VIBRATION PROBLEMS IN OIL WELLS

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ABSTRACT

The dynagraph animater is a device whereby we reproduce in miniature what happens at the pump as disclosed by the dynagraph. Guided by the dynagraph, we construct charts upon a cylindrical surface so that when viewed, while, rotating, through a fixed slot the motion of the rod system of the well and the pump action become animated - that is, they come to life and move, though on a reduced scale, quite as in the actual pumping well. Even blind men say, "I see" - meaning "I understand" - and so the animater was devised to aid the imagination in comprehending the complicated behavior of the enormous rod system and clarify our thinking on difficult problems.

INTRODUCTION

Every dynamic system has a definite fundamental natural frequency of vibration. This is true because all materials possess mass and elasticity and, therefore, are subject to vibration. Sometimes these vibrations are essential and desirable, sometimes of no consequence, and sometimes they are troublesome and not wanted. Usually it is desirable to know what the natural frequency of a system is, so that the design or operation, or both, may be governed to make use of or to avoid these vibrations as best suits the purpose.

It is apparent that in every-day engineering practice vibration problems are not given the consideration which is their just due. This is largely due to the fact that vibration problems have not been very well understood in the past. This need no longer be true; for the great work of Timoshenko, Vibration Problems in Engineering, is quite simple and clear. His exposition of the theoretical mathematics involved is unusually complete, and leaves little to be desired. Most vibration problems need only a wise application of the theory for their solution. It is the purpose of this paper to encourage a wider application of this engineering aspect to problems in the petroleum industry, and to present certain definite information for ready reference to that end.

PENDULUM FORMULA

For the purpose of clarity in following the discussion, some of the fundamental principles of vibration will first be reviewed. In order to get at the simplicity of the problem, we must go to simple illustrations and analogies. First, the fact must be kept in mind that all vibrations, regardless of their nature, have their root in the fundamental principle that pure vibrations are simple harmonic motion. Hence, any simple harmonic motion may be used for a study of the fundamental principles. The simple pendulum serves this purpose admirably. The formula for the time of one complete vibration of a simple pendulum is given by:

$$t = 2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

where:

t = the time, in seconds.
l = the length, in feet, from the support of the center of gravity.
g = the acceleration of gravity, in feet per second per second

The frequency (f) per second is:

$$f = \frac{1}{t}$$

The frequency (F) per minute is :

$$F = \frac{60}{t} = \frac{60}{2\pi\sqrt{\frac{l}{g}}}$$

$$= \frac{53.8}{\sqrt{l}} \text{ for } g = 32.2 \text{ ft. per sec. per sec.} \qquad (2)$$

Thus we see, for a fixed value of g, the time and the frequency are a function of the square root of the length l. In the general equation for vibrations, we have:

$$t = 2\pi\sqrt{\frac{W}{Kg}} \qquad (3)$$

where:

W = the weight, in pounds.
K = the spring constant of the system defined as the force necessary to produce a deflection of unit distance or the couple necessary to produce a deflection of one radian.

In the case of the simple pendulum for small displacements, the sine of the angle is numerically equal to the angle expressed in radians, and then:

$$K = \frac{W}{l} \qquad (4)$$

When this is substituted in the general equation (3), W disappears, and we have equation (1).

In the case of a concentrated weight suspended from a spring, the value of $\frac{W}{K}$ equals the deflection of the spring (by definition of K) due to the weight W. If the deflection is represented by d, then:

$$t = 2\pi\sqrt{\frac{d}{g}} \qquad (5)$$

This shows that the period of this system is equal to that of a simple pendulum whose length l is equal to the deflection d of the spring due to the weight W. This regards the spring as being of very small mass as compared to the total mass.

Thus knowing the deflection of such a system, its period may be calculated by substituting a simple pendulum whose length l is equal to the deflection d of the system.

In the case of a string of sucker rods, the weight is not concentrated, but distributed along its entire length. The motion of each section of the rods is different from the motion of every other section and, hence, this formula does not apply. It has been suggested that this formula be applied to determining the frequency of sucker rods by substituting their elongation due to their own weight in the simple pendulum formula. It is here pointed out that this result will be about 10 per cent in error for the reason just disclosed.

VELOCITY OF STRESS TRANSMISSION

There is another method which does apply, and has to do with the velocity of stress transmission in steel, which is another way of saying the velocity of sound.

If a sudden force be applied to the fixed end of a rod (fixed at one end only) that force is transmitted to the other end of the rod at the speed of sound in the form of a longitudinal wave. At the free end of the rod that wave is reflected and returns, but loses phase by one-half a wave. At the fixed end it is reflected without loss of phase, so that the rod contains only one-quarter of a wave length; hence, the length of the rod is one-quarter of a wave length. It is fundamental, of course, that the frequency is the velocity of stress transmission divided by the wave length:

$$F = \frac{V}{L} \qquad (6)$$

where:

V = velocity of stress transmittion, in feet per second.
L = length of wave.

But L is 4D, where D = length of rod.
Hence:

$$F = \frac{V}{4D}$$

Newton has shown that the velocity of stress transmission is:

$$V = \sqrt{\frac{E}{d}} \qquad (7)$$

where:

E = modulus of elasticity.
d = density, mass per unit volume.

In applying this to a rod string, fundamental units must be used.

Thus:

E = 29,0(
d = -
g(

Wr = weight p
D = length o
a = area, in

But:

$\frac{Wr}{a}$ = 3.8 for all

This reduces to:

V = $\sqrt{29,(}$

Changed to feet

15,800 ft.

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Equation (8) gi
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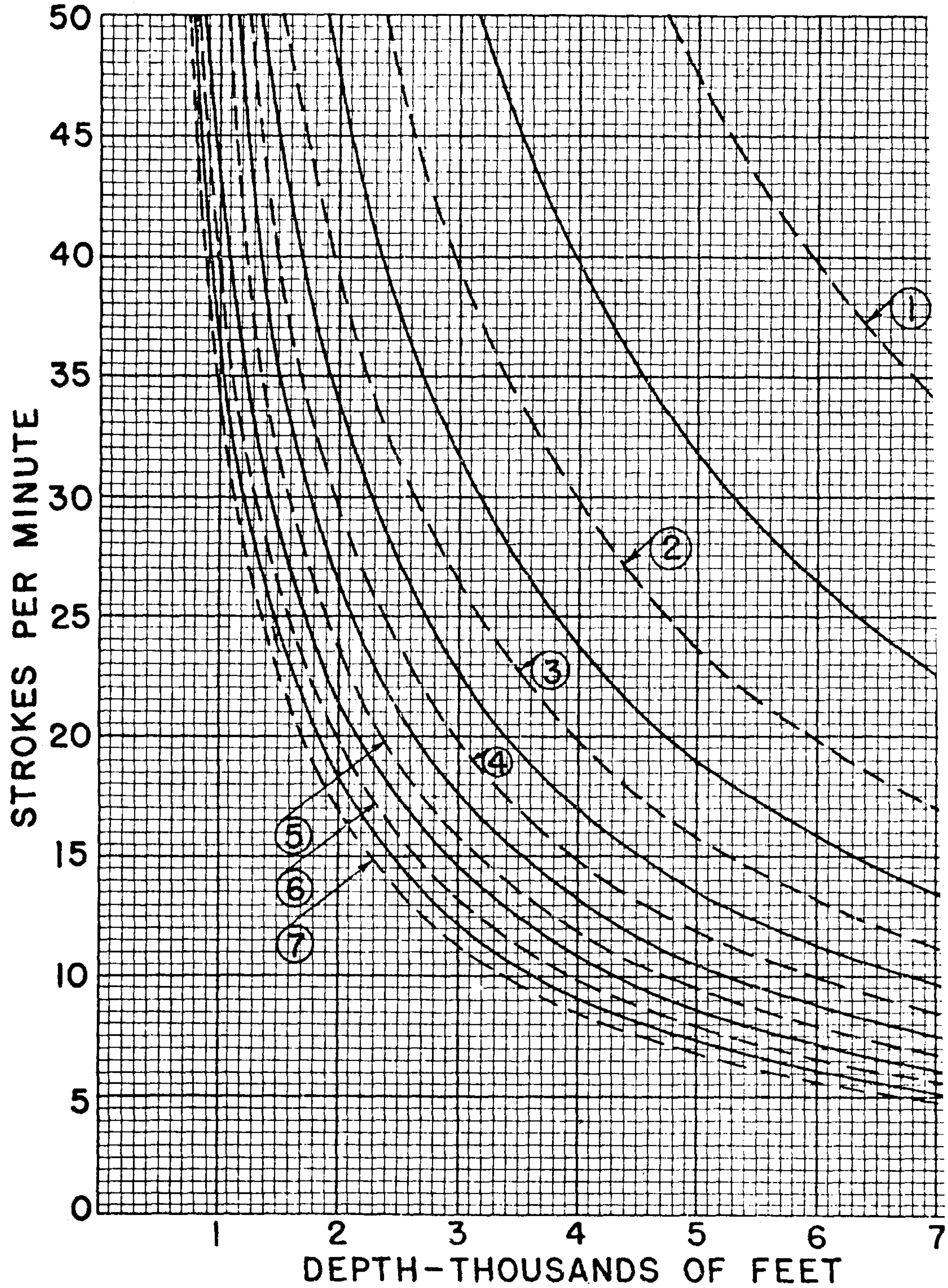


Non-synchronous ————
Synchronous — — — —

Pumping-Speed Curves.

FIG. 1

vibrations follow some other law. This force is commonly called the disturbing force, and more often is not a force due to harmonic motion. In the instance cited above it is a pure sine function; whereas in most mechanical problems it is an impure sine function, or a series of timed impulses. An excellent example of this is to be found in cable tooling, wherein the driller uses a very unsymmetrical motion (a very impure sine function, if you please) to produce a large vibration of the tools.

Again, if the timed impulses occur at intervals exactly equal to the natural frequency of the system, these are then known as "first-order" vibration. When the frequency of the impulses has exactly one-half the frequency of the system, the impulses are known as "second-order" vibration; one-third, "third-order," etc.

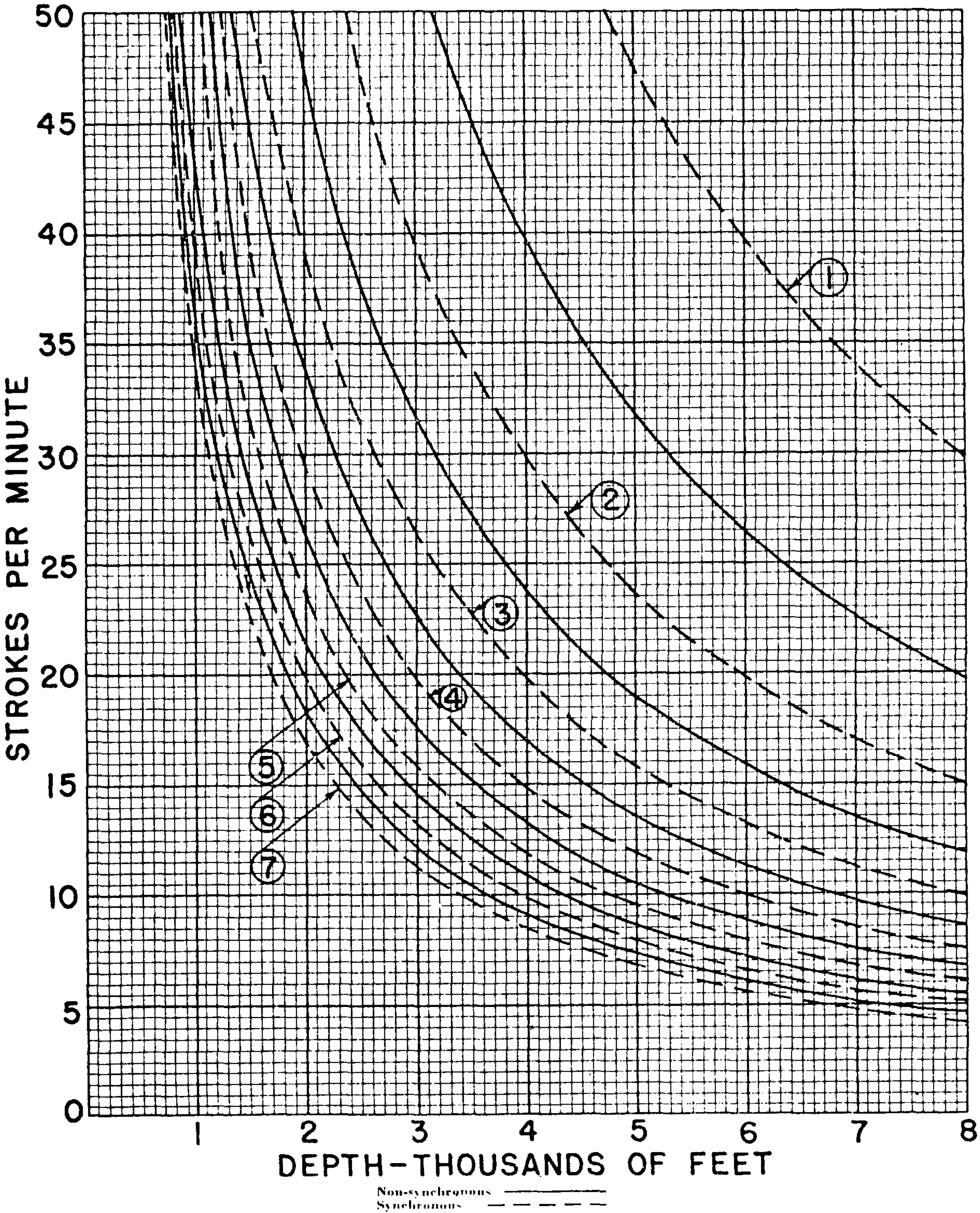
Obviously, other things being equal, the first-order vibration will be most severe, since energy is added to each wave. It is worthy of note that, as the frequency ofthe disturbing force becomes greater than the natural frequency of the system, its capacity to magnify vibrations very quickly disappears entirely. If, however, the frequency of the disturbing force is non-synchronous, then the impulses sometimes interfere with the vibrations, or rather start a second train of waves out of phase which partially interfere with the previously-excited waves.

DYNAMOMETER CARDS

Fig. 1 illustrates a series of curves, using depth for abscissa and frequency per minute for ordinates, showing the frequencies which are in synchronism with the natural frequency of the system based upon formula (8).

The dotted lines are first, second, third, etc., order frequencies as indicated. Approximately half-way between these curves are solid-line curves which are non-synchronous, or 1½-, 2½-, 3½-, etc., order vibrations.

The dynamometer card from a pumping well records the forces occurring in the system. These forces are the sum total of all forces including those due to the vibration of the rods. It is sometimes a difficult matter to separate or isolate the forces due to vibration. Perhaps it will be less confusing to an understanding of this matter to start from the opposite end, i.e., to start with simple vibrations, and develop them into a dynamometer card. Fig. 2 A shows a simple train of two complete waves, as shown by the solid line, plotted against equal intervals of time. If these two waves were exactly contained in one cycle, that cycle could be represented by folding one wave back upon the other, as indicated by the dotted line, thus making a continuous curve which would repeat itself cycle after cycle. This is accomplished in the dynamometer by reversing the motion of the card; however, the motion of the card represents equal displacements, and not time. The displacement at the beginning and end of the stroke is much smaller for equal intervals of time than at mid-stroke.
Fig. 2 B shows the distortion of the wave due to the non-uniform motion of the card.

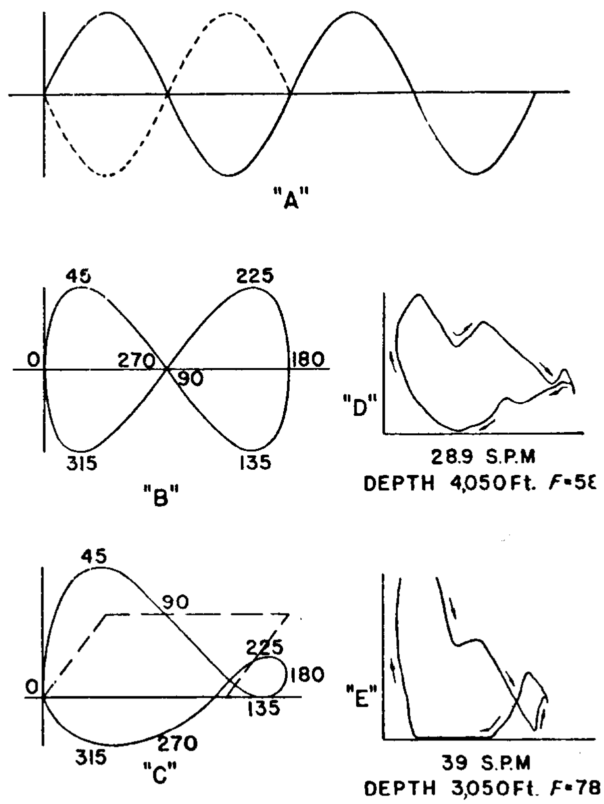


Pumping-Speed Curves.

FIG. 1

If a dynamometer card were taken from a string of rods suspended in a dry hole, and if the number of strokes per minute were exactly one-half the natural frequency of the rods, it would have the appearance of Fig. 2 B.

The net area of the card is zero, since no work is done. The horizontal line would represent the weight of the rods, but in a pumping well the force necessary to lift the fluid is also recorded in addition to the force due to vibration. In Fig. 2 C the dotted-line parallelogram represents the forces necessary to lift the fluid only. When these forces are added to the forces of vibration, a card is produced of the form shown by the solid-line curve in the same figure. In an actual dynamometer card, forces of friction and other disturbances due to many factors are superimposed upon this curve. They distort it, but do not alter its general form. We are not here concerned with the magnitude of the forces, but rather with the contour of the card. Cards of this contour can be identified quite readily as second-order vibrations.

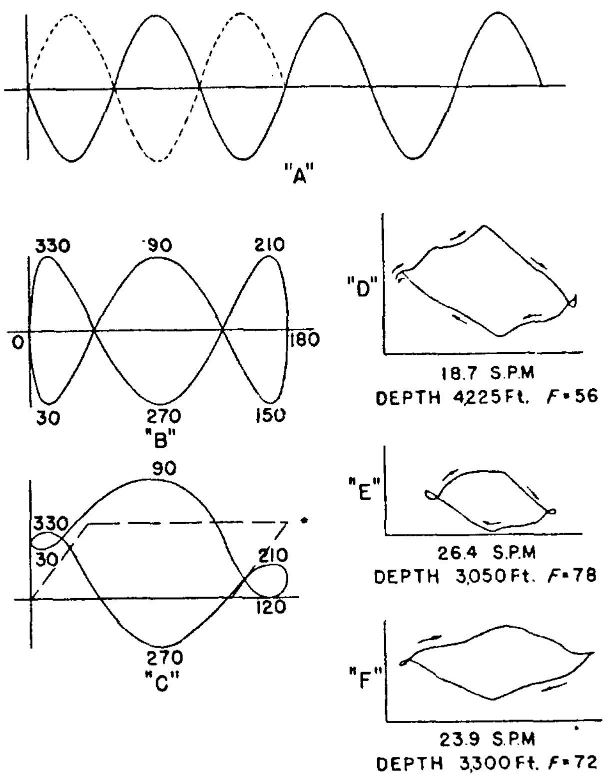


Second Order—Two Waves.

FIG. 2

Dynamometer cards of this class are shown at D and E in Fig. 2. The vibrations superimposed upon these cards are perhaps due to the vibration of the tubing or to a delayed action of the traveling valve, or the reaction of fluid waves in the tubing. This impulse peaks about in the middle of the up-stroke, and the waves from this impulse are damped out rather quickly. The characteristic small loop at the end of the upstroke has degenerated into a little tail due to interference or, rather, the superimposition of the impulse wave. It can be readily seen that a loop would be formed if this impulse wave was eliminated. In the case of the end E an impulse appears also, but it is not of sufficient magnitude to prevent the formation of the loop. The rods left the hanger on the downstroke; and, of course, the dynamometer had to stop at zero. On the upstroke the load for a time went beyond the scale of the instrument. In Fig. 3 a card is developed for the third-order vibrations by the same procedure as in Fig. 1. The steps A, B, and C in Fig. 2 correspond to the same steps in Fig. 1.

The small loop at both ends of the card is quite characteristic. The area of the card is concentrated in the center of the card, whereas in the second card the area of the card was concentrated at the beginning of the stroke.

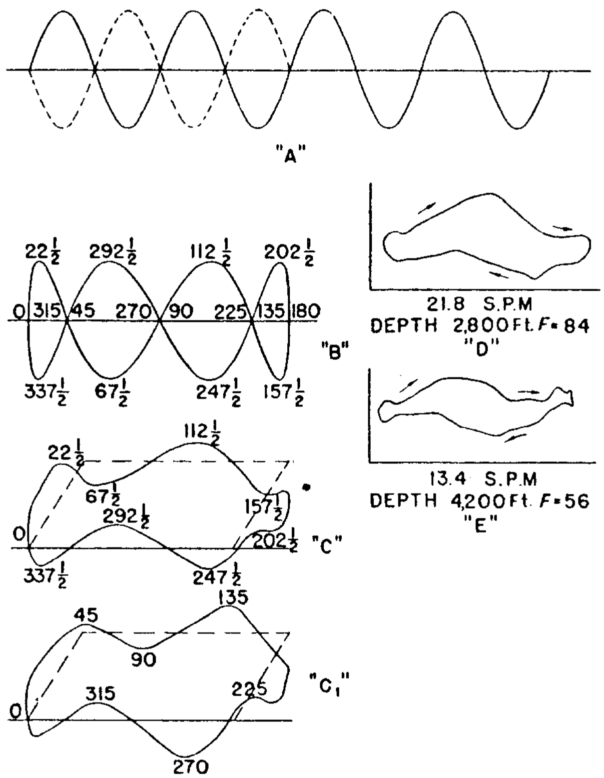


Third Order—Three Waves.

FIG. 3

Fig. 4 (**A**, **B**, and **C**) shows the development of a fourth-order card as before. **C** in the same figure shows the effect of a slightly different phase angle. The characteristics of this card are that the load at the end of the upstroke is as great as at the beginning of the stroke.

It is unfortunate that an exact fourth-order card was not available; however, card **D** is just a little faster than the fourth-order, and card **E** is a little slower. This shifts the phase angle, but does not wholly destroy the card. In the actual card the fact must be kept in mind that the time element on the upstroke is not of necessity the same as on the downstroke. This depends upon the



Fourth Order—Four Waves.

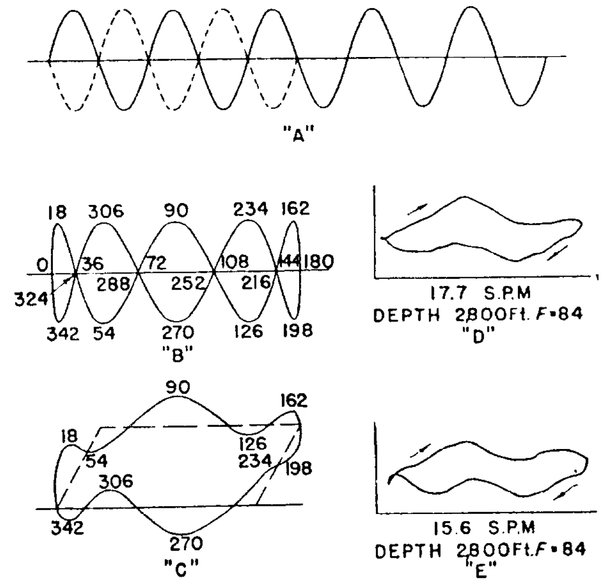
FIG. 4

degree of counterbalance. In cards of the higher orders this uneven motion tends to distort the shape of the card because of the greater number of waves in the card.

Fig. 5 (**A**, **B**, and **C**) shows the development of a fifth-order card. The finish of the upstroke is usually higher than the beginning. The examples given at **D** and **E** are not exact fifth-order cards and are, therefore, somewhat out of phase with the theoretical card; yet their form can be recognized.

While it is possible to develop the cards for the sixth-, seventh-, eighth-order, etc., it is deemed sufficient here simply to show only examples of these cards as taken by the dynamometer. Fig. 6 is such an exhibit, and needs no further comment.

At speeds other than that of an exact order, the card is modified by a change in phase angle. Let us suppose that we start with an exact-order card such as the second-order, and gradually diminish the speed until the



Fifth Order—Five Waves.

FIG. 5

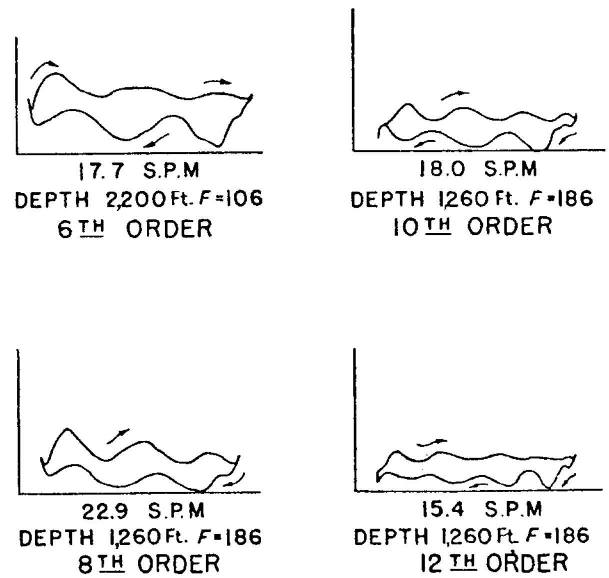


FIG. 6

third-order is reached. We should find that the second-order card form persisted in a modified form until the speed was the natural frequency divided by 2.4. For the want of a better terminology, let us call this the "2.4 order." At about the "2.5 order" the third-order form would begin to make its appearance. This form becomes

Also the loop at the beginning of the upstroke is usually higher than the loop at the end of the upstroke.

The card D in Fig. 3 is quite characteristic. The loop at the beginning of the upstroke is distorted by small disturbance, which may be engine impulse. There appear to be about 11 of these waves in the card, which corresponds to the number of explosions of the engine per revolution of the well crank. Card E is almost an exact duplicate of the theoretical card. Card F shows sufficient damping to prevent entirely the formation of the loop at the end of the upstroke, and maintain a higher load at that point rather than at the beginning of the upstroke. This card shows that the general form of a theoretical card is maintained.

Fig. 7 shows a series of cards from the second to the third order. These cards are all from the same well. It is not difficult to follow this transition.

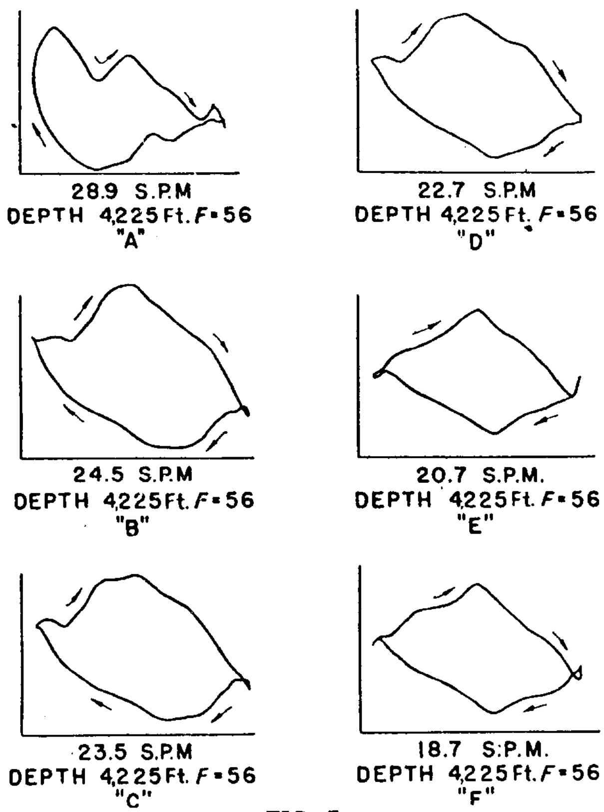


FIG. 7

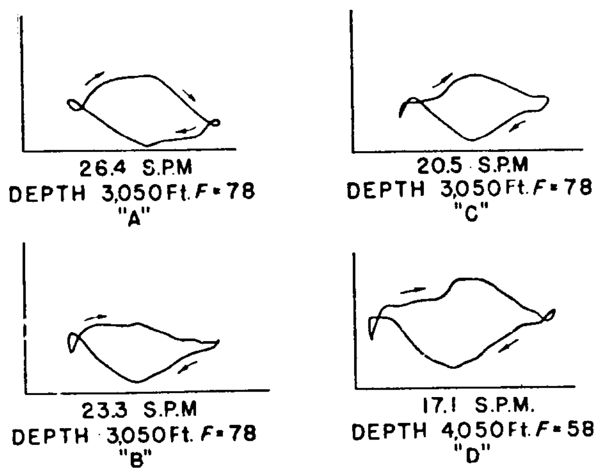


Fig. 8 shows the transition from a third order to the fourth order. The exact fourth-order card is missing in this series. This exact fourth-order card would no doubt have the appearance of theoretical card as shown in Fig. 3 C. The last card shown is a "3.4-order" card from another well.

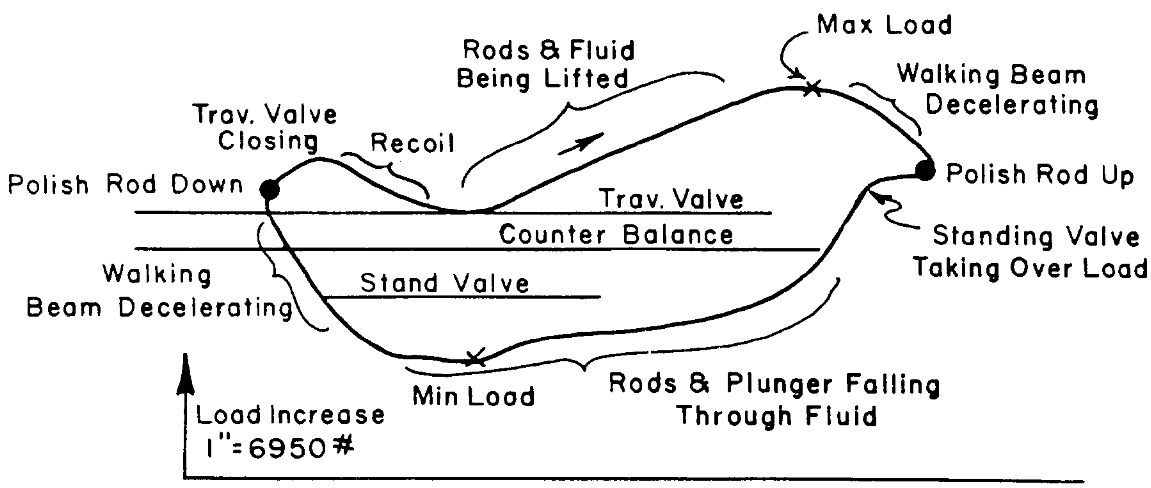


Figure 9 - Typical pumping cycle as represented on a dynamometer card. Various phases of polish rod travel and operation of the mechanical sub-surface pump are readily identified.

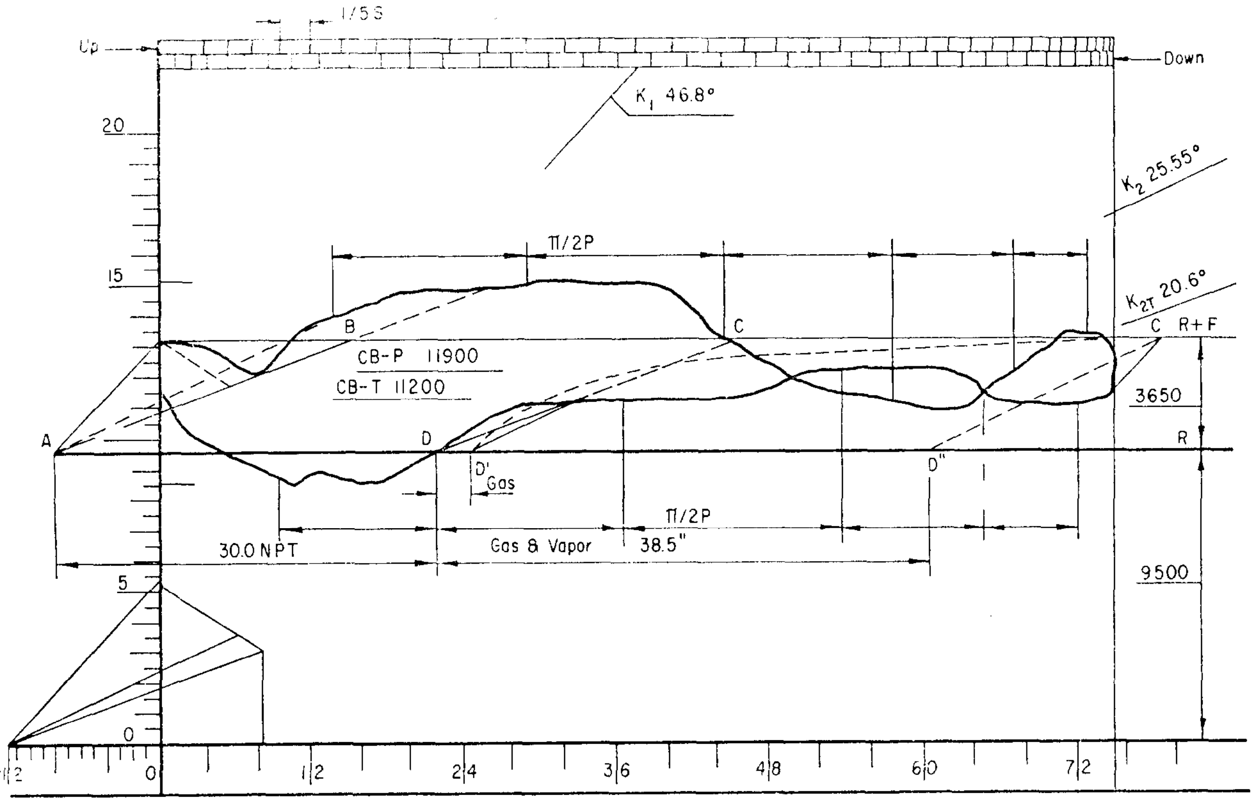


FIGURE 10 Graph is typical of gas pounding, but also presents confusing problem in analysis.

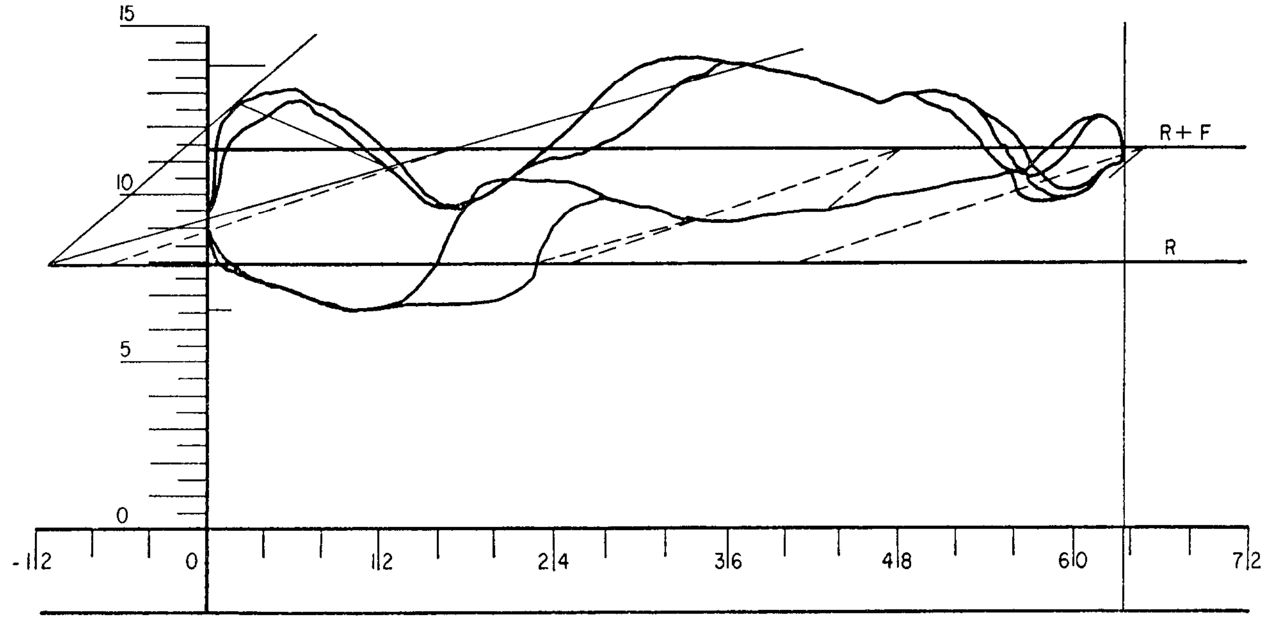


FIGURE 11 This graph and FIGURE 12 show how vapor interference progresses by degrees until the pump is completely vapor locked.

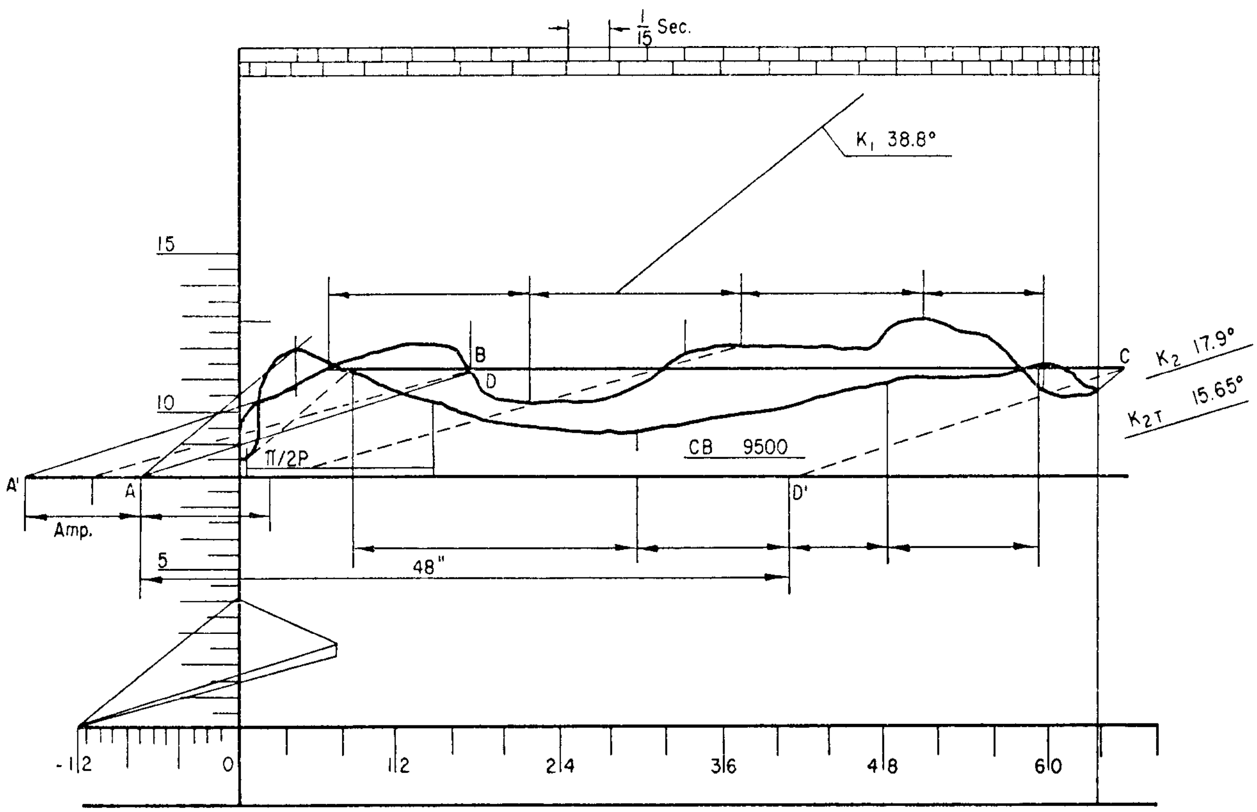


FIGURE 12 This graph was recorded 25 minutes after the one in FIGURE 11.

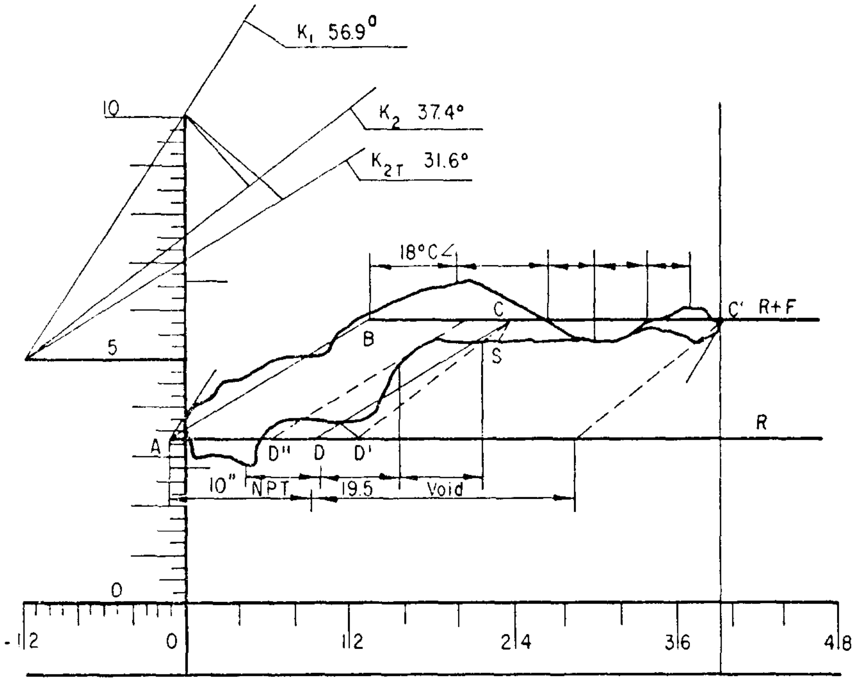


FIGURE 13 Graph illustrates pounding liquid.

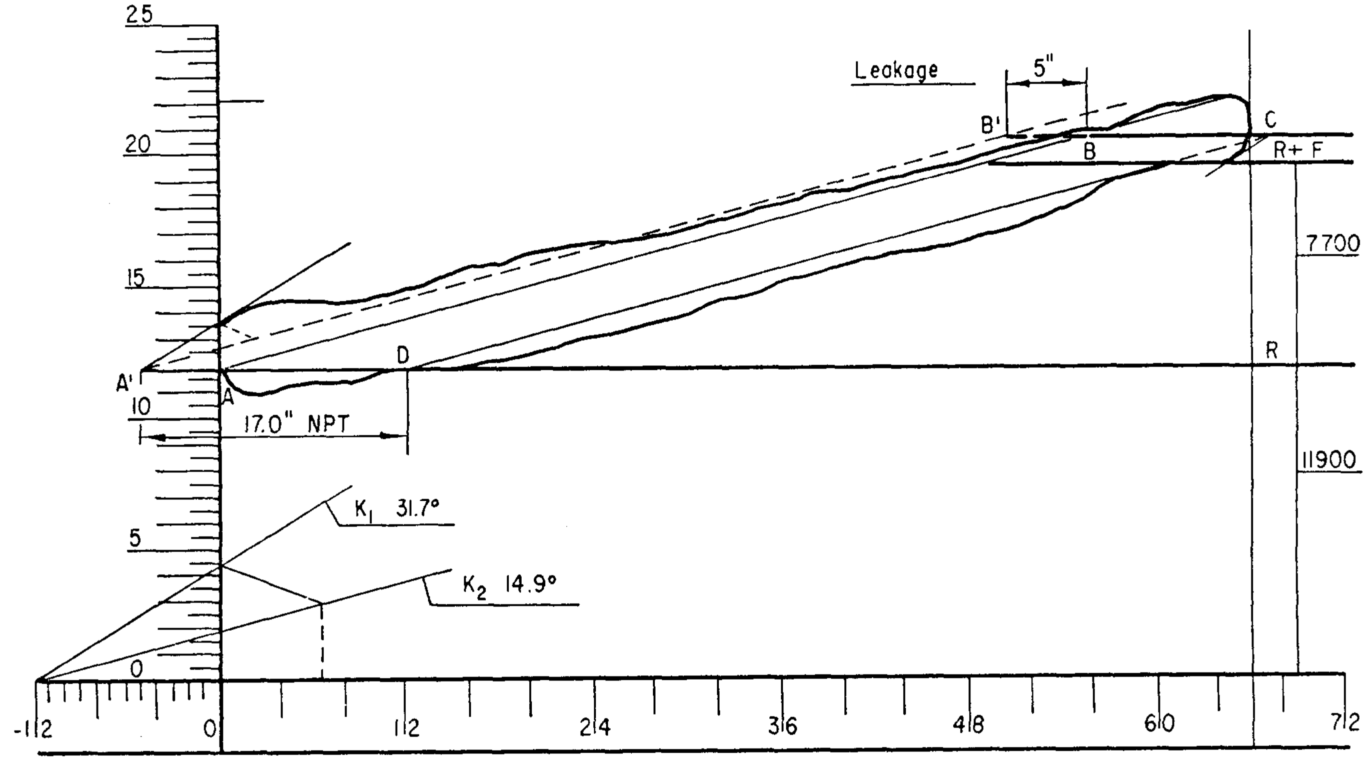


FIGURE 14 An example of excessive plunger load

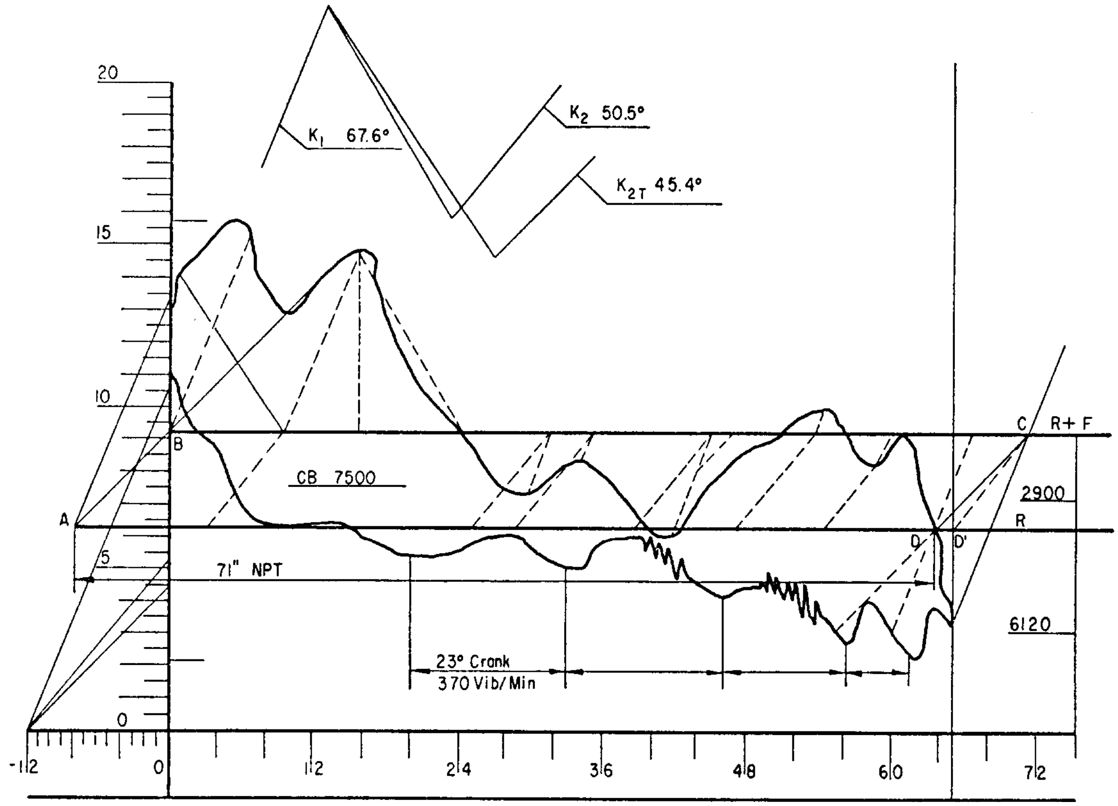


FIGURE 15 Graph in this case was accomplished by the crank-angle method.

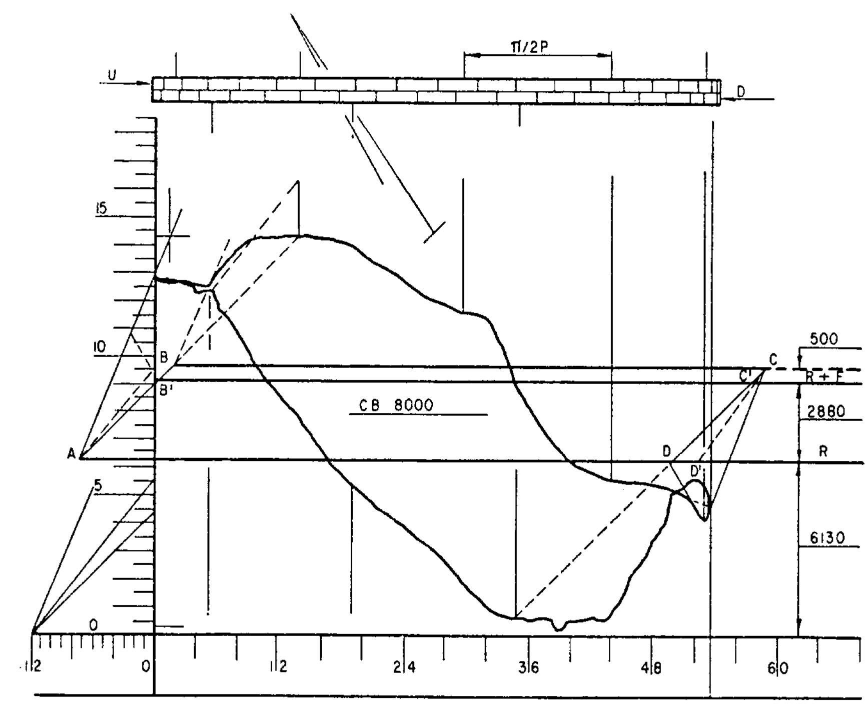


FIGURE 16 . Graph taken after making pumping cycle changes.

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