# Case Histories of Design and Implementation of Underbalanced Wells

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## Abstract

As operators strive to increase production from existing reservoirs with depleted reservoir pressure, the use of under-balanced drilling and underbalanced horizontal drilling is becoming more widely used. One of the primary problems to overcome in drilling under-balanced is designing a circulating "fluid" that has an equivalent circulating density below the reservoir pressure. This paper discusses the mathematical equations used to design air drilled "under-balanced" wells. It will also show how these equations were used to design four separate wells. The presentation of actual field data will validate these equations and computer modeling used. (See Table 1) The four wells will include 1) 17,000 TVD well drilled into the Ellenberger formation with reservoir pressure at 1200 psig, 2) horizontal well drilled into the Petit formation at 5900 ft. TVD with reservoir pressure at 210 psig, 3) horizontal wells drilled into the upper Penn Formation at TVD 7800 ft. and reservoir pressure of 1000 psig., and 4) deviated wells drilled into the Ellenberger formation below 13,000 feet TVD and reservoir pressure of 600 psig.

The mathematical equations used were developed by Dr. William C. Lyons of New Mexico Institute of Mining and Technology in Socorro, NM. Computer simulations for these wells were run by David Giffin on software developed by Boyum Guo and Dr. Lyons. Each of these wells was drilled using air drilling techniques. By measuring the pressure at both surface and bottom hole conditions, the validity of the mathematical models used has been established. Air or other compressed gas can be used to reduce equivalent circulating densities to accommodate almost any reservoir pressure.

## Introduction

Underbalanced drilling (UBD) can be defined as drilling with a fluid which while circulating exerts less equivalent circulating density (ECD) than the reservoir pore pressure being drilled. This paper concentrates on reservoirs that have pressures significantly lower than the ECD a fresh water fluid weighing 8.34 lbs/gal would exert. To achieve these lower ECD's it was necessary to introduce a gas, either air, membrane generated nitrogen, or natural gas, to the circulating fluid. And in fact, these gases were the major source of energy for the circulating system.

The use of air or natural gas as a circulating "fluid" in rotary drilling began in the early fifties'. The advantages of air drilling over conventional mud drilling have been long recognized and include faster penetration rates, longer bit life, preventing loss of circulation, continuous drill stem test, and less damage to the reservoir. The importance of maintaining adequate air flow is generally recognized in air drilling operations. Much disagreement exists in the industry, however, as to what constitutes "adequate flow rate". One of the earliest and still most widely used empirical models was developed by RR Angel[2]. Angel applied the Weymouth[3] friction factor to vertical flow within cased holed conditions to derive his annular pressure equation. More recent derivations on Angel's procedure have been done using Nikuradse's[4] friction factor correlation to provide models for annular pressure calculation. Boyum

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Guo and his colleagues extended the use of these formulas, applying them to both directional and horizontal wells. Most recently, Dr. Bill Lyons has derived that the differential pressure change, dP over incremental distance dh(ft), for the downward flow of a three phase system containing liquid, air and solids in the annulus is given by the expression:'

$$(1) \qquad dP = \gamma_{mix}\left[1 + \frac{fv^2}{2g\left(D_h^2 - D_p^2\right)}\right]dh$$

Substitution and integration of this equation into our air drilling system as shown in Appendix **A** yields:

$$(2) \qquad \int_{P_o}^{P_{bh}} \frac{dP}{\left[\dfrac{\omega_t}{\left(\dfrac{P_o}{P}\right)\left(\dfrac{T_{av}}{T_o}\right)Q_o + Q_m}\right]\left\{1 + \dfrac{f}{2g\left(D_h - D_p\right)}\left[\dfrac{\dfrac{P_o}{P}\left(\dfrac{T_{av}}{T_o}\right)Q_o + Q_m}{\dfrac{\pi}{4}\left(D_h^2 - D_p^2\right)}\right]^2\right\}} = \int_0^H dh$$

By substituting the von Karmin[6] "rough-pipe" flow equation for turbulent flow condition the friction factor equations:

$$(3) \qquad f = \left\{\frac{1}{\left[2\log\left(\dfrac{D}{e}\right) + 1.14\right]}\right\}^2$$

where

e for pipe is **0.00015 ft.**
e for open hole is 0.01 ft.

Equation (2) can be simplified for air and gas drilling by setting $Q_m=0$ and integrated to obtain a closed-form solution for $P_{bh}$. The closed-form solution for $P_{bh}$ is:

$$(4) \qquad P_{bh} = \left[\left(P_o^2 + b_a T_{av}^2\right)e^{\frac{2a_a H}{T_{av}}} - b_a T_{av}^2\right]^{0.5}$$

where

$$(5)\quad a_a = \left[ \frac{S}{R} + \frac{\frac{\pi}{4} D_h^2 (62.4)(2.7) K}{\frac{P_o Q_o}{T_o}} \right]$$

$$(6)\quad b_a = \frac{f}{2g\left(D_h - D_p\right)} \left(\frac{P_o}{T_o}\right)^2 \frac{Q_o^2}{\left(\frac{\pi}{4}\right)^2 \left(D_h^2 - D_p^2\right)^2}$$

$$(7)\quad f = \left[ \frac{1}{2 \log\left(\frac{D_h - D_p}{e}\right) + 1.14} \right]^2$$

Equation (2) must be integrated numerically for mist, aerated fluid, and foam drilling because $Q_m$ is no longer equal to zero.

$$(8)\quad \int_{P_o}^{P_{bh}} \frac{dP}{\left[\frac{\acute{\omega}_t}{\left(\frac{P_o}{P}\right)\left(\frac{T_{av}}{T_o}\right)Q_o + Q_m}\right]\left\{1 + \frac{f}{2g\left(D_h - D_p\right)}\left[\frac{\frac{P_o}{P}\left(\frac{T_{av}}{T_o}\right)Q_o + Q_m}{\frac{\pi}{4}\left(D_h^2 - D_p^2\right)}\right]^2\right\}} = \int_O^H dh$$

where

$$(9)\quad f = \left\{ \frac{1}{\left[2 \log\left(\frac{D_h - D_p}{e}\right) + 1.14\right]} \right\}^2$$

Computer simulations of these equations were used to model the wells described herein.

The minimum flowrate for all of the air and gas drilling models can be evaluated using the minimum kinetic energy (per unit volume) criteria. The criteria states that the minimum kinetic energy in the well (usually at its deepest and largest annular cross-sectional area) must be equal to or greater than the kinetic energy of sea level air with a velocity of 50 ft/sec (Angel's criteria).

The kinetic energy of sea level air is:

$$\text{(10)} \quad KE_{sl} = \frac{1}{2}\frac{\gamma_{mix}}{g}v_{sl}^2$$

Where $v_{sl}$ is the specific weight of air standard sea level conditions (lbs/ft) and $v_{sl}$ is the criteria reference velocity of 50 ft/sec.

The kinetic energy at the deepest and largest annular cross-sectional area is usually just above the collars or where the annulus increases in size such as at a liner top. Thus, the kinetic energy at these conditions is:

$$\text{(11)} \quad KE_h = \frac{1}{2}\frac{\gamma_h}{g}v_h^2$$

The computer derived solutions to the equations were used to model the wells listed as case histories prior to actual drilling. The computer simulation contains a ratio of Equation **(11)** to Equation **(1**0) as a non-dimensional kinetic energy index (KEI) to check minimum volumetric gas rate as stated herein step wise throughout the annulus of the wells. The aerated fluid and foam drilling models utilize a minimum cutting size instead of KEI.

Another technology that was used in several of these case histories was a nitrogen membrane separation unit. These separation units were first introduced to the UBD market in 1993[7]. In essence, they strip oxygen from the air stream and discharge an oxygen-depleted stream downstream to the booster compressor units. They have proven to be beneficial in reducing down hole burn off and oxygen induced corrosion'. The discharge stream is typically 50 - **60%** of the original input volume and contains 3-7% oxygen.

## Design Information

To design an underbalanced drilling fluid, the engineer must know the proposed well diagram, anticipated production while drilling, and reservoir pressure. Next, either Angel's Curves, prior UBD experience in the area, or Poettman and Berman's Two Phase Flow Chart' are used to determine a starting volumetric rate for the air, gas, and fluid. These starting flow rates, the well bore details, and the anticipated production rates are entered into the computer solution to these equations. Flowrates are then adjusted by trial and error to meet minimum lift criteria and volumetric rates available from compressors while checking for underbalance. The flow rate at which both minimum lift criteria and underbalance amounts are met is the starting flow rate for UBD. Rates are then adjusted for conditions monitored at the well site.

## Case History No. 1

**A** 17,000 feet TVD well drilled into the Ellenburger formation in Ward County, just south of Pyote, Texas. The reservoir pressure was expected to be 1200 psig. Original plans were to fluid drill the well to **15,900'** and then drill out with membrane generated nitrogen. Figure 1 shows the first proposed casing program and resulting minimum needed flow rates for adequate cleaning. The first proposed casing program was abandoned due to deviation problems and excessive wear on existing liners plus economics of supplying **3400** SCFM of membrane generated nitrogen. The next design involved running 5 ½ inch casing from surface to 15,900'. Upon drilling out, cones were lost off of the bit and never recovered. The results of this are the program represented by Figure 2.

The well was eventually drilled with mist instead of membrane generated nitrogen, because a complete dusting condition was never established. It should be noted that although there is published data for the suppression of down hole fires with mist', down hole fires have been experienced while mist drilling, and therefore some operators opt to use membrane generated nitrogen even while misting. Another phenomena seen while drilling this well was excess surface pressure when more than 1 gph of foamer was used. Surface pressure increased from 1300 - 1700 psig to 2500 - 3000 psig. This change of pressure was attributed to foam forming within the drill pipe. The pressure change will be discussed further in Case History #3. Table I shows pressures expected versus those measured while drilling for all of the case histories.

## Case History No. 2

The OXY USA Pirkle #2 was drilled in January of 1996[10]. The well was drilled into the Cretaceous Frost "A" zone of the lower Pettit limestone at 6,000 ft. TVD in the Carthage field, Panola County, Texas. A 1400 ft. lateral section was completed open hole with a bottom hole pressure of 185 psig. A 6 ¼ inch hole was drilled out of 7 inch casing landed at 84.25' 6252 ft. MD just into the top of the Frost "A" pay zone. A string of 3 ½ inch and 3 ½ inch heavy weight drill pipe was used to drill the 6 ¼ inch lateral.

The drilling rate peaked at an instantaneous rate of 318 ft/hr with an average penetration rate of 87 ft/hr. The well was drilled with 2500 SCFM and 2000 SCFM of membrane generated nitrogen to 7,002 ft MD. Surface pressures were 600 psig and 450 psig respectively. Computer modeling showed the surface pressures to be 580 and 490 psig respectively. From 7002 to 7625 ft. MD, the well was drilled with mist of 12 bph, 2-4 gph of foamer, and 2000 SCFM of membrane nitrogen. Surface pressures climbed from 450 to 900 psig on subsequent changes of BHA. Surface pressures predicted by the modeling were 440 and 900 psig respectively. Bottom hole pressures predicted by the model were 147 psig with 2500 SCFM membrane nitrogen, 119 psig with 2000 SCFM membrane nitrogen, and 147 psig with 2000 SCFM nitrogen and 12 BPH mist. The well flared gas while drilling with membrane nitrogen and mist at approximately 500 MSCFD.

Computer simulations of this well were later run to try to explain the changes in pressures, drilling rates, and gas flares observed while drilling. When nozzle diameters were reduced, a decreased volumetric fluid rate through the motor resulted which decreased power available to the downhole motor. These items are described in detail in Table 11.

TABLE 2
Equivalent Air Volume
Pirkle No. 2

| Stand Pipe pressure | EQGPM to Motor |
|---|---|
| 450 psig | 300 |
| 900 psig | 110* |

*change of nozzles from open to 6/32

Eq. (12) $$Q\,(scfm) = OF\,x\,\sqrt{TF}\,x\sqrt{DP\,x\,SP}$$

Eq. (13) $$V_2 = \frac{V_1 P_1 T_2}{P_2 T_1}$$

Eq. (14) $$V_2 = 0.455 \frac{V_1 P_1 T_2}{P_2 T_1}$$ 11

### *Case* History No. 3

Four horizontal wells with a total of six laterals were drilled for Marathon, in Eddy County, New Mexico. These wells were all whipstock exits of existing well bores drilling 4 ¾ inch holes. Lateral length and direction varied, and the example below uses the first well. This well had a 7,600 ft. TVD kick off point. The curve section was drilled out of the 7 inch csg using fluid with a radius of 130 ft.

Wells were drilled using 2-7/8 inch O.D., 10.4#, X-95 AOH drill pipe. Flow rates while drilling ranged from 1350 to 1450 SCFM of membrane generated nitrogen containing 3% oxygen. Misting fluid rates were 20 BPH containing 3-6 gph foamer, 3 gph corrosion inhibitor, and 2 gph of $H_2S$ scavenger. Surface pressures measured at the booster compressor ranged from 600 to 1450 psig. This pressure of 1450 psig was 620 psig higher than the anticipated pressure from the computer simulations. **A** Halliburton Energy foam program was able to roughly model the surface pressures seen while drilling.

Actual BHP monitoring showed 320 to 480 psi. This is compared to the 356-466 psig predicted by the model. The reservoir pressure in the field was thought to be 1000 psig. Production rates while drilling closely followed rates used to model the wells; therefore we are confident in the accuracy of all data input into the model. On the second well drilled, a large fracture was encountered, and circulation was lost. Tripping back to vertical and unloading the hole reestablished circulation; drilling continued 300 ft. MD past the first fracture with complete circulation until a second fracture was encountered and circulation was lost again. A jet sub and increased membrane nitrogen volume was recommended to re-establish circulation on the fourth well, but the well was TD before this method was attempted.

### Case History No. 4

Underbalanced drilling into the Pucket Field, Pecos County, Texas has produced some of the largest gas wells drilled in the Permian Basin within the past 10 years. The Chevron USA Robbins **A-8** was reported at 30 MMSCFD CAOF and 14 MMSCFD production[13]. The Puckett Field has a field wide BHP **of** approximately 550 – 800 psig. The depth to the Ellenburger varies slightly but is at approximately 13,100 to 14,100 ft. The case histories focus on three different well bore designs.

The first well bore design consisted of 5 ½ inch casing set at 14,914 feet, a window cut at 13,180 feet MD, 4 ½ inch hole drilled to 15,083' MD with 1320 SCFM of membrane generated nitrogen, and 7 BPH mist through 2-7/8 inch PH-6 and PX-95 DP.

The second well bore design consisted of 5 ½ inch casing set at 14,870 feet, a window cut at 13,200 feet MD and used 2350 SCFM of membrane generated nitrogen or 3.0 MMSCFD of 0.6 gravity natural gas with no mist through 2-3/8 inch drill pipe.

The third well bore design had 7 inch casing set at 14,100 feet, 6 ¼ hole drilled to 15,600 ft. using 3 ½ inch drill pipe, 2350 to 2700 SCFM of membrane generated nitrogen, and 18 BPH of mist with 4 gph of foamer.

The first well bore design showed surface pressures of 1150 psig on the model and 1450 psig in the field, but more importantly, pressure in the annulus was 550 to 650 psig which was above reservoir pressure. The well flared very little gas while being drilled and production was well below anticipated levels. The second well bore design showed surface pressure of 900 to 1110 psig and the measured pressure was 850 to 1100 psig with membrane nitrogen. The second design with natural gas showed surface pressures of 550 psig and those measured were 570 psig. The expected BHP were 305 and 310 psig respectively with membrane nitrogen and gas. The third well bore design showed surface pressures of 510 psig as compared to 510 psig measured. Calculated bottom hole pressure was 217 to 290 psig. All of the wells drilled with designs two and three had sizable flares while drilling (estimates from 6 – 20 MMSCFD) and are good producing wells.

## Conclusions

1. Updated models derived by Lyons for air and gas flow rates have been proven to be accurate. They model both expected surface pressure and bottom hole pressure quite accurately.
2. Pressure predictions for foam through small tubulars are still not very precise. Other models available need to be verified.
3. More reliable methods of reporting real time bottom hole pressures can prove invaluable for monitoring UBD system.

## Nomenclature

$a_a$= a function defined by eq. *(5)*.
$b_a$= a function defined by eq. *(6)*.
$D_h$= hole diameter, ft [m].
$D_p$= pipe outside diameter, ft [m].
e= absolute roughness of pipe, ft [m].
f= Fanning friction factor.
G= geothermal gradient, $^oF/ft$ [$^oC/m$].
g= gravitational constant, ft/sec$^2$ [m/sec$^2$]
h= depth below the surface to any point under consideration, ft [m].
H= vertical straight hole depth, ft [m].
K= ROP, rate of penetration, ft/hr [m/hr].
KE= kinetic energy.
$KE_h$= Kinetic energy at depth (h) in annulus eq. (12).
$KE_{SI}$= Kinetic energy at sea level eq. (11).
KEI= Kinetic energy index.
P= pressure, lb/ft$^2$ [Pa} absolute.
$P_{bh}$= pressure at the bottom of vertical hole, lb/ft$^2$ [Pa] absolute.
Ph= pressure at a depth (h) of vertical hole, lb/ft$^2$ [Pa] absolute.
$P_o$= pressure at surface, lb/ft$^2$ [Pa] absolute.
Q= circulation rate, scf/min [scm/sec].
$Q_m$= circulation rate of fluid, ft$^3$/sec [m/sec].
$Q_o$= circulation rate into compressor system, ft$^3$/sec, [cm/sec].
R= gas constant 53.3 ft-lb/lb-$^oR$.
S= specific gravity of gas related to air, dimensionless.
T= ambient temperature, $^oF$ ["C].
$T_{av}$= average temperature in well bore, $^oF$ ["C].

$T_o$= temperature at sea level, °F [°C].
V= velocity, ft/sec [m/sec].
$\acute{\omega}_g$= weight flow rate of gas or air into well, lbs/sec [Kg/sec].
$\acute{\omega}_m$= weight flow rate of fluid into well, lbs/sec [Kg/sec].
$\acute{\omega}_s$= weight flow rate of solids drilled, lbs/sec [Kg/sec].
$\acute{\omega}_t$= total weight flow rate in annulus, lbs/sec [Kg/sec].
$\gamma$ = specific gravity of item related to water, dimensionless
y, = specific gravity of fluid related to water, dimensionless
$\gamma_{mix}$ = specific gravity of mix related to water, dimensionless
$\gamma_o$ = specific gravity of air or other gas to compressor intake, dimensionless
$\gamma_h$ = specific gravity of mix at depth (h) in well bore, dimensionless

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## Appendix A

The differential pressure change, dp (lbs/ft$^2$, abs) over an incremental distance, dh (ft), for the downward flow of a three phase fluid in the annulus is given by the expression:

$$d P = \gamma_{mix}\left[1 + \frac{fv^2}{2g\left(D_h^2 - D_p^2\right)}\right] d h \ldots \text{(A-1)}$$

Where $\gamma_{mix}$ is the specific weight of the mixture of air (or gas), incompressible fluid (mud), and rock cuttings (lbs/ft$^3$), f is the fanning friction factor, v is the average velocity in the annulus (ft/sec), $D_h$ is the diameter of the borehole (ft), $D_p$ is the outside diameter of the drill pipe (ft), g is the acceleration of gravity (**32.2** ft/sec$^2$).

Note that the above is composed of two parts. The first term on the right side of Equation **(A-1)** describes the static pressure head of the mixed air (or gas), liquid (water, mud, or crude oil), and rock cuttings. The second term on the right side describes the pressure head due to friction loss.

The derivation will be carried out assuming a surface location at standard sea level conditions(i.e., API, $t_o = 60^oF$, and $p_o$ = **14.696** psia). The general form of the ideal gas law is:

$$\frac{P \quad RT}{\phantom{x}} \qquad \text{(A-2)}$$

Where R is the gas constant (**53.3** ft-lbs/lb- $^oR$), **S** is the specific gravity of the gas (**S** = **1.0** for air at standard conditions), T is temperature ($^oR$).

Using Equation **(A-2)** the specific weight of the atmospheric air or other gas going into the intake of the compressor system can be written as:

$$\frac{P_o S}{\phantom{x}} \qquad \text{(A-3)}$$

Thus the weight rate of flow of the air (or gas) into the well, $w_g$ (lbs/sec), can be written **as:**

$$\acute{\omega}_g = \gamma_o Q_o \ldots\ldots\ldots\ldots \textbf{(A-4)}$$

Where $Q_o$ is the volumetric flowrate of air (or gas) into the compressor system (ft$^3$/sec).

The weight rate of flow or drilling mud (or other incompressible fluid) into the well, $w_m$ (lbs/sec), can be written as:

$$\acute{\omega}_m = \gamma_m Q_m \ldots\ldots\ldots\ldots \text{(A-5)}$$

Where $Q_m$ is the volumetric flowrate of mud into the well.

The quantities $w_g$ and $w_m$ enter the well through the inside of the drill pipe at the top of the string and flow to the bottom of well through the inside of the string. They exit the inside of the string through the bit nozzles and entrain the drill bit cuttings and then the three phases (gas, incompressible fluid, and solids) flow to the surface in the annulus. The entrained weight rate of flow of solids, $w_s$ (lbs/sec), is:

$$\dot{\omega}_s = \frac{\pi}{4} D_h^2 (62.4)(2.7)k \text{ ..............................} \quad \text{(A-6)}$$

Where $k$ is the rate of penetration 9ft/sec). The quantity (62.4)(2.7) is the average sedimentary rock specific weight.

The total weight rate of flow, $w_t$ (lbs/sec), in the annulus from the bottom of the wells to the surface is:

$$\omega_t = \omega_g + \omega_m + \omega_s \text{ ....................................} \quad \textbf{(A-7)}$$

The specific weight of the gas at any position in the annulus can be written as:

$$\gamma = \frac{P\,S}{R\,T_{av}} \text{ ....................................................} \textbf{(A-8)}$$

Where $T_{av}$ is the average temperature of the well between at that position in the well annulus (over the length being considered). Therefore, the $w_g$ term can be written as :

$$\dot{\omega}_g = \gamma_o Q_o = \gamma Q \text{ .........................................} \text{(A-9)}$$

Where Q is the volumetric flowrate at any position in the annulus (ft$^3$/sec). Substituting Equations **(A-3)** and **(A-8)** into the two terms on the right side of Equation (A-9) yields:

$$\frac{P_o S}{R T_o} Q_o = \frac{PS}{R T_{av}} Q \text{ ..................................} \quad \text{(A-10)}$$

Solving the above for Q yields:

$$Q = \left(\frac{P_o}{P}\right)\left(\frac{T_{av}}{T_o}\right) Q_o \text{ ...................................} \quad \textbf{(A-11)}$$

Therefore, $\gamma_{mix}$ can be defined **as:**

$$\gamma_{mix} = \frac{\dot{\omega}_t}{\left(\frac{P_o}{P}\right)\left(\frac{T_{av}}{T_o}\right) Q_o + Q_m} \text{ ......................} \quad \textbf{(A-12)}$$

The velocity of the mixture at any position in the annulus can be written as:

$$v = \frac{Q + Q_m}{\frac{\pi}{4}\left(D_h^2 - D_p^2\right)} \quad \text{(A-13)}$$

Substituting Equations (A-11) into the above gives:

$$v = \frac{\left(\frac{P_o}{P}\right)\left(\frac{T_{av}}{T_o}\right) + Q_m}{\frac{\pi}{4}\left(D_h^2 - D_p^2\right)} \quad \text{(A-14)}$$

Substituting Equations **(A-**12) and **(A-14)** into Equation **(A-1)** gives:

$$dP = \left[\frac{\dot{\omega}_t}{\left(\frac{P_o}{P}\right)\left(\frac{T_{av}}{T_o}\right)Q_o + Q_m}\right]\left\{1 + \frac{f}{2g\left(D_h - D_p\right)}\left[\frac{\left(\frac{P_o}{P}\right)\left(\frac{T_{av}}{T_o}\right)Q_o + Q_m}{\frac{\pi}{4}\left(D_h^2 - D_p^2\right)}\right]^2\right\}dh \quad \text{(A-15)}$$

Separating variables in Equations **(A-1**5) and integrating form the surface to the bottom of the well (in a uniform cross-section annulus) yields:

$$\int_{P_o}^{P_{bh}} \frac{dP}{\left[\frac{\dot{\omega}_t}{\left(\frac{P_o}{P}\right)\left(\frac{T_{av}}{T_o}\right)Q_o + Q_m}\right]\left\{1 + \frac{f}{2g\left(D_h - D_p\right)}\left[\frac{\frac{P_o}{P}\left(\frac{T_{av}}{T_o}\right)Q_o + Q_m}{\frac{\pi}{4}\left(D_h^2 - D_p^2\right)}\right]^2\right\}} = \int_O^H dh \quad \text{(A-16)}$$

Equations (A-16) can be integrated from the surface, where the pressure $P_o$ is known, to the bottom of a uniform annulus well (depth H). In this manner, the pressure at the bottom of the well may be obtained. And under certain conditions, a closed-form solution may be obtained for $P_{bh}$. This can only be done for the situation where $Q_m$ is assumed to be negligible and can be set equal to zero. Equations (A-16) can also be used to successively integrate progressively deeper sections of the well (of uniform annulus sections) utilizing the pressures obtained from the previous solution. In this manner, the pressures in wells that have a number of different cross-sections in the annulus can be obtained.

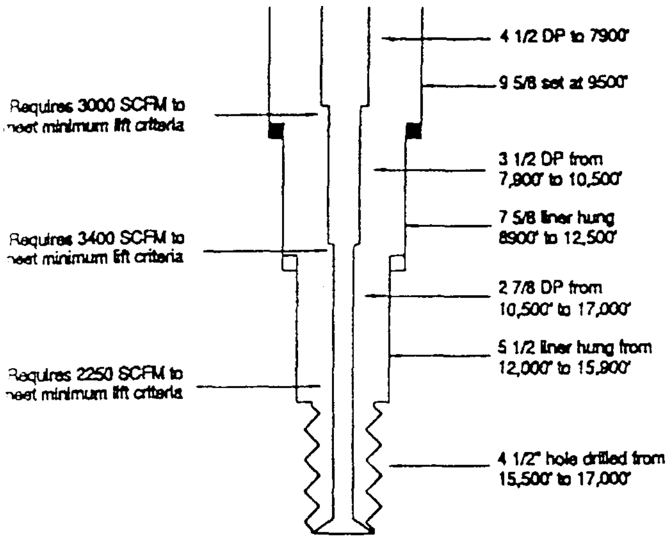


Figure 1 - Case History No. 1, Plan 1

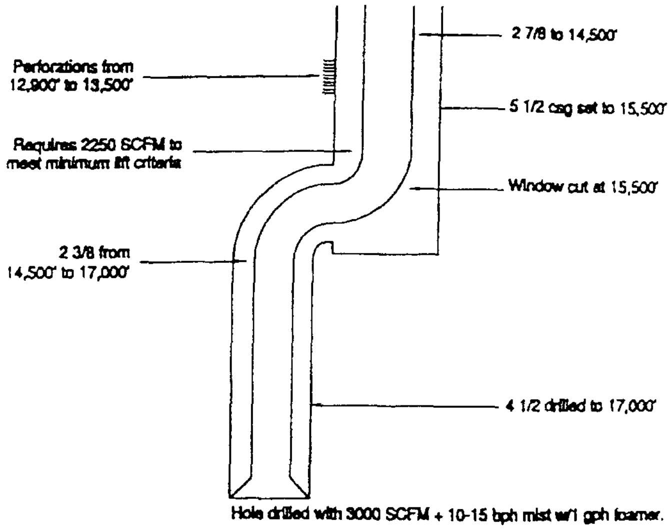


Figure 2 - Case History No. 1, Plan 2