A CRITICAL REVIEW OF TWO-PHASE FLOW GAS-OIL HYDRODYNAMICS FOR HORIZONTAL WELL APPLICATIONS

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ABSTRACT

The theory and experimental data of two-phase flow hydrodynamics currently available are mainly for the case of horizontal, inclined, and vertical flow. There have been some development in the understanding of two-phase flow hydrodynamics on the hilly terrain and the severe slugging cases in the case of subsea riser. Yet, for the case of horizontal well with the production in tubing, the impact of the tubing location in the horizontal well in the curve section connected between the vertical and horizontal zone are not quite clear. This work provides a review on the two-phase flow hydrodynamic characteristics for the case of horizontal well and provides a fundamental understanding on the impact of toe-up and toe-down of the well to the case of two-phase gas-oil flow. The review also emphasizes on the applicable two-phase flow steady-state and transient models that can be used to analyze the horizontal well production successfully.

INTRODUCTION

Multiphase flow has been an integral part of petroleum production systems since the discovery and production of petroleum reservoirs. The existence of hydrocarbons in reservoirs is often in the form of oil and/or gas. Typically, there is some connate water associated with these reservoirs. The production of these reservoirs would in most cases, result in the migration of at least two, if not all three, of these fluids through the well production system. Therefore, an understanding of the processes associated with the production of these reservoir fluids is by all means a necessity to the petroleum industry.

A lot of research has been done on multiphase flow in various fields of study and industries such as Nuclear energy, Bioengineering, and Process engineering. The petroleum industry's cognizance of multiphase flow began in the late 1950s. Most literature has chronologically categorized development of research based on flow equations into: Empirical Period; Awakening Years; and Modeling Period (Brill and Arlrachakaran, 1992). Some others, based on engineering application, into: Intuition; Graphical; Steady-State; Transient; and Coupled (Shippen and Bailey, 2012). Regardless of the classification method, researchers have tried to predict the behavior of multiphase flow by finding a relationship between pressure drop, flow rates, and the flow path geometry. To do this, an understanding of heat, mass, and momentum conservation principles and petroleum fluid phase behavior are required (Brill, 1987). This is by no means an easy task.

Petroleum fluids pose a major complication to the flow analysis and prediction process because of the complexity of the fluids. Petroleum fluids are complex mixtures made up of multiple components whose properties vary across a wide range of temperature and pressures. Therefore, during production process, these fluids will change from one phase to another as the pressure and temperature of the production system vary over a broad range. The production system is typically made up of the reservoir, the completion system, wellbore tubulars, and the surface facilities (separation system and pipeline).

The applications of multiphase flow in the petroleum industry are vast. Nodal analysis which is the constant analysis of production systems and the design optimization process requires an understanding of multiphase flow throughout the life of the well. Also, the design and application of Artificial Lift systems such as Gas Lift, require multiphase flow for good design and installation. Determining the right location for gas lift valves, and the quantity of gas to be injected are important (Poettmann, F, and Carpenter, P., 1952). Other applications of multiphase flow include: (1) drilling applications like, preliminary design and on site optimization of underbalanced drilling operations. In this drilling operation, gas-liquid mixture are used in place of traditional drilling mud (Gregory et al., 2000; Guo et al., 1996; Mykytiw et al., 2004); (2) the prediction of the onset and size of flow assurance issues such as severe slugging, paraffin deposition, and hydrate formation.

In 1988, Joshi showed that under the right conditions of well length, reservoir thickness and anisotropy, horizontal wells productivity are 2- to 6-times greater than an unstimulated vertical well with the same drawdown pressure and reservoir conditions (Joshi, 1988). At that time, there were only thirty horizontal wells in the world, compared to now, that number has proliferated, resulting in increased research in horizontal drilling and completion since the shale revolution. Unfortunately, as will be shown in this study, a close observation of the existing literature showed that there is a very limited understanding on the multiphase flow inside a horizontal well. Most research have often been focused on flow through vertical wellbores and flow in horizontal or inclined pipelines (Shoham, 2005). Most of these established and existing models are developed based on a constant mass flux through the pipe. Horizontal well technology is completely different from these models. These wells are typically made up of three sections, a vertical section, a deviated section and a lateral section. The existing models can be applied to the vertical and deviated sections but not to the lateral section. The main difference in this section is the presence of perforations and the possibility of influx and out flux of fluid from the pipe. This results in changes in the pressure drop in the system and even changes in the fluid flow patterns (Ihara et al., 1995) (Ouyang et al., 1998).

The complete understanding on the multiphase flow in the horizontal wells is needed for a further systematic development of the horizontal well technology. Yet, a comprehensive review of the multiphase flow topics which are specifically applicable for the horizontal well cases is unavailable. Therefore, this study presents the available literature and technological advancements so far in the field of multiphase flow for the horizontal well cases and the closely related areas. It also highlights the current areas of research and the future research needs.

PREDICTING MULTIPHASE HYDRODYNAMIC BEHAVIOR IN HORIZONTAL WELLS

Based on the geometry of a horizontal well, it would require the utilization of all pressure predictions techniques: Vertical, inclined, and horizontal. Therefore a review of the existing literature is necessary for better understanding and development of flow models. The literature will be categorized based on sections of the well. Under each section we will go over the technological development, from the empirical, to the mechanistic, to the state of the art computer simulators.

The prediction of pressure drop in multiphase flow through pipes has been a challenge for many years in the petroleum industry. It is very complex because of the large number of variables involved. Apart from these variables, there are a lot of flow patterns and mechanisms that vary based on the interaction of the fluids and flow geometry (Hagedorn and Brown, 1965).

Like single phase flow pressure drop prediction, the foundation of the multiphase flow pressure prediction requires the utilization of energy balance on flowing fluid between two points (Ros, 1961). According to Ros, the two commonly used methods are: energy-balance method; and pressure-balance equation. The difference between multiphase flow and single phase pressure drop prediction is the addition of pressure drop which occurs as a result of slippage between the phases (Hagedorn and Brown, 1965).

Poettmann and Carpenter (Poettmann, F, and Carpenter, P., 1952) whose work is one of the more prominent ones and commonly used in the petroleum industry used the energy-balance equation method. Their equation is given by:

$$\frac{dP}{dh} = \bar{\rho} + f_p \frac{Q^2 M^2}{7.413 \times 10^{10} \bar{\rho} D^2} + Acceleration\ Gradient \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where $\frac{dP}{dh}$ is the Pressure gradient (lbs/ft^2/ft),

$\bar{\rho}$ is the Gas-liquid mixture density, which would occur under no slip conditions (lbs/ft^3)

QM is the Total mass flow rate (lbs/ft^3)

D is the Inside diameter of tubing (ft)

f_p is the Dimensionless correlating function for the total energy loss

Ros (Ros, 1961) suggested the pressure-balance equation, similar to the Poettmann and Carpenter energy-balance equation, given as:

$$\frac{dP}{dh} = Static\ Gradient + Friction\ Gradient + Acceleration\ Gradient$$

$$\frac{dP}{dh} = \rho g + \frac{4f\rho v^2}{2D} + Acceleration\ Gradient \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

Where $\frac{dP}{dh}$ is the Pressure Gradient

ρ is the Density of fluid

g is the Gravity Acceleration

v is the Fluid velocity

D is the Inside diameter of the pipe

f is the Friction factor

Early Empirical Period

The empirical approach typically involved the development of correlations based on data gotten from series of experiments carried out on a lab scale model or in some cases, field scale models. First a dimensional analysis is carried out on all the variables involved in the multiphase flow, then the relevant dimensionless groups were identified and studied in a series of experiments. The data typically captured based on the selected dimensionless groups are fluid flow rates, densities, liquid viscosities, pipe diameter and inclination angles, and pressure drop between points on the pipeline. Also, the flow patterns and liquid holdup were measured in some cases. These were then used to develop correlations the calculation of pressure drop based on the assumption that the fluids, gas and liquid, flowed as a homogeneous mixture.

As a result of the complications involved in pressure gradient prediction, most of the early researchers attempted to solve the problem by developing empirical or semi-empirical correlations which assumed that the fluids flow as a homogeneous mixture (Hagedorn and Brown, 1965). Like most correlations, these were only applicable to the scenarios with the same or similar set of conditions, therefore, limiting their range of applicability.

Vertical Section

Empirical Models for vertical Flow

Initially most researchers approached the problem by assuming there was no slippage between fluids. This approach meant, both slippage-losses and friction-losses were lumped into a single energy-loss factor similar to the friction factor in single phase flow (Hagedorn and Brown, 1965; Poettmann, F, and Carpenter, P., 1952; Ros, 1961). This resulted in errors and inaccuracies in prediction, due to the fact that the effects of flow defining parameters like the holdup, fluid viscosities, and interfacial tension between fluids were not properly reflected in the correlations (Baxendell and Thomas, 1961; Fancher Jr. and Brown, 1963; Poettmann, F, and Carpenter, P., 1952; Tek, 1961). Some of the prominent researchers who followed this approach include:

- Poettmann and Carpenter (1952): they developed a correlation for the prediction of pressure traverse of flowing oil wells and gas lift wells, with knowledge of only the surface data. The correlation was developed based on data from real wells using the energy-balance approach and the assumption that the flow of fluids was always turbulent homogenous steady flow with no slip, and no flow pattern prediction is necessary.
- Baxendell and Thomas (1961): developed energy-loss correlation for the prediction of pressure drop in flowing wells at relatively higher rates (above Nine-hundred barrels per day) than those applicable to Poettmann and Carpenters (1952) correlations. The correlation is applicable to both vertical and horizontal wells, and to the tubing flow and flow through the annulus.

These models and similar ones had errors as much as $\pm$20% in prediction of pressure drop (Brill, 1987).

Generalized Empirical Correlations

These correlations also known as the Separated models, assumes that the gas and the liquid phases have different velocities, therefor slip is taken into account (Behnia, 1991). It was observed that at low flow rates, “Slippage” occurs between the flowing phases. There are significant differences between velocities of liquid and gas phases, hence, it will be wrong to assume that the fluids flow as a homogeneous mixture. The difference in flow velocity and geometry results in the development of different geometrical configurations or Flow patterns (Brill, 1987; Orkiszewski, J., 1967).

Researchers who followed this approach include:

- Ros (1961): developed separate correlations for liquid holdup and wall friction for the prediction of pressure gradient using laboratory scale experimental data. Using the pressure-balance approach and dimensional analysis he was able to determine the important parameters that affect holdup and wall friction, thereby narrowing down the number of parameters to be focused on experimentally and reducing the amount of experiments to be done. Data gathered from experiments was used to develop one of the earliest flow maps and the pressure gradient correlations related to the various flow regimes.
- Hagedorn and Brown (1963 and 1965): Developed pressure traverse curves for the prediction of pressure gradient for a wide range of low rates, pipe sizes and oil viscosities (Hagedorn and Brown, 1965, 1964). The correlations were developed using data gathered from experiments carried out on an experimental well. The correlations did not require flow maps for them to be applicable. These curves and correlations are still commonly used nowadays in designing gas lift applications.
- Orkiszewski (1967): Examined all the available correlations at that time, and selected the most accurate correlation and modified it to cover the range of conditions where it had limitations. Based on his analysis, the most accurate correlation was Griffith and Wallis Correlation for certain flow regimes, and to make up for its limitations he proposed using Duns and Ros Correlation for those range of conditions. He made an algorithm using these two correlations and called it the Modified Griffith and Wallis Correlation. He concluded that in order to get the best prediction of pressure gradient, it necessary to know the flow regime and the liquid distribution(Orkiszewski, J., 1967).

Lateral Section

The lateral section is usually horizontal or near horizontal. The existing literature on multiphase flow in horizontal pipes has been focused mainly on flow in pipelines, with very limited research on flow in horizontal wells. The analysis of multiphase flow in a horizontal well compared to flow in a horizontal pipeline are quite similar, so an understanding of multiphase flow in pipeline will be helpful in the analysis of horizontal wells. The analysis of multiphase flow in these pipes follow the same principles as those followed in vertical sections. Conservation of mass, momentum and energy. The only difference being that in the horizontal wells the mass flow rate will not be constant due to the presence of perforations which result in the influx and outflux of fluids from the flow line.

In the petroleum industry, research on multiphase flow in horizontal pipelines came to prominence with development of offshore oil and gas plays. The produced oil and gas are transported via pipelines from the offshore wells to centralized gathering and separation systems. These long flowlines typically exhibit huge pressure changes as result of changes in the ambient surrounding conditions and also as a result of the phase behavior of the fluids. These pressure losses affect the performance of the producing well. Most of the research categorized as horizontal pipeline flow have inclinations in the range of +10 to -10 from the horizontal. Like vertical flow we categorize them as follows:

Empirical Models for Horizontal Flow

Some of the commonly used empirical models for the determination of pressure drop include Lockhart and Martinelli correlations (1949), Dukler et al (1964) and Eaton et al (1967).

- Lockhart and Martinelli (1949) developed correlations for pressure gradient and liquid holdup using data obtained from the simultaneous flow of air and liquids in various pipes. The correlation is developed based on a parameter-X which is the square root ratio of the pressure drop in the pipe of liquid to that of gas, with the assumption that only that single phase flowed. Based on this parameter, four flow mechanisms were described, depending on the flow regime. These were turbulent-turbulent, turbulent-viscous, viscous-viscous and viscous-turbulent (Lockhart and Martinelli, 1949).
- Dukler et al (1964) used similarity analysis to develop correlations from field data for friction factor and liquid-holdup. (Mandhane, J.M. and Gregory, G.A. and Aziz, K., 1977)
- Eaton et al (1967) Used experimental data to develop correlations for liquid holdup, energy-loss factor and flow-pattern map. Pressure gradient is calculated using the energy balance approach, and its calculation is independent of the existing flow regime (Eaton et al., 1967).
- Dukler (1964)-Eaton (1967)-Flanigan (1958): this correlation is a combination of the Dukler et al (1964) and Eaton et al (1967) correlation. The friction factor is calculated using the Dukler correlation and the Holdup

is calculated using the Eaton correlation. In the case of inclination, the Flanigan elevation term is introduced when calculating the pressure gradient, otherwise, it is neglected (Yuan and Zhou, 2008).

- Beggs and Brill (1973): developed correlations for holdup and pressure gradient using data from experimental test facility. They developed correlations for holdup for different horizontal flow patterns. The friction pressure was then calculated using the holdup independent of the flow regime (Beggs and Brill, 1973).
- Mandhane et al (1975 and 1977): They evaluated all the available correlations for Holdup and Pressure gradient at that time and came up with the following conclusions: Mandhane et al. (1974) had the most accurate flow pattern map and simplest to use(Mandhane et al., 1975). Dukler et al (1964), and Beggs and Brill method were the most accurate for the calculation of friction pressure drop of all those tested (Mandhane, J.M. and Gregory, G.A. and Aziz, K., 1977).
  Mandhane et al (1975 and 1977) proposed a two-step (three-step if holdup is required) process for the prediction pressure gradient in horizontal pipes. The process required the combination of the best methods as listed above for the calculation of various components required for pressure drop prediction (Mandhane et al., 1975; Mandhane, J.M. and Gregory, G.A. and Aziz, K., 1977).

Deviated (Inclined) Section

Incline flow is encountered very often in gathering lines and offshore pipelines laid along the uneven seabed. It is also common in deviated wells and directional wells. Research in this area began in the 1970s, because a lot of the existing correlations for vertical and horizontal multiphase flow were inaccurate when applied to inclined pipelines. A lot of the research has been done in this area, some under the guise of multiphase flow in hilly terrain. It is typically neglected in single-phase flow, as the energy lost moving up hill is recovered in the downhill section. A lot of the researchers have showed that this is not true for multiphase flow. Due to the fact that the liquid holdup and consequently the mixture density are much lower in the downhill section, pressure recovery is negligible (Beggs and Brill, 1973; Payne et al., 1979).

Empirical Models for Inclined Flow

- Flanigan (1958) developed correlation for holdup in hilly terrain in the uphill section. He suggested that the downhill section angle had no effect on the pressure drop and that pressure recovery in that section is negligible. He developed a correlation for pressure drop using the panhandle equation and showed the effect of inclination by using a term called elevation factor (Payne et al., 1979).
- Guzhov et al (1967) developed correlations for pressure drop in pipes on hilly terrain with slight inclination angles ($\pm 9^{o}$). The correlations are limited to only plug flow and stratified flow. The correlation accounts for pressure drop on the downhill section of the hill. During stratified flow, the holdup terms are calculated for both uphill and downhill sections. While for plug flow, the same holdup term used for the uphill section is used for the downhill section (Beggs and Brill, 1973; Payne et al., 1979).
- Beggs and Brill (1973): The correlations developed for horizontal flow regimes were applied to inclined pipes by the introduction of an inclination correction factor for the horizontal holdup. When holdups are determined, they are corrected for the inclination angles and then used in pressure gradient calculations.
- Griffith et al (1973) developed a simple method for the determining the pressure gradient in oil and gas wells. Their correlation neglects pipe roughness, viscosity of fluids and entrainment effects. They also ignore the downhill pressure drop (P. Griffith, C. W. Lau, P. C. Hon, 1973).

Modern Modeling Era

With the introduction of computers and better numerical computation techniques, better data gathering and analysis were possible. This led to the development of models and programs for the prediction of multiphase flow behavior. This period has been referred to by some authors as the Awakening years (Brill and Arlrachakaran, 1992). This is because of the realization of the fact that the empirical flow-patterns maps and correlations did not really give enough insight on the phenomena associated with multiphase flow. This meant that no improvements could be done to get better predictions, unless the fundamental mechanisms behind the phenomena was understood (Shippen and Bailey, 2012).

These programs are typically made up of several models. First a model is developed for the prediction of the existing flow patterns and for the transition between flow patterns. The transitions are determined based on description of the physical phenomena occurring at the start and the end of each flow regime. Next, individual models are developed for prediction of pressure gradient, holdup or void fraction for each individual flow pattern (Ansari et al., 1994; Hasan and Kabir, 1988; Ouyang and Aziz, 1999; Petalas and Aziz, 2000; Xiao et al., 1990). Initially, most of these models were developed for steady state Isothermal conditions. With better numerical solution and faster computer processing speeds, modern models now incorporate transient conditions into the models.

There are several approaches to modelling multiphase flow, the petroleum industry has two approaches which are commonly used either individually, in combination of the two approaches namely:

1. Mechanistic Approach in which the **Multi- or Two-Fluid Model** is commonly utilized approach.
2. Homogeneous with Slip Approach commonly known as the **Drift-Flux model**

The main difference between both approaches is the constitution of the conservation of momentum equations in the mathematical model (Spesivtsev et al., 2013). The multi-fluid model has a momentum equation for each fluid phase present in the multiphase flow system, while the drift-flux model has one lump momentum equation for the mixture of the existing phases in terms of the volume-average velocity of the mixture.

Mechanistic Models

Mechanistic modeling involves the determination of the cause mechanism and then the mathematical modelling of the phenomenon (Ansari et al., 1994; Barnea, 1987; Gomez et al., 2000; Ouyang and Aziz, 1999; Xiao et al., 1990). Also called two-fluid or multi-fluid models, they involve the writing of separate mass, momentum and energy conservation equations for the individual phases. The combined conservation equations will then be solved numerically to determine the pressure drop, average holdup and velocities(Brill and Arlrachakaran, 1992; Shippen and Bailey, 2012).

The mechanistic modeling approach began in the 1980s. It was added by the introduction of computers and advances in the data gathering systems. More quality data meant better observation and better studying which improved the development of mathematical models that sufficiently describe the physical mechanisms governing multiphase flow behavior in pipes (Brill, 1987). Mechanistic models typically require the identification of the existing flow pattern for the system being analyzed, then individual models are developed for each pattern for the prediction of the hydrodynamics and heat transfer.

Several approaches are available for the development of multiphase mechanistic models, but the most prominent approach is the Two-Fluid model. The Two-Fluid Model requires the writing of conservation of mass, momentum and energy for each fluid. Consider a one dimensional isothermal two fluid flow in a pipe, the momentum equation can be written as:

$$-A_L \left.\frac{dp}{dL}\right|_L - \tau_{WL}S_L + \tau_I S_I - \rho_L A_L g \sin\theta = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$-A_G \left.\frac{dp}{dL}\right|_G - \tau_{WG}S_G - \tau_I S_I - \rho_G A_G g \sin\theta = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Where, g is the gravity acceleration term, θ is the pipe inclination angle, A_i is the cross sectional area of the pipe occupied by the phase i, S_i is the wall perimeter for a cross section of the wall wetted by phase i and S_I is the length of the interface between the phases, ρ_i is the density of phase i, τ_{Wi} is the wall shear stress of phase i, and τ_I is the interfacial shear stress between the two phase. Subscript "*i*" represent the phase G for gas and, L for Liquid.

Mechanistic models are to some extent, empirical. Empirical closure relationships are required for the determination of some parameters such as the friction factors, liquid holdup, etc. Closure relation may be referred to as an operator whose mechanism cannot be defined using basic physics, but it is required to solve a related system of equations. Based on this fact, the mechanistic models are semi-empirical. Closure relations will be need for the friction losses, interfacial shear stress and the holdup.

Combining the mass balance equations and the above momentum balance equations for the two fluid phases would result in four equations, and four unknowns. These unknowns will have to be determined using numerical solutions. Furthermore, with mechanistic models it is possible to model transient processes like changes in the flow rates or pressures at certain points of interest along the flow channel.

Mechanistic modeling approach was used in many of the early works. These include Taitel and Dukler (1976), Taitel et al 1980, and Barnea et al (1985 – 1987) in mechanistic modeling for the development of flow pattern prediction models (Xiao et al., 1990). Using these models, it is possible to predict flow patterns by defining transition boundaries between flow regimes (Ansari et al., 1994).

Furthermore, the one dimensional two-fluid model is used in the development of the individual hydrodynamic models for the prediction of flow parameters like the liquid holdup and the pressure gradient for each flow pattern. This is typically the disadvantage of mechanistic models because of discontinuities that may occur in the hydrodynamic models as it approaches transitions boundaries between flow patterns.

Some of the popular mechanistic Two-fluid models include:

- Taitel and Dukler (1976) developed a mechanistic model for the prediction of multiphase flow pattern transitions for all flow regimes in horizontal two phase flow. The predictions of the transition boundaries between flow regimes was based on physically realistic mechanisms (Taitel and Dukler, 1976). A generalized flow regime map was developed based on the model. When tested, predictions were more accurate compared to the existing flow regime correlation maps of the time. In 1980, taitel et al (1980) developed models for vertical flow pattern transition for steady gas-liquid flow in vertical tubes based on physical mechanisms suggested for each transition (Taitel et al., 1980). These models were the foundation for most of the subsequent models developed after.
- Barnea et al (1985) modified the exiting models for vertical and horizontal multiphase flow pattern predictions. They extended the existing model to account for upward flow at pipe inclinations between vertical and horizontal flow (Barnea et al., 1985).
- Barnea (1987) developed a unified model for flow pattern transition prediction for all range of pipe inclinations during steady state gas-liquid flow in pipes (Barnea, 1987).

These models serve as the base for defining flow regime boundaries in modern modeling process for multiphase flow pattern prediction.

For the prediction of hydrodynamic properties like the pressure gradient, holdup or void fraction, several authors developed individual models for each flow regime while others developed comprehensive models containing all possible flow regimes for the flow conditions. Some of the more prominent models include:

- Xiao et al (1990) developed comprehensive mechanistic model for predicting two-phase flow in horizontal and near horizontal pipelines. They developed a model for prediction of flow pattern based on transition boundaries defined mainly from works by Tailtel and Dukler (1976), and Barnea (1982-1987). Then they developed individual models using the steady state one-dimensional two-fluid model approach and individual models developed by other researchers, for stratified flow, intermittent flow, annular flow and dispersed bubble flow. The model was tested and shown to have overall better performance over a wide variety of data compared to any of the commonly used empirical correlations. It had some draw backs as well. These include uncertainties in determination of interfacial friction factor for stratified flow and annular flow(Xiao et al., 1990).
- Ansari et al (1994) developed a comprehensive steady state one-dimensional mechanistic model for predicting upward two-phase flow in wellbores. The model was made up of a model for flow pattern prediction and several individual models for the prediction of hydrodynamic parameters during bubble flow, slug flow and annular flow. The model was evaluated over a wide range of flow scenarios for vertical wells and deviated wells, and over different flow regimes. In comparison to other existing models and correlations, it performed better than a large majority, and had comparable performances to that of Hagedorn and Brown, Aziz et al., Duns and Ros correlations, and Hasan and Kabir models. It had limitations in the prediction of bubble rise velocities and film thickness in deviated wells (Ansari et al., 1994).
- Ouyang and Aziz (1999) developed a mechanistic model for two-phase flow in horizontal or near horizontal pipes with mass exchange through the pipe wall. The influx and outflux of fluid have effects on the wall friction, acceleration and flow pattern transition. Individual models were written for stratified flow, annular-mist flow, bubble flow and intermittent flow. Furthermore, a flow pattern transition prediction model was also developed, which showed the effects of mass exchange at the wall (Ouyang and Aziz, 1999). The momentum equations in this model compared to that of a regular horizontal pipe with constant mass flux, had an addition term for the mass flux.

$$-A_L \frac{dp}{dL}\bigg|_L - \tau_{WL} S_L + \tau_I S_I - \rho_L A_L g \sin\theta - 2\rho_L U_L q_{inL} = 0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5)$$

$$-A_G \frac{dp}{dL}\bigg|_G - \tau_{WG}S_G - \tau_I S_I - \rho_G A_G g \sin\theta - 2\rho_G U_G q_{inG} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

For perforated horizontal wells,

$$q_{inL} = nA_{in}U_{insSL} \qquad q_{inG} = nA_{in}U_{insSG} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (7)$$

The above equations (5) and (6) represent the momentum equations for stratified flow using the steady state one dimensional two-fluid model for stratified flow. Compared to (3) and (4) previously written by Taitel and Dukler (1976), we observe that the acceleration term is not neglected, but rather it incorporates the mass exchange term representing perforations in horizontal wells as denoted in equation (7). The introduction of the mass exchange terms meant the determination of wall friction was different from previous models. The wall friction was defined as a function of the flow rates from the perforations (Ouyang and Aziz, 1999).

The flow pattern transition model also showed significant differences from those typically observed in regular horizontal pipe models within mass exchange. The interfacial friction factor correlation was shown to affect the pattern transition. Also, flow pattern transition boundaries were strongly dependent fluid properties and on pipe inclination. They showed that the slightest change in inclination angle significantly altered the flow pattern map (Ouyang and Aziz, 1999). Finally, the mass flux exchange at the wall affected the wall friction factors, interfacial friction factor and the fraction of liquid entrainment which ultimately affects the holdup, film thickness, liquid velocity and other quantities, which could result in a shift of the transition boundaries (Ouyang and Aziz, 1999).

A comparison with existing models revealed that significant improvements were made in the prediction of horizontal wellbore pressure drop and flow pattern using the new model. As mentioned earlier only a limited amount of researchers have investigated the pressure drop along a horizontal well. These include (Clemo, 2006; Comm, 1991; Ihara et al., 1995; Penmatcha et al., 1999; Schulkes et al., 1998; Su and Gudmundsson, 1994)

- Gomez et al (2000) developed a unified steady-state two-phase flow mechanistic model for the prediction of flow pattern, and prediction of multiphase flow hydrodynamic parameters for all flow patterns and all range of inclinations from horizontal to upward vertical flow. First, they developed a unified flow pattern prediction model based on the Barnea et al (1987) model. Then, they developed unified models for stratified flow using Taitel and Dukler (1976) model, but incorporated Ouyang and Aziz (1999) liquid wall friction factor and Baker et al (1988) interfacial friction factor. Next, they developed individual unified models for slug flow, annular flow, and bubble flow. They also incorporated a fluid property model as proposed by Beggs and Brill (1991). The model was tested and validated using a wide range of data and several individual flow pattern unified models. The model performed well without any tuning (Gomez et al., 2000).

Drift Flux

The drift-flux model is a multiphase flow homogeneous mixture model that takes slip (phases moving at different velocities) into consideration (Zuber & Findlay, 1965). Some authors (Aziz et al., 1972; Shi et al., 2005) refer to it as a mechanistic model because it is used to model the physical mechanism of a flow phenomenon. Shi et al (2005) describes Drift-Flux model as a simple mechanistic model for intermittent flow, and it is used within general mechanistic models when the flow pattern is predicted to be bubble or slug (Shi et al., 2005). Though mechanistic models are believe to be more accurate in general, the have limitations such as discontinuities when approaching flow pattern transition boundaries which cause convergence issues, and they are difficult to couple with large reservoir simulators. Drift-flux models are simple, continuous and differentiable, so issues of discontinuities or convergence are limited. Furthermore, these attributes makes coupling with reservoir simulators convenient(Shi et al., 2005).

Just like mechanistic models, individual conservation equations are written for each fluid phase in the multiphase flow system with the exception of the momentum conservation equation. A single momentum conservation equation is written for a homogeneous mixture of the phases in terms of the volume average velocity of the mixture (Spesivtsev et al., 2013). The momentum conservation equation for a drift flux model can be written as:

$$\frac{\partial v_m}{\partial t} + v_m \frac{\partial v_m}{\partial x} = -\frac{1}{\rho_m}\frac{\partial p}{\partial x} + gCos\theta + 2\frac{f{v_m}^2}{d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (8)$$

Where: f is the Friction Factor

d is the Pipe Diameter

v_m is the Mixture Velocity

$$v_m = \alpha_L v_L + \alpha_G v_G \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (9)$$

ρ_m is the Mixture Density

$$\rho_m = \alpha_L \rho_L + \alpha_G \rho_G \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (10)$$

According to Shi et al. (2005), slip between phases is described using two mechanisms: One of the mechanism is due to non-uniform velocity and phase distribution in the pipe cross section; and the second mechanism is due tendency of gas to rise vertically through the liquid due to buoyancy (Shi et al., 2005). The formulation of both mechanisms gives a gas velocity:

$$v_g = C_O v_m + v_d \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (11)$$

Where C_O is the Profile Parameter

v_d is the Drift Velocity

The Profile Parameter represents the non-uniform distribution of the gas phase and the velocity profile over the pipe cross-section , while the drift velocity describes the local slip due to the difference in densities of the gas and liquid phases (Aziz et al., 1972; Spesivtsev et al., 2013).

n the general case, profile parameter and the drift velocity are functions of the mixture velocity, the void fraction, and pressure. Assuming steady state, substituting equation (11) into the conservation equations will result in the liquid velocity being expressed as a function of mixture velocity, gas volume fraction, and pressure. Combining the system of equations for mass conservation and momentum conservation, would result in a total of three equations and three unknowns. Compared to the two fluid model with four equations and four unknowns, the drift-flux formulations will require less computing time (Aziz et al., 1972; Spesivtsev et al., 2013).

Just like the mechanistic models, the drift flux model requires closure relations, and has to be fine-tuned with experimental data. The closure relation is required for determining the phase velocities. The drift-flux closure relation can record both negative and positive velocities, which implies that it will be suitable for the simulation of counter-current flow during gravity segregation in vertical pipes or in unsteady flow conditions in pipeline (Spesivtsev et al., 2013).

The drift-flux model can be summarized to have the following advantages (Spesivtsev et al., 2013):

- It has fast computing speed compared to multi-fluid model.
- It has numerical stability and robustness because it is smooth and differentiable.
- It is free from singularity errors at small values of void fractions.
- It is capable of predicting terrain slugs.

Though it was originally developed for dispersed bubble flow, its application has been extended to cover all flow regimes. Despite all of its advantages, the drift-flux model has major setbacks such as its inability to model hydro-dynamically induced slugs developing as a result of Kelvin-Helmholtz instability at the interface between fluids during stratified flow. Also, it is not calibrated for horizontal flow and downward inclined flows. Finally, it is based on the assumption of a non-inertia slip interface (Spesivtsev et al., 2013).

Some of the researchers who have applied the drift flux model include:

- Aziz et al (1972): Developed a simple mechanistic scheme for the calculation of pressure drop in oil wells that have two-phase flow of oil and gas. The aim was to develop a computer program which was a lot faster than any of the existing methods of the time. The model was focused on predicting pressure drop during single phase flow, and during multiphase bubble flow and slug flow patterns. Like most mechanistic models, the flow pattern will first be identified using flow pattern map developed by Govier et al (1957), then in-situ volume fraction of the gas phase (void fraction) and pressure are calculated using the mechanistic scheme. The method was compared with the most accurate method of the time, Orkiszewski (1967) method, using data from forty-eight wells. The results were in good agreement with Orkiszewski (1967) method in forty-four out of the forty-eight wells data sets (Aziz et al., 1972).

- Hasan and Kabir (1988): Developed a model for the prediction of flow regime, void fraction, and pressure drop during multiphase flow in deviated wells. The model works for bubble, slug, churn and annular flow patterns. They showed that the void fraction is significantly affected by deviations from the vertical position, as a result, it was found that the local void fraction was 0.25 at the transition from bubble to slug flow. The model was validated by comparing its results with those obtained from works by other researchers, and was found to be in good agreement with theirs. It was also tested using filed data from 10 wells reported by Griffith et al (1973). Its predictions were compared to those obtained using the Beggs and Brill (1973) method, which were found to be in agreement as well (Hasan and Kabir, 1988).
- Petalas and Aziz (2000): developed a robust mechanistic model for the prediction of multiphase flow behavior in pipes. The model is made of up several models developed using either the multi-fluid model or the drift flux model. The drift-flux model was used to develop the intermittent flow model for pattern prediction, and also used to develop almost all the pressure prediction models except stratified flow and annular-mist flow. They included froth flow in their model. Froth flow was said to exist at the transitions between bubble flow and annular-mist flow, and between slug flow and annular-mist flow (Petalas and Aziz, 2000).

Multiphase Flow Simulators

With the availability and continuous development of mechanistic models, the modern petroleum industry design their production facilities using mechanistic models, rather than empirical correlations. This has been supported by the fast growth rate and continuous innovation of the computer industry. High speed computers and personal computers have made run of the mechanistic models a routine practice for the modern production engineer. These mechanistic model now come in simulation software packages that run lots of program along with multiphase flow models for production system design purposes or for optimization of existing production systems. There is large amount of simulation packages available in the market, so only a few of the more prominent packages will be discussed. These include:

- OLGA
- PEPITE & WELLSIM
- TACITE
- TUFFP
- LEDAFLOW

All of these simulator are steady-state multiphase flow while a couple of them are also transient-state simulators. They all have their benefits and their downsides, researchers (Dhulesia and Lopez, 1996; Ellul and Reservoir, 2010) have done good comparison of some the above listed simulators. An attempt will be made to give a brief overview of each one.

- OLGA: It was developed by the Norwegian Institutes SINTEF and IFE. It was developed based on a modified one-dimensional two-fluid mechanistic model. The modification to the model is the introduction of liquid droplets as a third phase in addition to gas and bulk liquid (Bendiksen et al., 1991; Dhulesia and Lopez, 1996; Ellul and Reservoir, 2010). The liquid droplet phase only has a mass conservation equation, unlike the gas and bulk liquid phases that have all conservation equations. This implies that for a steady state one-dimensional gas-liquid flow, we would have five equations made up of three mass conservation equations and two momentum conservation equations, instead of a total of four. Bendiksen et al (Bendiksen et al., 1991) gives a comprehensive description of the theory and development of OLGA.
- PEPITE & WELLSIM: These simulators are developed based on the steady-state drift-flux model. PEPITE was developed for pipeline (horizontal or near horizontal) flow, while WELLSIM was developed for wellbore (vertical or deviated) flow. They were developed in France as a joint venture project sponsored by IFP, Elf-Aquitaine, and TOTAL between 1974 and 1985. For more information about these models, refer to publication by Lagiere et al. (Lagiere et al., 1984) and Corteville et al. (Corteville et al., 1984) for PEPITE and WELLSIM respectively.
- TACITE: It was also developed in France by IFP, Elf-Aquitaine, and TOTAL in 1994. It is developed based on the Two-fluid/Two-flow-pattern model developed by Fabre et al (Fabre et al., 1989). It is capable of

running transient multiphase flow simulation. For detailed description of the simulator refer to the article written by Pauchon et al (Pauchon et al., 1994).

- TUFFP: This is Tulsa University Fluid Flow Projects' model. It is developed based on the two-fluid model for multiphase flow in pipelines. A full description is given in the publication by Xiao et al (Xiao et al., 1990).
- LEDAFLOW: This is one of the state of the art pipeline simulators. It is capable of simulating transient flow, and both one- and three-dimensional flow. It is a three-phase simulator developed based on a multi-fluid/multi-field model. Similar to OLGA, it considers sub-phases as well as the bulk phases for the mass conservation equation. LedaFlow has nine mass conservation equations, three momentum and three energy conservation equations (Danielson et al., 2005; Ellul and Reservoir, 2010). For a detailed description of the model, Danielson et al (Danielson et al., 2005) gives a great description.

These simulators are not just limited to multiphase flow hydrodynamics, they are also used for other applications related to or affected by multiphase flow such as flow assurance, corrosion control etc. Also, not mentioned are Computational Fluid Dynamics (CFD) simulators. CFD can be used for multiphase flow modeling, but it is not commonly used in the petroleum industry because of the large amount of computing time required for prediction. Though CFD may be more accurate, the computation time and computing power required for prediction are several folds more than the commercial multiphase simulators.

Future Research

Despite the large amount of work that has been done so far, there are still a lot of questions yet to be answered. A lot of the current research is focused on transient modeling, improvement of closure relationship, flow physical mechanism description, three dimensional modeling, prediction of flow assurance issues such as wax deposition and severe slugging.

Research on completion related issues is being done. These include:

- Establishing rules on the best way to complete horizontal wells in relation to the possibility of multiphase flow occurring in the wells.
- The location of the production tubing inlet has not been determined.
- The spacing between fractures in relation to pressure drop along the well bore.
- The recommended degree of inclination of the lateral section of a horizontal well.

Also, a lot of research is focused on improving the experimental processes associated with multiphase modeling. These include:

- The development of better measuring instruments
- Better data gathering and capturing modules
- model upscaling and downscaling, etc.

Research on better numerical scheme for solving the conservation equations. Faster, more robust, and stable schemes are required for better prediction of multiphase flow process.

Research on integrated and robust systems. A lot of research into techniques of coupling multiphase pipe flow simulators with reservoir simulators are being conducted.

CONCLUSION

A review of multiphase flow in horizontal wells has been conducted. An insight into the various methods and approaches for the determination of pressure drop was given. The technological development of multiphase hydrodynamic parameters such as flow pattern, liquid hold up and pressure drop were discussed. An insight into the available modeling techniques and their development so far, was given. The available commercial simulators and their capabilities were discussed. Not much insight was given about transient models, only steady state models were discussed. Fluid mass-exchange between the phases and PVT properties were considered to be beyond the scope of this review. Finally, a highlight of the current and future research to be done was also given.

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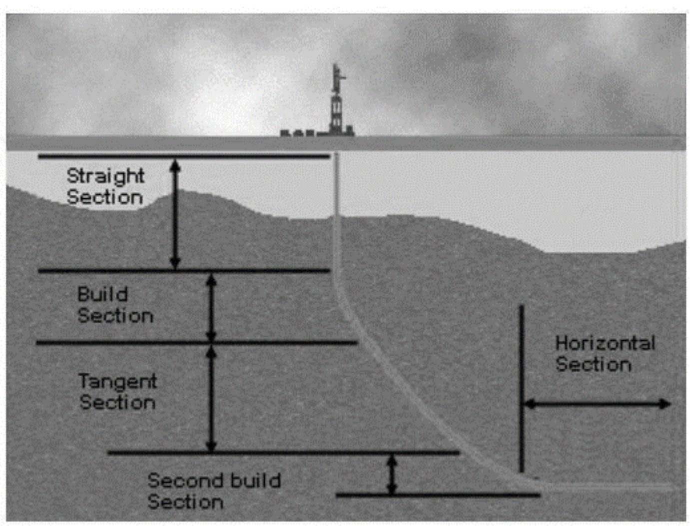


Figure 1: Horizontal Well Geometry (Courtesy: Wordpress)

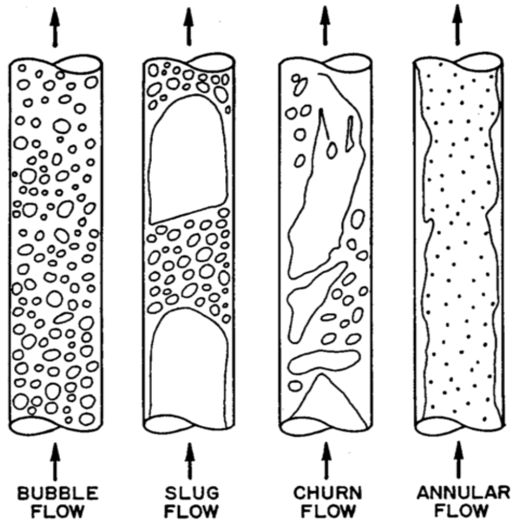


Figure 2: Vertical Flow Patterns (Courtesy: (Brill, 1987)

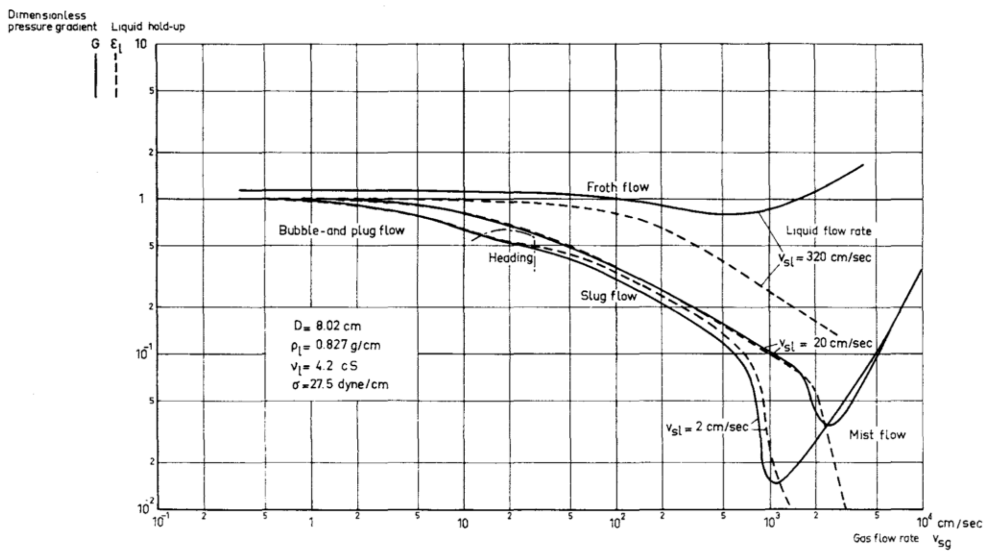


Figure 3: Pressure Gradient and Liquid Hold-up against gas Flow and Liquid Flow. (Courtesy:(Ros, 1961)

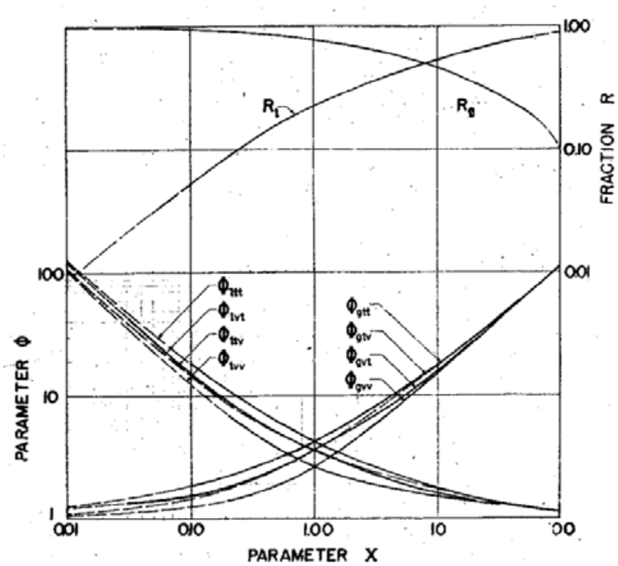


Figure 4: Lockhart and Martinelli Parameter-X Flow Curves.
(Courtesy: (Lockhart and Martinelli, 1949))

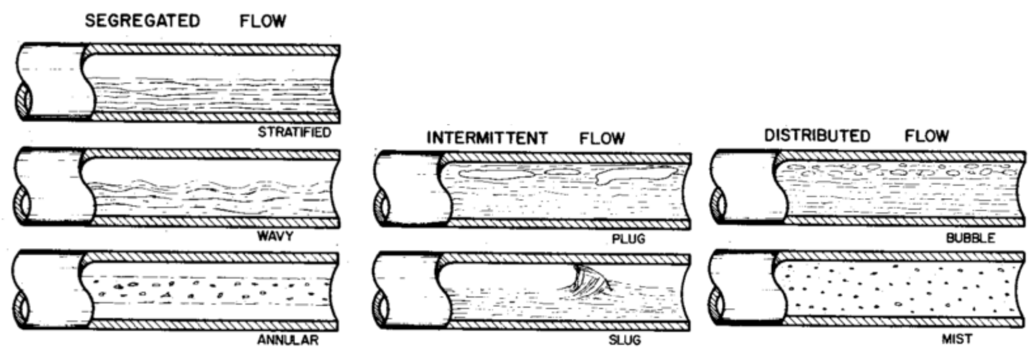


Figure 5: Horizontal Flow Patterns. (Courtesy:(Beggs and Brill, 1973))

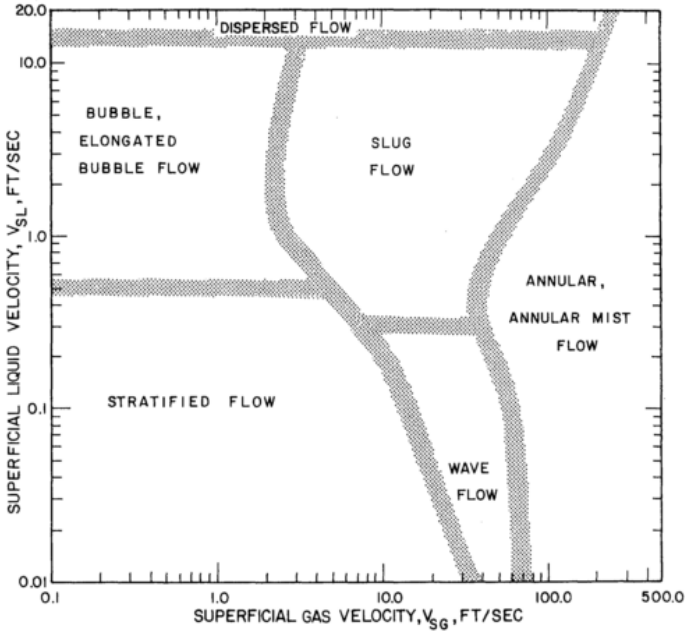


Figure 6: Flow Pattern Map (Courtesy:(Mandhane, J.M. and Gregory, G.A. and Aziz, K., 1977))

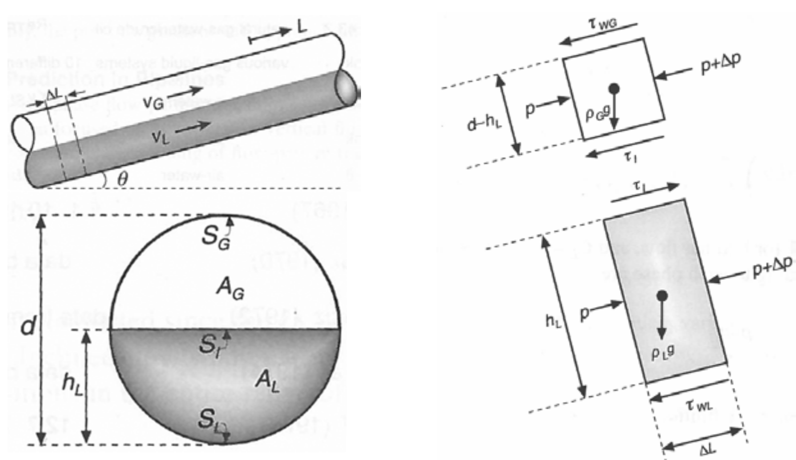


Figure 7: Equilibrium Stratified Flow Two-Fluid Model momentum Balances (Courtesy: (Shoham, 2005))

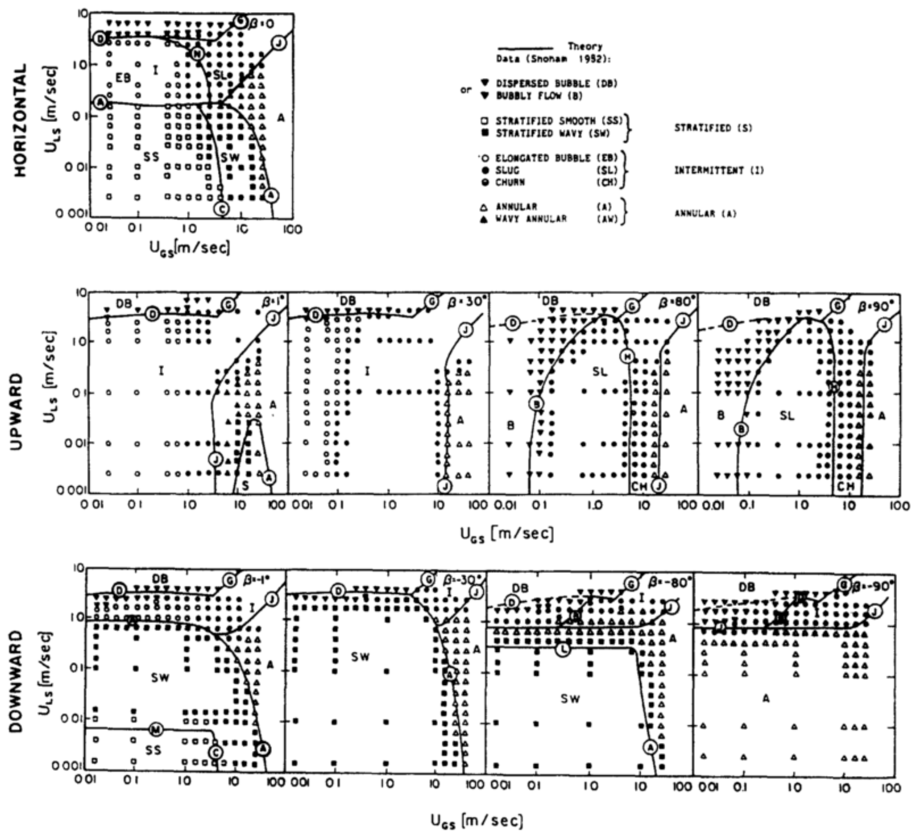


Figure 8: Unified Flow Pattern Map (Courtesy: (Barnea, 1987))

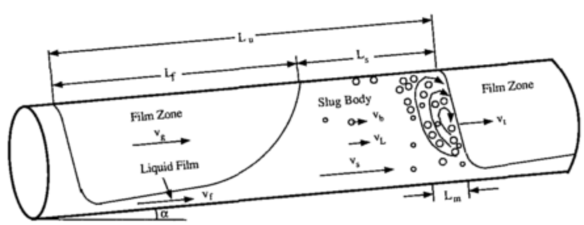


Figure 9: Intermittent Flow Model (Courtesy: (Xiao et al., 1990))

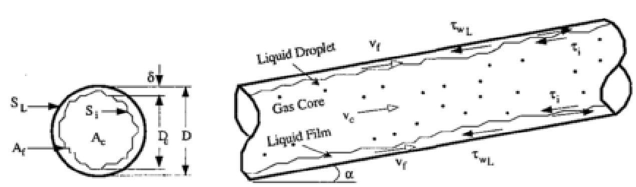
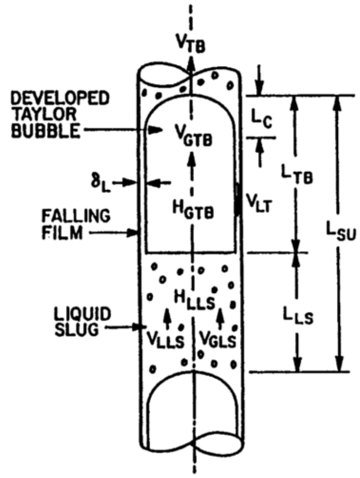


Figure 10: Annular Flow Model (Courtesy: (Xiao et al., 1990))



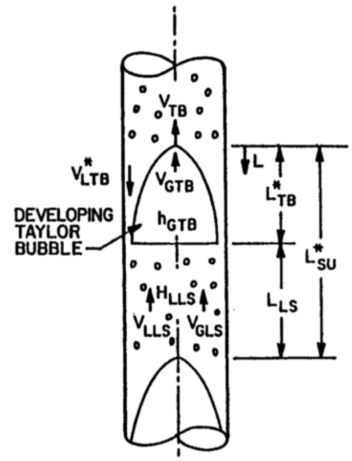


Figure 11: Developed Slug Unit (Courtesy:(Ansari et al., 1994))

Figure 12: Developing Slug Unit (Courtesy: (Ansari et al., 1994))

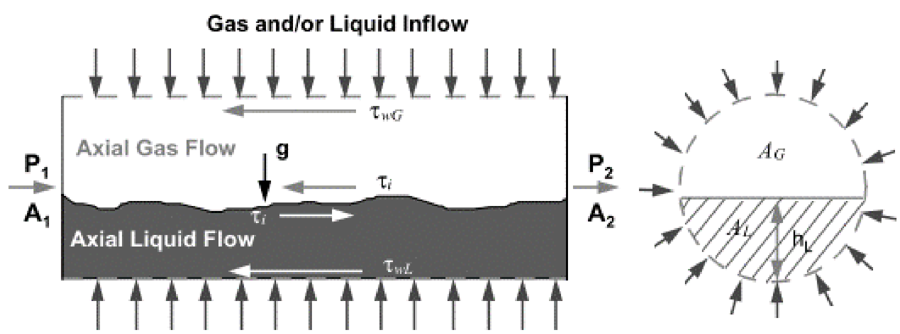


Figure 13: Two-Phase Stratified Flow in a Horizontal Wellbore
(Courtesy: (Ouyang and Aziz, 1999))

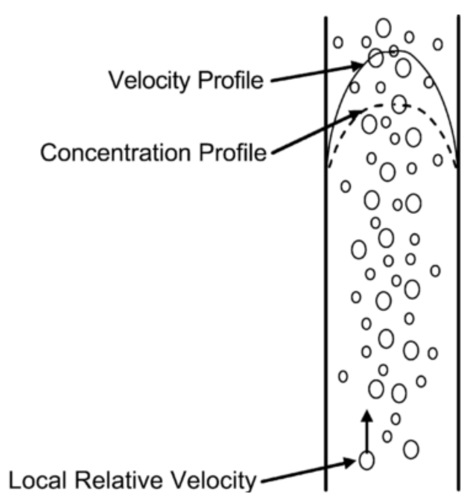


Figure 14: Profile and Local Slip mechanisms in drift flux model (Courtesy: (Shi et al., 2005))