft	2,239 ft	1,987 ft	0.829	0.993
RODSTAR	1,825 ft	2,200 ft	1,975 ft	0.801	0.908

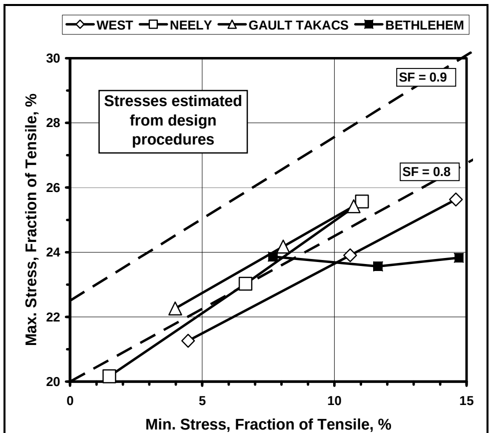

Figure 1

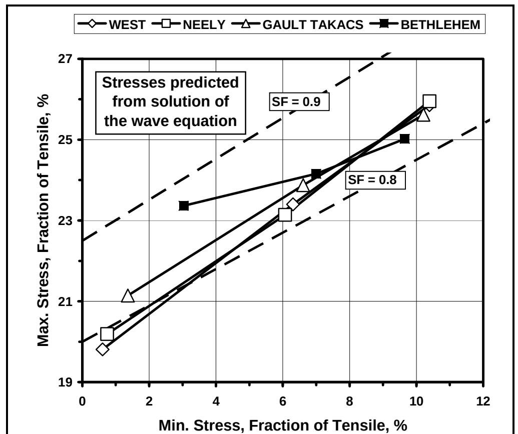

Figure 2

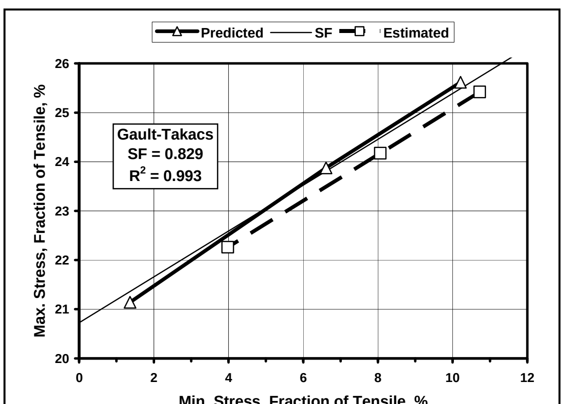

Figure 3

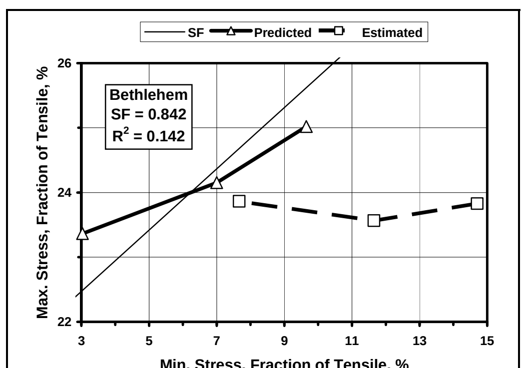

Figure 4

Table 3

WEST

	Plunger Diameter											
	1.5"				2"				2.5"			
4,000'	API 76				API 76				API 86			
		86	100	120		86	100	120		86	100	120
	4	0.87	0.87	0.86	4	0.92	0.90	0.88	4	x	x	x
	6	0.98	0.95	0.93	6	0.91	0.91	0.92	6	x	0.81	0.76
	8	0.79	0.78	0.94	8	0.90	0.93	0.76	8	0.78	0.74	0.70
	10	0.99	0.90	0.51	10	0.89	0.78	0.72	10	0.70	0.72	0.69
	12	-11.9	-18.4	-4.05	12	-9.89	-1289	-1619	12	0.90	0.81	0.72
6,000'	API 86				API 86							
		86	100	120		86	100	120				
	4	0.91	0.89	0.87	4	0.95	0.94	0.92				
	6	0.89	0.88	0.89	6	0.95	0.95	0.93				
	8	0.91	0.94	0.94	8	0.97	0.97	0.96				
	10	0.76	0.70	0.57	10	0.98	0.97	0.94				
	12	1.00	0.99	1.00	12	0.37	0.49	0.72				
8,000'	API 86				API 86							
		86	100	120		86	100	120				
	4	0.96	0.97	0.98	4	0.99	0.98	0.98				
	6	0.97	0.97	0.95	6	0.99	0.98	0.97				
	8	0.98	0.95	0.72	8	1.00	-	-				
	10	0.13	-1.85	-15.1	10	0.69	-	-				
	12	0.76	0.85	-	12	-	-	-				

	Plunger Diameter											
	1.5"				2"				2.5"			
6,000'	API 97				API 97				API 97			
		86	100	120		86	100	120		86	100	120
	4	0.89	0.88	0.89	4	0.93	0.91	0.92	4	0.96	0.96	0.95
	6	0.88	0.90	0.94	6	0.94	0.93	0.92	6	0.97	0.97	0.96
	8	0.96	0.92	0.88	8	0.97	0.97	0.99	8	0.99	0.97	0.99
	10	0.40	-0.82	-20.1	10	0.94	0.89	0.70	10	0.98	0.94	0.95
	12	0.40	-0.73	0.66	12	0.63	0.62	0.60	12	0.01	-0.60	-1.05
8,000'	API 97				API 97				API 97			
		86	100	120		86	100	120		86	100	120
	4	0.98	0.99	0.99	4	0.98	0.99	0.98	4	1.00	0.99	-
	6	0.96	0.96	0.97	6	0.99	0.99	0.99	6	0.99	-	-
	8	0.95	0.86	0.23	8	1.00	1.00	0.99	8	0.93	-	-
	10	-1.09	-5.30	-15.0	10	0.40	0.00	-	10	0.73	-	-
	12	0.96	0.94	0.88	12	0.97	-	-	12	-0.24	-	-

Table 4

GAULT - TAKACS

	Plunger Diameter											
	1.5"				2"				2.5"			
4,000'	API 76				API 76				API 86			
		86	100	120		86	100	120		86	100	120
	4	0.93	0.51	0.50	4	0.95	0.96	0.98	4	0.94	0.93	0.96
	6	0.99	0.49	0.47	6	0.98	1.00	0.97	6	0.97	0.97	0.98
	8	0.89	0.46	0.44	8	0.99	0.98	1.00	8	0.93	0.93	0.90
	10	####	0.46	0.46	10	0.85	0.99	0.98	10	0.82	0.67	0.50
	12	-4.08	0.54	0.53	12	-8.48	-4.35	-2.21	12	-6.44	####	####
6,000'	API 86				API 86							
		86	100	120		86	100	120				
	4	0.90	0.90	0.88	4	0.96	0.96	0.95				
	6	0.91	0.91	0.92	6	0.97	0.98	0.99				
	8	0.95	0.98	1.00	8	0.98	0.99	0.99				
	10	0.39	-2.81	-4.40	10	0.85	0.77	0.53				
	12	0.98	0.89	0.71	12	-0.84	-0.70	-0.52				
8,000'	API 86				API 86							
		86	100	120		86	100	120				
	4	0.95	0.97	0.98	4	1.00	0.99	0.99				
	6	0.97	0.98	0.96	6	1.00	1.00	1.00				
	8	0.97	0.89	0.32	8	0.98	-	-				
	10	-0.35	-8.37	####	10	0.65	-	-				
	12	0.61	0.83	-	12	-	-	-				

	Plunger Diameter											
	1.5"				2"				2.5"			
6,000'	API 97				API 97				API 97			
		86	100	120		86	100	120		86	100	120
	4	0.89	0.88	0.89	4	0.93	0.93	0.93	4	0.98	0.98	0.98
	6	0.89	0.92	0.98	6	0.95	0.97	0.97	6	1.00	0.99	1.00
	8	0.93	0.61	0.42	8	0.99	0.99	0.90	8	0.99	0.97	0.96
	10	-1.08	####	-54.0	10	0.77	0.35	-0.84	10	0.81	0.16	0.20
	12	-0.81	-3.90	-3.37	12	-0.84	-0.92	####	12	-1.36	-7.69	-8.71
8,000'	API 97				API 97				API 97			
		86	100	120		86	100	120		86	100	120
	4	0.98	0.99	0.99	4	0.98	0.99	0.99	4	0.99	1.00	-
	6	0.96	0.96	0.99	6	0.99	1.00	1.00	6	0.96	-	-
	8	0.95	0.77	-0.65	8	1.00	0.99	0.99	8	0.81	-	-
	10	-1.98	####	####	10	0.36	-0.52	-	10	0.67	-	-
	12	0.96	0.95	0.82	12	0.95	-	-	12	-0.87	-	-

Table 5

NEELY

	Plunger Diameter											
	1.5"				2"				2.5"			
	API 76				API 76				API 86			
		86	100	120		86	100	120		86	100	120
4,000'	4	0.73	0.74	0.75	4	0.84	0.84	0.85	4	x	x	0.88
	6	0.83	0.82	0.83	6	0.87	0.93	0.95	6	0.90	0.90	0.90
	8	0.68	0.71	0.80	8	0.89	0.95	0.82	8	0.86	0.83	0.80
	10	0.92	0.93	0.59	10	0.92	0.82	0.79	10	0.79	0.86	0.86
	12	####	####	0.01	12	####	####	####	12	0.94	0.92	0.72
	API 86				API 86							
		86	100	120		86	100	120				
6,000'	4	0.80	0.79	0.78	4	0.91	0.91	0.91				
	6	0.79	0.79	0.81	6	0.93	0.94	0.94				
	8	0.82	0.85	0.85	8	0.97	0.97	0.95				
	10	0.96	0.95	0.98	10	1.00	0.98	0.96				
	12	0.87	0.87	0.89	12	0.61	0.68	0.87				
	API 86				API 86							
		86	100	120		86	100	120				
8,000'	4	0.91	0.93	0.95	4	1.00	0.99	1.00				
	6	0.92	0.94	0.92	6	1.00	1.00	1.00				
	8	0.96	0.99	0.96	8	0.98	-	-				
	10	0.75	0.45	-0.7	10	0.61	-	-				
	12	0.98	0.98	0.96	12	-	-	-				

	Plunger Diameter											
	1.5"				2"				2.5"			
	API 97				API 97				API 97			
		86	100	120		86	100	120		86	100	120
6,000'	4	0.77	0.76	0.77	4	0.86	0.85	0.87	4	0.96	0.97	0.96
	6	0.78	0.81	0.83	6	0.89	0.89	0.90	6	0.98	0.98	0.99
	8	0.90	0.92	0.92	8	0.93	0.94	0.97	8	1.00	1.00	0.99
	10	1.00	1.00	0.8	10	0.99	0.97	0.90	10	0.98	0.77	0.90
	12	0.95	0.97	0.95	12	0.97	1.00	0.98	12	-0.02	-0.67	-1.73
	API 97				API 97				API 97			
		86	100	120		86	100	120		86	100	120
8,000'	4	0.93	0.94	0.95	4	0.97	0.98	0.98	4	0.98	-	-
	6	0.91	0.91	0.92	6	0.98	0.99	1.00	6	0.90	-	-
	8	0.99	1.00	0.86	8	0.99	0.99	0.99	8	0.68	-	-
	10	0.92	0.21	-0.93	10	0.73	0.46	-	10	0.22	-	-
	12	0.95	0.88	0.83	12	1.00	0.96	-	12	-1.63	-	-

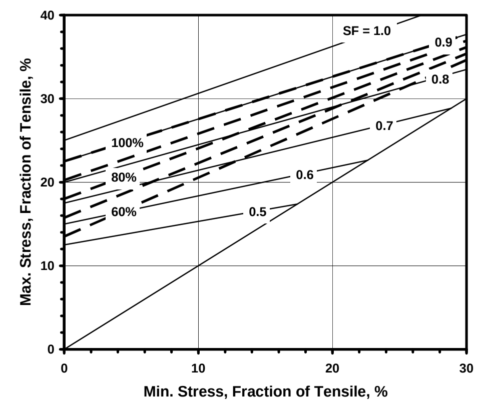

Figure 5

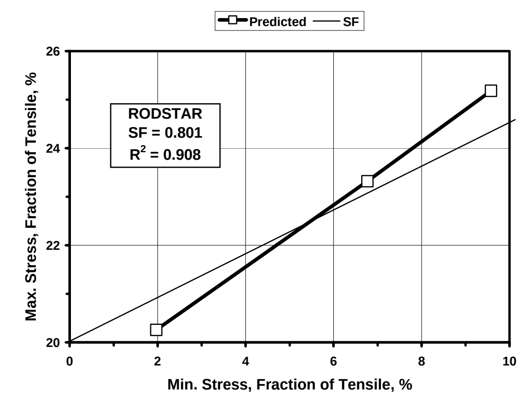

Figure 6