BOUNDARY CONDITIONS USED WITH DYNAMIC MODELS OF BEAM PUMP PERFORMANCE

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ABSTRACT

Several studies have appeared in the literature concerning details of dynamic models of beam pump and rod string performance. Many of these studies deal with the equations and models presented by S. G. Gibbs.[2-6] Also, some other work has indicated techniques of modeling and solutions to governing equations.[12-18]

An equation is developed here which is used to generate a "diagnostic" and a "design" type of beam pump analysis program. The equations needed at the interface of rods of different properties are presented and discussed. This method allows an explicit statement of the equal force boundary condition at the interface of rods of different properties. No averaging of properties is required across the interface. Also, any change in the speed of sound from one rod to another is accounted for.

INTRODUCTION

S. G. Gibbs published the initial work and concepts for dynamic beam pump modeling. He presented a finite difference model for the so-called "design" or "predictive" model.[3,6] Also, he presented a technique for the "diagnostic" model[4] using a Fourier series solution to the governing equation.

Knapp[12] indicated how finite differences can be used to generate both the predictive and the diagnostic models.

Doty and Schmidt[13] outline a model that includes the effects of fluid motion.

Chacin and Purcupile[17] have presented a model which is a discrete digital model of the analog model that was used to develop the API method. Modeling of dampening from viscous and coulomb effects eliminates the need for input of dampening coefficiencies.

Shafer and Jennings[18] published a parametric study of input parameters needed for models presented by Gibbs. A constant distance increment is used for the model studied with averaged properties at locations where rods change properties between nodes.

Thesis work by Nicol,[14] and Chacin,[15] Schafer[16,18] provides detailed studies of both diagnostic and predictive models of beam pump analysis.

METHOD OF ANALYSIS

The equation of motion (Eqn. (1)) for the constant property sucker rod string is presented in several references (3, 4, 12, etc.) to include viscous dampening. The symbols commonly used for this analysis are defined in the Nomenclature.

$$\frac{\partial^2 U}{\partial t^2} = \frac{Eg}{\rho} \frac{\partial^2 U}{\partial x^2} - \frac{c\pi}{2L} \sqrt{\frac{Eg}{\rho}} \frac{\partial \bar{U}}{\partial t} \qquad (1)$$

To include the relationship for changing rod properties, the equation is written below:

$$\frac{\partial^2 U}{\partial t^2} = \frac{g}{\rho A} \frac{\partial}{\partial x} \left[EA \frac{\partial U}{\partial X} \right] - C\,V \frac{\partial U}{\partial t} \qquad (2)$$

where: $C = \frac{c\pi}{2L}$

V = speed of sound in the rod material.

In the following development, only Eqn. (1) is used; however, a method of solution to handle changing rod properties is still developed.

In order to generate an equation suitable for passing from rods of one type of properties to rods of another type of properties, equation (1) is used and solved by finite differences. The condition of passing from rods of condition (a) to rods of condition (b) is shown schematically in Figure (1) and must be modeled for a solution with a tapered rod string.

One boundary condition at the interface of rods a and b is that of equal force:

$$F_a = F_b \text{ (at point i, Fig. 1)} \quad (3)$$

or

$$(AE)_a\left(\frac{\partial U}{\partial X}\right)_{ia} = (AE)_b\left(\frac{\partial U}{\partial X}\right)_{ib} \quad (4)$$

This method also satisfies the conditions of equal velocity, displacement, and acceleration at the rods interface. Following a procedure in Ref. 19, and expanding U_{i+1} and U_{i-1} about U_i gives:

$$U_{i-1} = U_i - \left(\frac{\partial U}{\partial X}\right)_{ia}\Delta X_a + \left(\frac{\partial^2 U}{\partial X^2}\right) \frac{(\Delta X_a)^2}{2} \ldots \quad (5a)$$

$$U_{i+1} = U_i + \left(\frac{\partial U}{\partial X}\right)_{ib}\Delta X_b + \left(\frac{\partial^2 U}{\partial X^2}\right)_{ib} \frac{(\Delta X_b)^2}{2} \ldots \quad (5b)$$

Solving Eqns. (5a) and (5b) for force at i gives:

$$F_{ia} = (AE)_a\left(\frac{\partial U}{\partial X}\right)_{ia} = \left(\frac{\partial^2 U}{\partial X^2}\right)_{ia}\left(\frac{\Delta X_A}{2}\right)(AE)_a + \left(\frac{U_i - U_{i-1}}{\Delta X_a}\right)(AE)_a \quad (6a)$$

$$F_{ib} = (AE)_b\left(\frac{\partial U}{\partial X}\right)_{ib} = -\left(\frac{\partial^2 U}{\partial X^2}\right)_{ib}\left(\frac{\Delta X_b}{2}\right)(AE)_b + \left(\frac{U_{i+1} - U_i}{\Delta X_b}\right)(AE)_b \quad (6b)$$

All U's are at time j in Eqns. (5) and (6).

Representing the time derivatives with finite difference expressions for rods a and rods b in Eqn. (1) gives:

$$\left(\frac{\partial^2 U}{\partial X^2}\right)_{ia} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(V_a \Delta t)^2} + \left(\frac{C}{V_a \Delta t}\right)(U_{i,j+1} - U_{i,j}) \qquad (7a)$$

$$\left(\frac{\partial^2 U}{\partial X^2}\right)_{ib} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(V_b \Delta t)^2} + \left(\frac{C}{V_b \Delta t}\right)(U_{i,j+1} - U_{i,j}) \qquad (7b)$$

where j and i index time and distance.

Now equating (6a) and (6b) and substituting for $(\partial^2 U/\partial X^2)$ from Eqns. (7a) and (7b) gives:

$$\alpha_a(U_{i,j+1} - 2U_{i,j} + U_{i,j-1}) + \beta_a(U_{i,j+1} - U_{i,j})$$

$$+ \gamma_a(U_{i,j} - U_{i-1,j})$$

$$= -\alpha_b(U_{i,j+1} - 2U_{i,j} + U_{i,j-1}) - \beta_b(U_{i,j+1} - U_{i,j})$$

$$+ \gamma_b(U_{i+1,j} - U_{i,j}) \qquad (8)$$

where: $\alpha = \frac{\Delta XAE}{2(V\Delta t)^2}$, $\beta = \frac{\Delta XAEC}{2V\Delta t}$, $\gamma = AE/\Delta X$

In Eqn. (8) all the groupings are either groups of a or b properties. No averaging of properties is required as the calculation proceeds from a to b. Also the X increments can be equal or unequal across the boundary, or for that matter anywhere in the solution, if this algorithm is used.

Solving for $U_{i,j+1}$ gives an algorithm for the "design" or "predictive" type of analysis.

$$U_{i,j+1} = \frac{U_{i,j}(2\alpha_s + \beta_s - \gamma_b - \gamma_a) - (U_{i,j-1})(\alpha_s) + U_{i+1,j}(\gamma_b) + U_{i-1,j}(\gamma_a)}{(\alpha_s + \beta_s)} \quad (9)$$

where:

$$\alpha_S = \alpha_a + \alpha_b$$

$$\beta_S = \beta_a + \beta_b$$

This completes the algorithm for calculating across the boundary from rod (a) to rod (b) for the predictive solution. However, the equation can be used to make the calculation from one X location to another location across a ΔX anywhere in the rod string. Of course, a simpler equation could be used where the advance is made across constant properties (Ref. 3, 11).

The technique of solution of the "design" or "predictive" problem is covered elsewhere.[3,6,12,13] The solution involves evaluating the rod wave equation over the complete length of the rod string for each time in a complete pump cycle. The surface motion is supplied at each time by a geometric formula.[8,9] At each time, a pump boundary condition that specifies pump loading is used. The solution requires repetitive complete cycles until the solution has converged usually requiring at least 3-4 cycles.

The stability of the solution is affected by the choice of the time increment. For a constant X increment, the solution is stable[12] for $\Delta t \leqq \Delta X/V$. For a variable ΔX, a stable solution can generally be found by $\Delta t \leqq \Delta X/V$ where Δt is the smallest value for any segment.

Another type of rod pump analysis is to input dynamometer card loads and positions and calculate rod conditions down the rod string to above the pump. This is referred to as the "diagnostic" problem. Instead of using a Fourier series solution[4] to develop the diagnostic type of beam pump analysis, Eqn. (8) solved for $U_{i+1,j}$.[12]

$$U_{i+1,j} = U_{i,j+1} \left(\frac{\alpha_S + \beta_S}{\gamma_b}\right) + U_{i,j-1} \frac{\alpha_S}{\gamma_b}$$

$$- U_{i-1,j} \frac{\gamma_a}{\gamma_b} - U_{i,j}\left(\frac{2\alpha_S + \beta_S - \gamma_b - \gamma_a}{\gamma_b}\right) \qquad (10)$$

Using this algorithm at the boundary of changing rod properties allows the solution of the so-called "diagnostic" problem. Again, a simpler form can be developed for continuous rod properties. The diagnostic program consists of an outer X loop summing up the ΔX's to depth with an inner time loop summing up the Δt's around a complete pump cycle. The input includes a surface dynagraph from which loads and positions at equal time intervals are obtained at the surface. This algorithm is stable[12] for $\Delta t \geqq \Delta X/V$ so if too many equal time points are input, the solution could be unstable. On the other hand, if too few points are input, the solution becomes progressively more ragged as the number of points becomes too few to well define the input dynagraph. This solution is direct without some number of iterative cycles being required to calculate the resultant downhole rod string conditions. Once the distance (X) loop sums to the bottom of the rod string, the conditions in the rod string just above the pump are defined, along with all intermediate rod conditions.

The method of using finite differences to obtain a diagnostic solution was previously outlined by Knapp[12] although specifying boundary conditions at dissimilar rod junctions was not presented.

Note that this method can accept direct load-position (equal time spaced) points without them being fit as series first. Because of the smoothing required by any Fourier Series type solution, the series solution may appear to be a smoother solution to the problem with fewer points input, although it may or may not be a more accurate final solution of the input data. However, if the user suspects that the input data needs a degree of smoothing (due to, perhaps, instrumentation noise) before input to a finite difference solution, it would be a simple matter to Fourier Series represent load and position points and then input the curve fit points to the finite difference model as surface conditions. The final solution could also be series fit before presentation, but no smoothing at all should be necessary if a sufficient number of accurate points are input.

SAMPLE RESULTS

A data set is used to generate results from both a "design" and a "diagnostic" routine. The design routine (Table 1) uses the algorithm in Eqn. (9) throughout. The pump boundary conditions are for complete fillage.[6,12] The diagnostic routine (Table 2) uses the algorithm in Eqn. (10) only at the rod junctions and uses a simpler algorithm from node to node when rod properties are unchanged.

Example Data

Fluid level = pump depth = 3000 ft

Rod string: .750 in dia. 1956 ft length
.625 in dia. 1044 ft length

WRF (wt of rods in fluid) = 3819 lbs
FO (fluid load) = 6375 lbs

SPM = 8.975 Tubing, anchored
Stroke Length = 49.57 inches

These dimensions are chosen only to give the familiar API terms[11] F_o/Sk_r = 0.4 and N/No' = 0.1

For simplicity, a simple equation with .06 second harmonics from Ref. 10 is used to generate motion calculations.

Figure 2 illustrates some of the results obtained. The predictive output is composed of both the surface card and bottomhole card at the pump.

The PPRL = 11372 lbs and the MPRL = 2410 lbs. This compares to 11309 and 3100 for a corresponding calculation using API RP11L.[11] The API method uses the motion equation used here. However, 10% rod dampening and 10% dampening at the pump is used by the API method for results in RP11L. This is somewhat different than allowed by the method developed here. The routine used here applies dampening using input up and down stroke dimensionless coefficients equal to .05 and .09. However, no pump dampening was used (although it could be)so the correspondence is not entirely one-to-one with the API method. However, examination of the cards in Ref. 10, for F_o/Sk_r = 0.4 and N/No' = 0.1 shows a great deal of similarity.

Now using the surface load and position and points developed from the predictive program, the pump card is duplicated in Fig. 2 by using a diagnostic routine with the equations developed here. Note that the diagnostic program duplicates the pump card very well. If the input data is first series fit, before being input to the diagnostic program, the calculated pump card will be even smoother. Note that the method of Ref. 4 requires the data to be fit by using Fourier series before it is used in a solution.

Fig. 3 shows the bottom card from a predictive program plotted with several cards generated from a diagnostic program. As the number of points decreases, the calculated card becomes progressively more erratic.

Figure 4 overlays one of the cards digitized from Ref. 10 by the calculated surface card. Both cards are scaled to fit the same graph. Again, F_o/Sk_r and N/No' are the same, but there are necessarily some differences in some other dampening parameters previously indicated. The character of the cards is very similar, however.

SUMMARY

A method of evaluating conditions at the junction of dissimilar rods for both the diagnostic and predictive type of dynamic beam pump analysis is presented. Although several methods now exist in the literature for these types of analyses, this method is straightforward, requires no property averaging at rod junctions and has an explicit form for the force boundary condition at the rod junctions. The diagnostic solution checks the predictive solution and are near API examples that have nearly the same conditions input. The simple routines discussed here could be made more sophisticated to include motor slip,[7] actual unit geometries,[8,9] use inertial forces[2] and calculate gearbox torque to become more useful for general analysis. Using the above-mentioned additions, when appropriate, will produce models which have been found to compare very well to field data.

NOMENCLATURE

A = Rod cross section area, ft^2 [m^2]

c = Input dampening factor, dimensionless

C = Dampening factor grouping, $\frac{c\pi}{2L}$, $\frac{1}{ft}$, [$\frac{1}{m}$]

E = Modulus of elasticity (of sucker rods), lbf/ft^2 [Pa/m^2)

F = Force internal to rods due to dynamic stretch, lbf[Pa]

g = $32.17 \frac{lbm}{lbf} \frac{ft}{sec^2}$, $\left[\frac{k_g - m}{N - sec^2}\right]$

L = Length of rod string, ft, [m]

t = Time, sec [S]

Δt = Increment of time, sec [S]

U = Dynamic rod displacement as a function of position, X and time, t, ft [m]

X = Downward positive location from the surface to a position on the rod string, ft [m]

ΔX = Increment of position, ft [m]

ρ = Density of rod materials, $\frac{lbm}{ft^3}$ [Kg/m^3]

V = Velocity of sound in rods, ft/sec [m/S]

α, β, γ = Groupings of terms, Eqn. (8), lbf/ft, [Pa/m]

Subscripts

a - Signifies rod properties for "a" rods

b - Signifies rod properties for "b" rods

i - Index for distance X

j - Index for time t

s - Indicates sum of groupings

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Table 1
Example Boundary Conditions in the Predictive Mode

```
1   DIM UO(61),U(61),UOO(61)
2   OPEN "BOT.DAT" FOR OUTPUT AS #1:OPEN"TOP.DAT"FOR OUTPUT AS #2
3   N=2:DEP=3000:UDAMP=.05:DDAMP=.09:SPM=8.975:WRF=3819
4   RD(1)=.750:RO(1)=492:LNG(1)=1956:XN(1)=4:E(1)=4.43E+09
5   RD(2)=.625:RO(2)=492:LNG(2)=1044:XN(2)=2:E(2)=4.43E+09
6   TS=0:STK=4.13:FO=6375:G=32.17:PI=4!*ATN(1):IF TS<.05 THEN TS=.05
7   F1=PI/(2!*DEP):K=1:NCY=4:IBC=0:IC=1:DXS=DEP:REM DX=LNG/XN
8   IXN=XN(1)+XN(2):XL(1)=LNG(1):XL(2)=XL(1)+LNG(2):FOR I=1 TO N
9   A(I)=PI*RD(I)¬2/576!:DX=LNG(I)/XN(I): IF DX<DXS THEN DXS=DX
10  NEXT I:VS=SQR(E(1)*G/RO(1)):DTH=SPM*6!*(DXS/VS)
11  REM:USE SMALLEST (OR LESS) DXS/VS OF ALL SEGS ABOVE FOR STABILITY
12  IPTS=360!/DTH:DTH=360!/IPTS:DT=DTH/(SPM*6!):ANG=TH:IR=1
13  FOR L=1 TO NCY:TH=0!:REM  TIME LOOP FOR MAIN EQN
14  FOR JJ=1  TO IPTS:ANG=TH:IR=2 :REM SP=CONV UNIT MOTION (API)+DWN
15  ARG=TH*2!*PI/360!: SP=-STK/2!*(COS(ARG)+.06*COS(2!*ARG))
16  U(1)=-SP-STK*.53:TH=TH+DTH:DX=LNG(1)/XN(1):AM=A(1):EMN=E(1)
17  ROM=RO(1):DXM=DX:X=0:K=1:REM:X LOOP FOR MAIN EQN BELOW
18  FOR I=2 TO IXN:X=X+DX:IF ABS( X-XL(K) )<.2  THEN K=K+1
19  IF  I>2 THEN DXM=DXP:AM=AP:EMN=EP:ROM=ROP
20  DXP=LNG(K)/XN(K):DX=DXP:AP=A(K):EP=E(K):ROP=RO(K)
21  VSA=SQR(G*EMN/ROM):C1=UDAMP:IF U(1)>UO(1) THEN C1=DDAMP
22  BETA=DXM*AM*EMN*C1*F1/(2!*VSA*DT):ALPA=DXM*AM*EMN/(2!*VSA*VSA*DT*DT)
23  GAMA=AM*EMN/DXM:VSB=SQR(G*EP/ROP)
24  BETB=DXP*AP*EP*C1*F1/(2!*VSB*DT):ALPB=DXP*AP*EP/(2!*VSB*VSB*DT*DT)
25  GAMB=AP*EP/DXP:AAS=ALPA+ALPB:BS=BETA+BETB
26  V=UO(I)*(2!*AAS+BS-GAMB-GAMA)-UOO(I)*AAS
27  U(I)=(V+UO(I+1)*GAMB+UO(I-1)*GAMA)/(AAS+BS):NEXT I : REM <MAIN EQN
28  UIN=U(IXN):UINM=U(IXN-1):UOIN1=UO(IXN+1):IF IBC=0 GOTO 46:REM START
29  :
30  IF IBC<>1 GOTO 35:REM PMP LOAD 0->FO OVER TS,UPSTK,SV,TV CLOSED
31  IBC=1:G1=FO/(EP*AP*TS):UIN1=(G1*U1*DX+2!*UIN-.5*UINM)/(1.5+G1*DX)
32  U2=UOIN1:IF UIN1>U1 GOTO 46
33  IF (1.5*UIN1-2!*UIN+.5*UINM)/DX*EP=>FO/AP GOTO 36:REM T>MAX CHG BC
34  :
35  IF IBC<>2 GOTO 39:REM PMP LOADED TO FO, UPSTK,SV OPEN, TV CLOSED
36  IBC=2:UIN1=(DX*FO/(AP*EP)+2!*UIN-.5*UINM)/1.5 :IF UIN1>U2 GOTO 31
37  U3=UOIN1:IF UIN1>UOIN1 GOTO 40: REM PLUNGER STARTS DWN, CHANGE BC
38  :
39  IF IBC<>3 GOTO 45:REM UNLOAD FROM FO->0 OVER TS,DWNSTK,SV,TV CLOSED
40  IBC=3 : G1=FO/(TS*EP*AP):G2=G1*TS
41  UIN1=((G1*U3+G2)*DX+2!*UIN-.5*UINM)/(1.5+G1*DX):U4=UOIN1
42  IF UIN1<U3 GOTO 36 :REM IF PAST U3 THEN NO LOAD
43  T=(1.5*UIN1-2!*UIN+.5*UINM)/DX*EP:IF T<=0! GOTO 46:REM TO NEXT BC
44  :
45  IF IBC<>4 GOTO 49  :REM PMP NO LOAD, DWNSTK, SV CLOSED, TV OPEN
46  IBC=4 : UIN1=(2!*UIN-.5*UINM)/1.5:U1=UOIN1: IF UIN1<U4 GOTO 40
47  :
48  IF UIN1<=UOIN1 GOTO 31:REM PLUNGER STARTS UP, CHANGE BC
49  U(IXN+1)=UIN1:IF IC=NCY GOTO 50 ELSE GOTO 54
50  WTOP=-A(1)*(U(3)*.5-U(2)*2!+1.5*U(1))/( LNG(1)/XN(1) )*E(1)+WRF
51  WBOT=AP*(1.5*UIN1-2!*UIN+.5*UINM)/DX*EP:IF JJ=1 THEN X1=U(1):X2=UIN1
52  PRINT#2,USING"#####.#####   ";-U(1)+X1;WTOP  :REM PRT TOP POS,LOAD
53  PRINT#1,USING"#####.###  ";-UIN1+X2;WBOT     :REM PRT BOTTOM DATA
54  FOR J=1 TO IXN+1:UOO(J)=UO(J):UO(J)=U(J):NEXT J:NEXT JJ
55  IC=IC+1:PRINT IC-1:NEXT L:END
```

Table 2
Example Boundary Conditions in the Diagnostic Mode

```
1  DIM U(12,300),POSI(300),XLOD(300)
2  OPEN "CARD.DAT" FOR INPUT AS #1:OPEN "BOT.DAT" FOR OUTPUT AS #3
3  INPUT #1,NPTS:FOR I=1 TO NPTS:INPUT #1,POSI(I),XLOD(I):NEXT I
4  NS=2:DEP=3000:UDAMP=.05:DDAMP=.09:SPM=8.975:WRF=3819
5  RD(1)=.750:RO(1)=492:LNG(1)=1956:XN(1)=4:E(1)=4.43E+09
6  RD(2)=.625:RO(2)=492:LNG(2)=1044:XN(2)=2:E(2)=4.43E+09
7  PI=4!*ATN(1):G=32.17:K=1:IXN=XN(1)+XN(2):XL(1)=LNG(1)
8  FOR I=1 TO NS:A(I)=PI*RD(I)¬2/576!:IF I>=2 THEN XL(I)=XL(I-1)+LNG(I)
9  NEXT I : DT=(60!/SPM)/NPTS:REM DT MUST BE > ALL DX/VS FOR ALL SEC
10    FOR J=1 TO NPTS:U(1,J)= -POSI(J):DXP=LNG(1)/XN(1):D1=DXP
11    NEXT J: AP=A(1):EP=E(1):ROP=RO(1):X=X-DX :F1=PI/(2!*DEP)
12    FOR I=1 TO IXN:X=X+DX:IF ABS(X-XL(K))<.2 THEN K= K+1
13    DXM= DXP:AM=AP:EMN=EP:ROM=ROP:DXP=LNG(K)/XN(K)
14    DX= DXP:AP=A(K):EP=E(K):ROP=RO(K):XR=ROM*DXM*EMN*AM-ROP*DXP*EP*AP
15  FOR J=1 TO NPTS : XR=ABS(XR) : VSA=SQR(G*EMN/ROM)
16      IF J=1   THEN UJM1= U(I,NPTS)  ELSE UJM1=U(I,J-1)
17      IF J=NPTS   THEN UJP1= U(I,1)  ELSE UJP1=U(I,J+1)
18      IF U(1,J)>U(1,J-1) THEN C1= DDAMP  ELSE C1=UDAMP
19      BETA= DXM*AM*EMN*C1*F1/(2!*VSA*DT)
20      ALPA= DXM*AM*EMN/(2!*VSA*VSA*DT*DT):VSB=SQR(G*EP/ROP)
21      BETB= DXP*AP*EP*C1*F1/(2!*VSB*DT)
22      ALPB= DXP*AP*EP/(2!*VSB*VSB*DT*DT):BS=BETA+BETB
23      GAMB= AP*EP/DXP:GAMA=AM*EMN/DXM:AAS=ALPA+ALPB
24  IF I<>1 THEN UIM1=U(I-1,J) ELSE UIM1=U(1,J)-D1*XLOD(J)/(E(1)*A(1))
25  IF XR<1 GOTO 28
26  V= UJP1*(AAS+BS)+UJM1*AAS-UIM1*GAMA-U(I,J)*(2!*AAS+BS-GAMB-GAMA)
27  U(I+1,J)=V/GAMB :GOTO 30
28  BX=C1*F1*VSA: RA=(DXM/(VSA*DT))¬2: RB=RA*DT
29  U(I+1,J)=RA*(UJP1-2!*U(I,J)+UJM1)+BX*(UJP1-U(I,J))*RB+2*U(I,J)-UIM1
30  IF I<>IXN GOTO 34
31  TEN= (1.5*U(IXN+1,J)-2!*U(IXN,J)+.5*U(IXN-1,J))/DXP*EP
32  PBOT=-U(IXN+1,J):IF J=1 THEN Q=PBOT
33  PRINT#3,USING"#####.## ";PBOT-Q;A(NS)*TEN-WRF:REM <PRT LOAD,POS
34  NEXT J:NEXT I:STOP:END
```

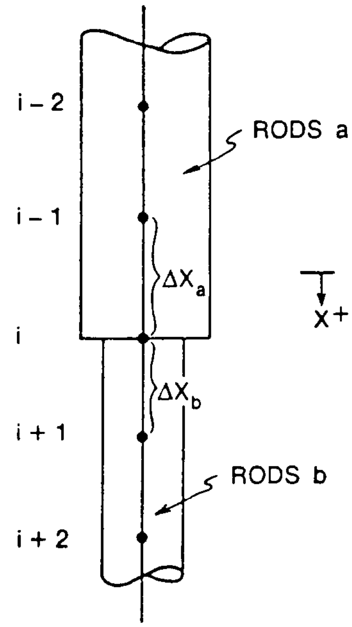

Figure 1

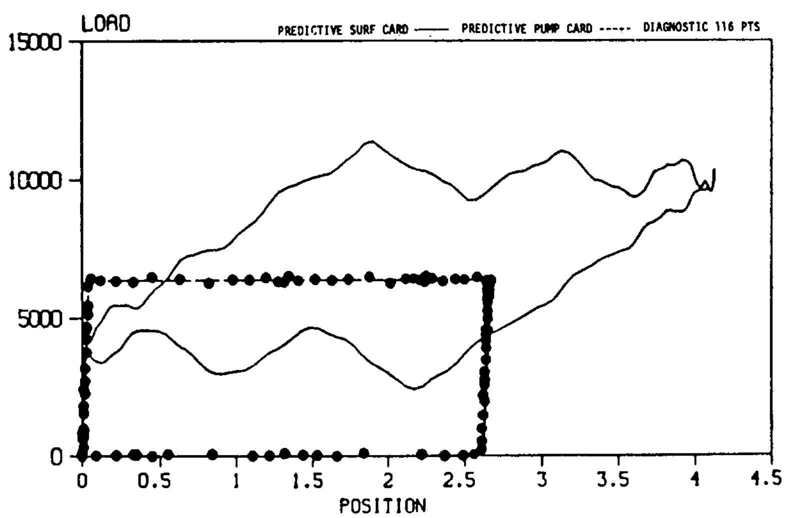

Figure 2 - Design and diagnostic output

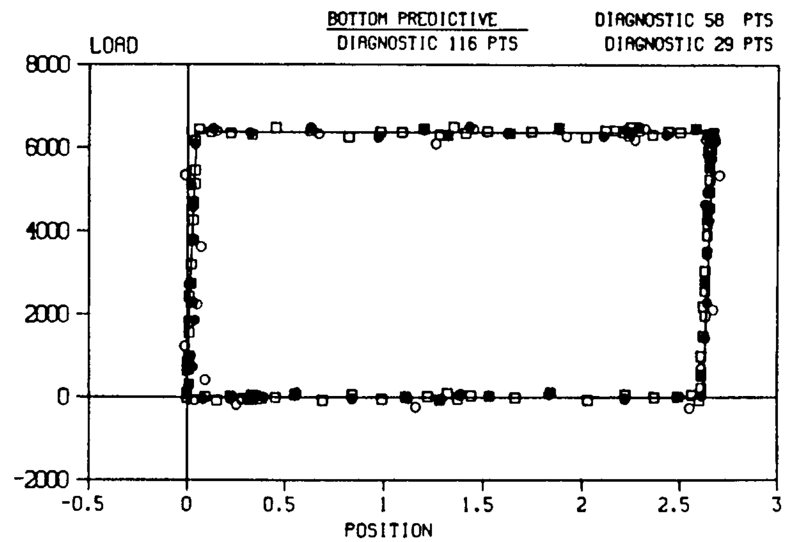

Figure 3 - Diagnostic pump cards vs points used

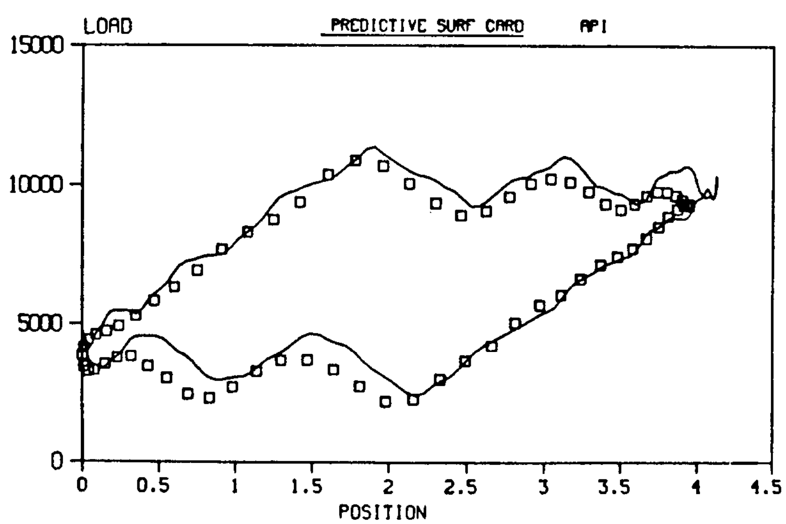

Figure 4 - API vs predictive F_o/Sk_r = 0.4
and N/No' = 0.1