COEFFICIENT OF ISOTHERMAL OIL COMPRESSIBILITY FOR RESERVOIR FLUIDS BY CUBIC EQUATION-OF-STATE

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ABSTRACT
Coefficients of isothermal oil compressibility are usually obtained from reservoir fluid analysis. Reservoir fluid analysis is an expensive and time consuming operation that is not always available when the volumetric properties of reservoir fluids are needed. For this reason correlations have been developed and are being developed for predicting fluid properties including the coefficient of isothermal oil compressibility.

This study developed a mathematical model for predicting the coefficient of isothermal oil compressibility based on Peng-Robinson Equation of State (PR EOS). A computer program was developed to predict the coefficient of isothermal compressibility using the developed model. The predicted coefficient of isothermal oil compressibility closely matches the experimentally derived coefficient of isothermal compressibility.

INTRODUCTION
The coefficient of isothermal compressibility is defined as the fractional change of fluid volume per unit change in pressure at constant reservoir temperature. The coefficient of isothermal oil compressibility, c_o, is usually determined from the pressure-volume measurements of reservoir fluids. These data are usually obtained from reservoir fluid analysis. Isothermal compressibility is used in a wide range of calculations involving production and exploration of hydrocarbon reservoirs. Some of the reservoir engineering applications for the isothermal compressibility include well testing analysis, material balance calculations and metering[18].The coefficient of isothermal oil compressibility can be expressed mathematically as shown in equation 1[29].

$$c_o = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \qquad 1$$

The isothermal oil compressibility is a point function and it can be calculated from the slope of a pressure versus specific volume curve, or from the differentiation of an equation of state, or a correlation involving compressibility factor z, density or formation volume factor [26, 10]. There are also a number of correlations that are available to calculate the isothermal compressibility.

Laboratory studies data are required to construct a curve of pressure versus specific volume. Such data are not always available due to the cost of taking a reservoir fluid sample and the accuracy of the data depends on the reliability of the collected reservoir fluid samples. Correlations are good alternative when experimental values are not available, but most correlations are only applicable with a good degree of accuracy for a well-defined range of reservoir fluid characteristics or a given geographical area.

The significance of this research is the development of a purely empirical mathematical equation based on a cubic equation-of-state (EOS) to predict the isothermal compressibility of reservoir fluids.

The cubic equation-of-state used in this research is the Peng-Robinson EOS.

Coefficient of Isothermal Oil Compressibility above the Bubble Point Pressure

Above the bubble point pressure, the coefficient of isothermal oil compressibility does not vary significantly and can be defined by equation 1. The approach to this research is to use the Peng-Robinson EOS to determine of the volume and to determine the differential term in equation 1. The volume term is defined from the volume expansion of the Peng-Robinson EOS and is expressed in equation 2.

$$V^3 - \left(\frac{RT}{p} - b\right)V^2 + \left(\frac{a}{p} - 3b^2 - \frac{2bRT}{p}\right)V + \left(b^3 + \frac{RTb^2}{p} - \frac{ab}{p}\right) = 0 \qquad 2$$

Equation 2 is cubic in volume and could yield three real roots. From a typical pressure-specific volume curve, the highest value of the volume corresponds to the vapor volume, while the lowest volume corresponds to the liquid. The middle root is of no physical significance.

The differential term of equation 1 is derived form the Peng-Robinson EOS and expressed in equation 3.

$$\left(\frac{\partial V}{\partial p}\right)_T = \frac{(V-b)^2\left(V^2+2bV-b^2\right)^2}{2a(V+b)(V-b)^2 - RT\left(V^2+2bV-b^2\right)^2} \qquad 3$$

Combining equations 2 and 3, gives the mathematical model for predicting the coefficient of isothermal oil compressibility at pressures above the bubble point pressure. Thus, the equation for predicting the coefficient of isothermal compressibility for pressures above the bubble point is expressed in equation 4.

$$c_o = \left(-\frac{1}{V}\right)\frac{(V-b)^2\left(V^2+2bV-b^2\right)^2}{2a(V+b)(V-b)^2 - RT\left(V^2+2bV-b^2\right)^2} \qquad 4$$

Coefficient of Isothermal Oil Compressibility below the Bubble Point Pressure

At pressures below the bubble point pressure, there are free gases in the reservoir fluids. There is need to account for the free gas in the system in order to predict the coefficient of isothermal oil compressibility. To determine the volumes of gas and liquid at pressures below the bubble point pressure, flash calculations were conducted to simulate the constant composition experiment. The volume of gas computed from the flash calculations was used to calculate the coefficient of isothermal gas compressibility. The defining equation for the coefficient of isothermal compressibility is given as [29, pp173]

$$c_g = \frac{1}{p} - \frac{1}{z}\left(\frac{\partial z}{\partial p}\right)_T \qquad 5$$

The partial differential of compressibility factor with respect to pressure at constant temperature for the PR EOS is derived as,

$$\left(\frac{\partial z}{\partial p}\right)_T = \frac{\frac{bp}{(RT)^3}\left(2a-3pb^2\right) + \frac{1}{(RT)^2}\left(6zpb^2-2pb^2-za\right) + \frac{zb}{(RT)}(2-z)}{\frac{p}{(RT)^2}\left(a-3pb^2\right) + \frac{2bp}{(RT)}(z-1) + z(3z-2)} \qquad 6$$

Thus the coefficient of isothermal gas compressibility can be computed using this developed model,

$$c_g = \frac{1}{p} - \frac{1}{z}\frac{\frac{6zpb^2}{(RT)^2} + \frac{2zb}{RT} + \frac{2abp}{(RT)^3} - \frac{z^2b}{RT} - \frac{za}{(RT)^2} - \frac{2pb^2}{(RT)^2} - \frac{3p^2b^3}{(RT)^3}}{\left[3z^2 - 2z + 2z\frac{bp}{RT} + \frac{ap}{(RT)^2} - \frac{3(bp)^2}{(RT)^2} - \frac{2bp}{RT}\right]} \qquad 7$$

The coefficient of oil compressibility at pressures below the bubble point is computed form equation 4 using the volume of oil computed from the flash calculations. The total coefficient of isothermal compressibility below the bubble point is computed by combining the coefficients of isothermal oil and gas compressibility below the bubble point pressure. The total coefficient of isothermal oil compressibility below the bubble point is computed from equation 8 using the fractional volume of the gas and liquid computed from flash calculations.

$$c_o = \frac{V_g}{(V_L + V_g)} c_g + \frac{V_L}{(V_L + V_g)} c_L \qquad (8)$$

Prediction Results

Figures 1 through 3 compares the predicted and the experimental values of the coefficient of isothermal compressibility.

Figure 1 through 3 shows the accuracy of the developed model in predicting the coefficient of isothermal oil compressibility. At pressures below the bubble point the developed model closely matches the experimentally derived values. The average relative error is 12.53 percent. The pressures below the bubble point pressure the deviation of the predicted model from the experimental increases. The increase is expected based on the accumulating error of the flash calculations used in computing the volumes of gas and liquid at pressures below the bubble point pressure. The result from this study gave a very good prediction of the coefficient of isothermal oil compressibility at pressures above the bubble point pressures compared with prediction methods in literature[38]. This study also provides a very good model to predict the coefficient of isothermal oil compressibility at pressures below the bubble point pressure.

CONCLUSIONS

This study presents a mathematical model based on PR EOS to predict the coefficient of isothermal compressibility at pressures below and above the bubble point pressure. The study also presented a mathematical model to predict the coefficient of isothermal gas compressibility. The results shows that the developed model predicts the coefficient of isothermal compressibility with a good degree if accuracy.

Nomenclature

a	Attraction Parameter Term of EOS
b	Van der Waals co-volume
AAPD	Average Absolute Percent Deviation $AAPD = \frac{1}{N}\sum\left\lvert\frac{\text{Expt} - \text{Predicted}}{\text{Exptal}}\right\rvert$
c_g	Isothermal Gas Compressibility
c_L	Isothermal Liquid Compressibility
c_o	Isothermal Oil Compressibility
p	Pressure
p_b	Bubble Point Pressure
R	Gas Constant
T	Absolute Temperature
V	Molar Volume
z	Compressibility Factor

Subscript

g	Gas
o	Oil

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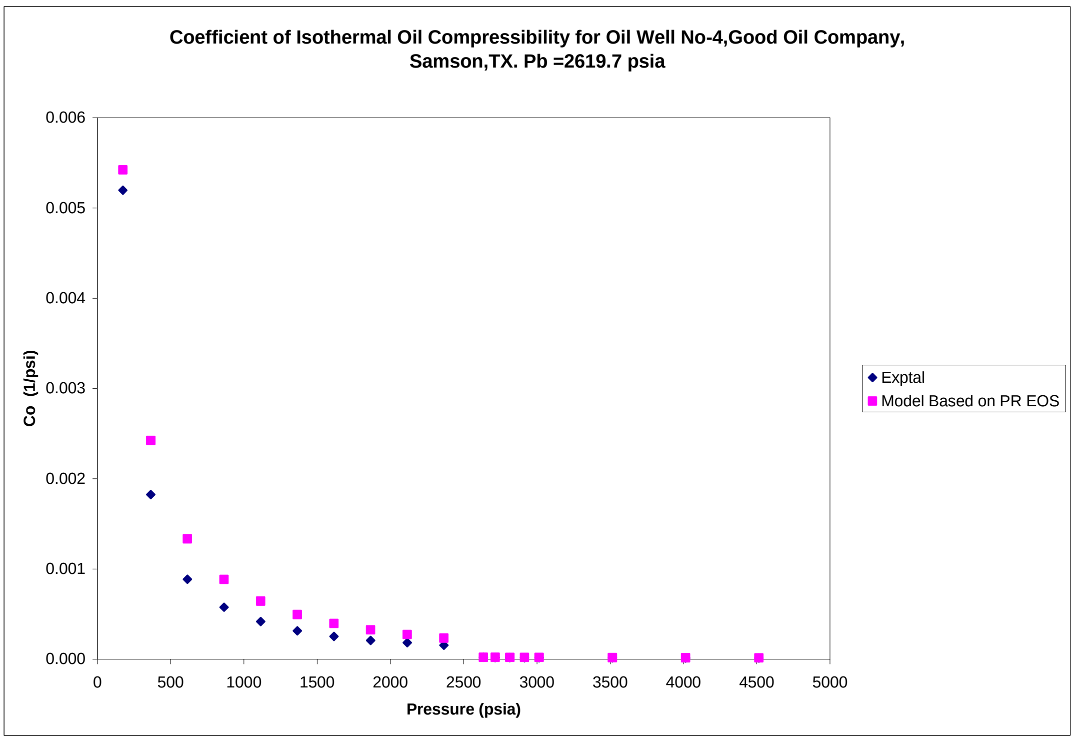

Figure 1 - Predicted Coefficient of Isothermal Oil Compressibility for Oil Well No. 4, Good Oil Company. Samson, Texas.

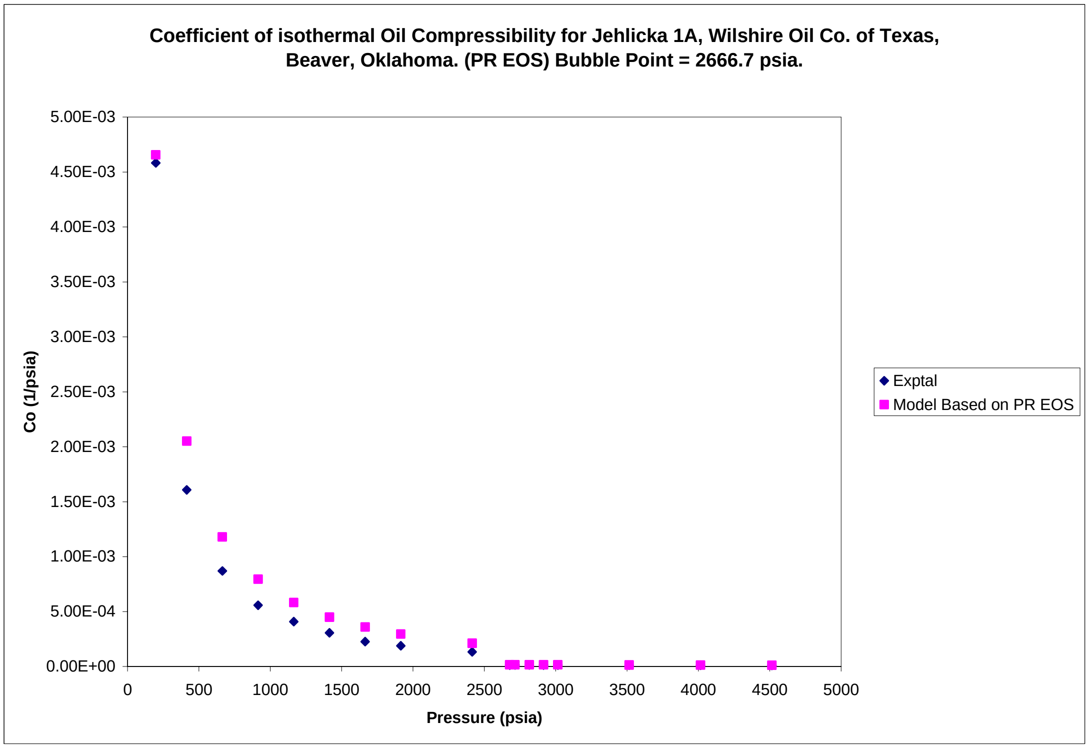

Figure 2 - Predicted Coefficient of Isothermal Oil Compressibility for Jehlicka 1A, Wilshire Oil Co. of Texas, Beaver, Oklahoma.

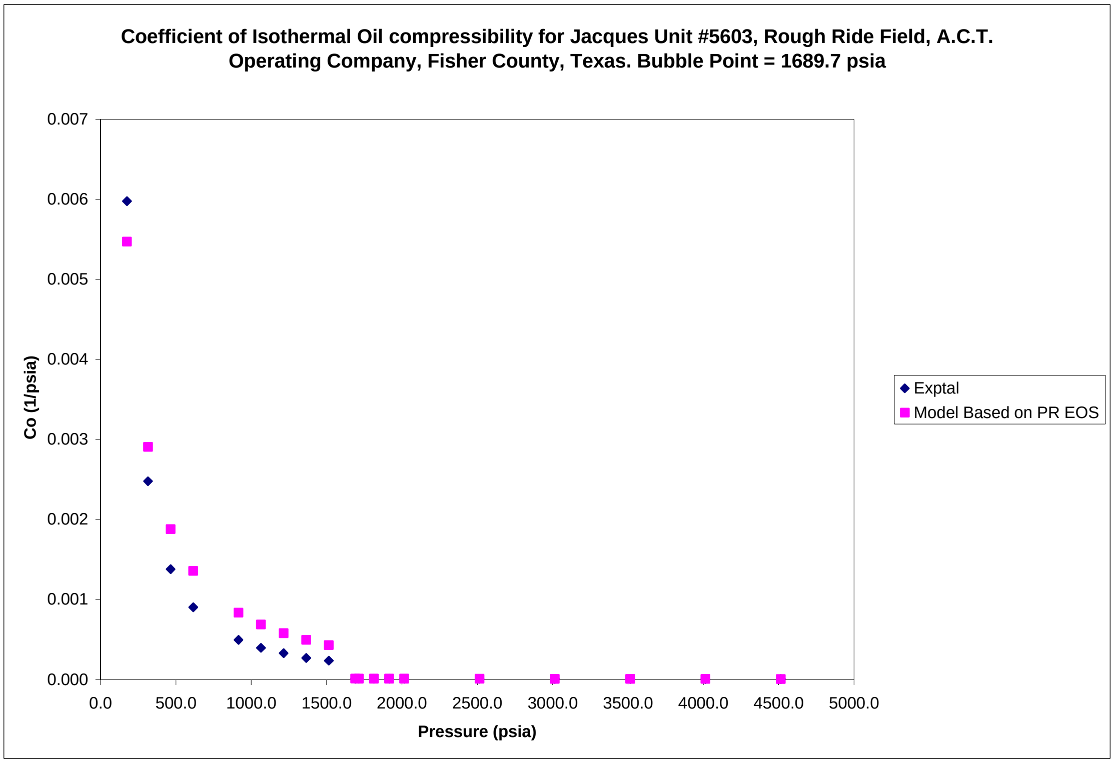

Figure 3 - Predicted Coefficient of Isothermal Oil Compressibility for Jacques Unit #5603, Rough Ride Field, A.C.T. Operating Company, Fisher County, Texas.