USE OF SHORT-TERM, MULTIPLE-RATE FLOW TESTS TO PREDICT PERFORMANCE OF WELLS HAVING TURBULENCE

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INTRODUCTION

A new analytical procedure is described here for predicting well performance and analyzing completion effectiveness of wells which have a significant pressure drop from turbulence. (In this discussion we use the term turbulence to describe both turbulence and all other rate dependent deviations from darcy flow such as inertial effects.[1]) The procedure is more applicable to gas well completions but has been applied to a high-rate oil well. In particular, the new procedure should provide a powerful analytical tool in areas where most wells have high production potential.

The procedure can be used in wells requiring sand control measures and in hydraulically fractured wells to determine if the cross-sectional area open to flow into the wellbore is sufficient. It also provides an indication of perforation effectiveness in normally completed wells because an abnormally high turbulence coefficient indicates too few open perforations. Incidental to determination of a turbulence coefficient, the new procedure provides a laminar flow coefficient which includes skin effect. If permeability thickness is known, an estimate of skin effect can be made from the laminar flow coefficient. Also included in the theory is an explanation of the effects of partial completion or a change in completion geometry on pressure buildup results when the turbulence pressure drop is significant.

The analysis procedure permits determination of turbulence effects on completion efficiency irrespective of skin effect and laminar (darcy) flow. The required data are either: (1) two or more stabilized flow tests; or (2) two or more isochronal flow tests. Flow rates and bottomhole flowing pressures must be known in either case. Transient pressure data are not needed and bottomhole flowing pressures calculated from surface pressures may often be sufficient.

The previous means of determining the turbulence coefficient have required some means of obtaining kh; usually a buildup test coupled with several production tests or a series of buildup tests.[2,3] These were used to determine a total skin effect, s′, which included the turbulence term. The s′ values were then plotted versus flow rates. Actual skin effect and turbulence coefficients were then calculated from the intercept and slope. The new method avoids the necessity for transient data from a buildup or drawdown test and permits a direct plot of pressure data versus rate. The added simplicity should make the new procedure much more useful in direct field applications.

THEORY

Steady state radial and linear flow equations, including turbulence flow, are available in the literature[4,5] for both oil and gas wells. These equations apply directly to stabilized producing wells; i.e., wells which have produced long enough so that the pressure transient has reached a long distance into the reservoir. The equations are readily adapted to nonstabilized wells by using isochronal tests, i.e., short flow tests of equal time duration, with each separate flow test followed by a shut-in period to allow wellbore pressure to rise nearly back to initial reservoir pressure before the next flow test is performed. The isochronal testing procedure should prove particularly useful in determining whether the fracture width is sufficient in fractured wells. A series of very short tests may be sufficient where otherwise a long-term buildup would be required.

Equations

A simple diagnostic procedure can be used to analyze completion effectiveness of both gas and oil wells. Using Forchheimer's equation,[5] it is shown in the Appendix that flow rate and pressure drop can be related by the following equations:

For gas wells:

$$\frac{P_{ws}^2 - P_{wf}^2}{Q_g} = C + DQ_g \quad (1)$$

For oil wells:

$$\frac{P_{ws} - P_{wf}}{q} = C + Dq \quad (2)$$

Where:

P_{ws} = average formation pressure, psi
P_{wf} = flowing well pressure, psi
q = oil flow rate, STB/D
Q_g = gas flow rate, SCF/D
C = laminar flow coefficient for oil or gas wells (defined in the Appendix)
D = turbulence coefficient for oil or gas wells (defined in the Appendix)

From Eq. (1) it is apparent that for gas wells, a plot of

$$\frac{P_{ws}^2 - P_{wf}^2}{Q_g}$$

versus Q_g has a slope of D, and

$$C = \lim_{Q_g \to 0} \frac{\Delta p^2}{Q_g}$$

For oil wells, a plot of

$$\frac{P_{ws} - P_{wf}}{q}$$

versus q has a slope of D, and

$$C = \lim_{q \to 0} \frac{\Delta p}{q}$$

where Δp is pressure difference

The plots apply to both linear and radial flow. With these values alone, flow rates can be predicted for these completions at other arbitrarily chosen flowing pressures.

However, the major value of the analysis will probably be in obtaining values for D and using them to determine whether a completion can be improved. In calculating potential benefits to be obtained by changing completion geometry, it is necessary to define D in terms of the factors which can be affected by changes in completion procedures. The following cases are those most likely to be of interest.

Linear Flow - Gas or Liquid

For linear flow, it is shown in the Appendix that

$$\frac{D_1}{D_2} = \frac{\beta_1 L_1 A_2^2}{\beta_2 L_2 A_1^2} \quad (3)$$

If, as often is the case, only the area is changed,

$$\frac{D_1}{D_2} = \frac{A_2^2}{A_1^2} \quad (4)$$

In these equations,

β = turbulence factor, ft^{-1}
L = flow path length, ft
A = cross-sectional area open to linear flow, ft^2

Subscripts 1 and 2 refer to conditions before and after the change in flow geometry.

Radial Flow - Gas or Liquid

Radial flow is the base case for determination of completion effectiveness in ordinary wells. The following equation is developed in the Appendix:

$$\frac{D_1}{D_2} = \frac{\beta_1 h_{p_2}^2 r_{w_2}}{\beta_2 h_{p_1}^2 r_{w_1}} \quad (5)$$

where h_P is the length of the completed interval and r_w is the well radius. If only the completion length is altered,

$$\frac{D_1}{D_2} = \frac{h_{P_2}^2}{h_{P_1}^2} \quad (6)$$

EXAMPLES

Example 1 — Analysis of Gas Well Flow Test Date

Either a sequence of completely stabilized back-pressure tests or a series of isochronal tests provide the ideal data for evaluation of gas wells. Cullender's[6] excellent paper provides the data used here to illustrate the analysis technique for stabilized and unstabilized wells. Fetkovich[7] has presented similar data for oil wells.

Stabilized Well Test Data

Data from Cullender's Well No. 5 are given in Table 1. The data are plotted on Fig. 1. The points all fall on the straight line except for the lowest rate where an error of two tenths of a pound in reading absolute pressure will account for the deviation.

From the slope of the line, $D = 1.26 \times 10^{-10}$, and from the intercept, $C = 0.00028$. The low value of C is indicative of an exceptionally high value of kh for this particular well. The D value of 1.26×10^{-10} is quite high for such a low value of C and probably indicates that flow into the wellbore is either through a small number of perforations or a short producing interval.

With C and D known, a true calculated open flow potential can be determined with Eq. (1). For an average reservoir pressure of 439.0 psi,

$$\frac{(439.0)^2}{Q_g} = 0.00028 + 1.26 \times 10^{-10} Q_g$$

where Q_g is now the calculated open flow potential of 38.0 MMSCFD.

TABLE 1—PERFORMANCE DATA OF GAS WELL NO. 5

Shut in Pressure psia	Flow Rate MCF/D	$P_{ws}^2 - P_{wf}^2$	$\frac{\Delta p^2}{Q_g}$
439.6	2,231	1,010	.000453
439.8	4,841	4,320	.000892
439.0	8,373	11,270	.00134
439.0	12,484	23,250	.00186
439.0	16,817	40,600	.00241

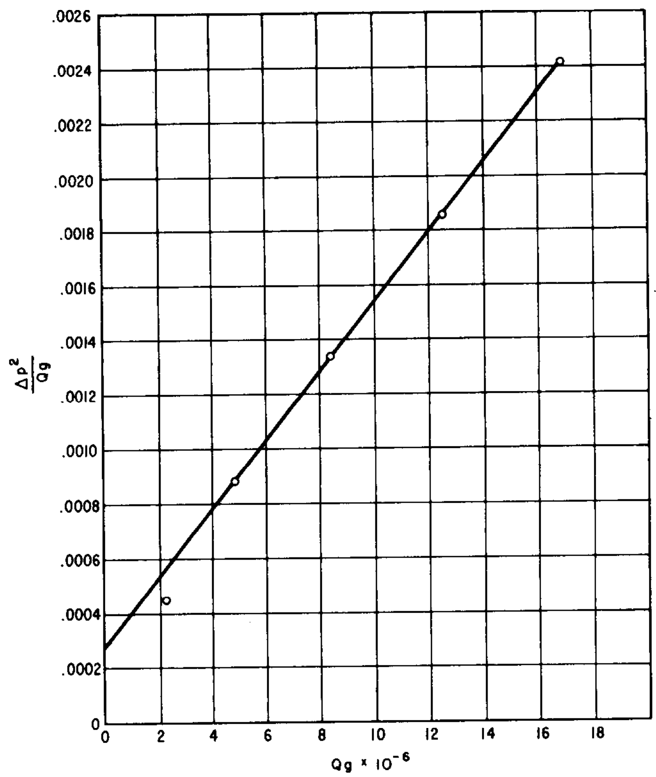

FIG. 1—ANALYSIS OF STABILIZED WELL TEST DATA, WELL NO. 5.

Since Well No. 5 has a large calculated absolute open flow potential, there probably is no reason for a workover or stimulation. However, it illustrates the type of test results obtained for wells which are good candidates for improvement. Whenever the slope of the plot is large in comparison to the intercept, the efficiency of the completion is suspect. When the value of $\Delta p^2/Q_g$ at maximum Δp^2, ($\Delta p^2 = P_{ws}^2$), is more than two or three times as large as the value at the intercept, the indication is that area open to flow near or at the well is smaller than desirable. For certain completions, such as inside casing gravel packs or fractured wells in thin producing formations, this is expected. Then it is necessary to use Eqs. (3) and (4) to check whether performance is acceptable. In ordinary cased and perforated wells, a large slope is not expected, and its occurrence indicates that the number of effective perforations is small. Such a completion has excess pressure drop from both turbulence and a large skin effect. Improvement will reduce both the slope and the intercept, and the increase in productivity can be dramatic.

Isochronal Flow Test Data

Cullender's isochronal flow test data for Well No. 1 are given in Table 2. The results are plotted on Fig. 2. The slope, D, is approximately the same for each isochronal set, with a value of 1.70×10^{-10}. The C value varies from 0.0027 for the 0.2-hr isochronal set to 0.00775 for the 24-hr set. The increase in C is due to the change in drainage radius with time, and C will continue to increase until the well is "stabilized". The value of D for this well is within the expected range, and reperforating probably will not cause a dramatic change in productivity.

The fact that D can be obtained from a series of very short isochronal tests is significant. The time required for pressure recovery after a 10 or 15-

TABLE 2—ISOCHRONAL DATA OF GAS WELL NO. 1

Isochronal 0.2 hr. data			Isochronal 1 hr. data		
$Q_g \times 10^3$	$\Delta p^2 \times 10^3$	$\Delta p^2/Q_g$	$Q_g \times 10^3$	$\Delta p^2 \times 10^3$	$\Delta p^2/Q_g$
2009	5.68	.00283	1994	8.26	.00414
2997	9.21	.00307	2937	12.87	.00438
4130	12.72	.00308	2941	13.28	.00452
7327	26.09	.00356	4052	18.40	.00454
			4656	22.52	.00484
			7092	35.52	.00500

3 hr. data			24 hr. data		
1980	10.38	.00524	1447	14.56	.00748
2905	15.72	.00541	4440	38.67	.00871
4587	27.26	.00594	9900	97.70	.00987
6887	42.99	.00624			

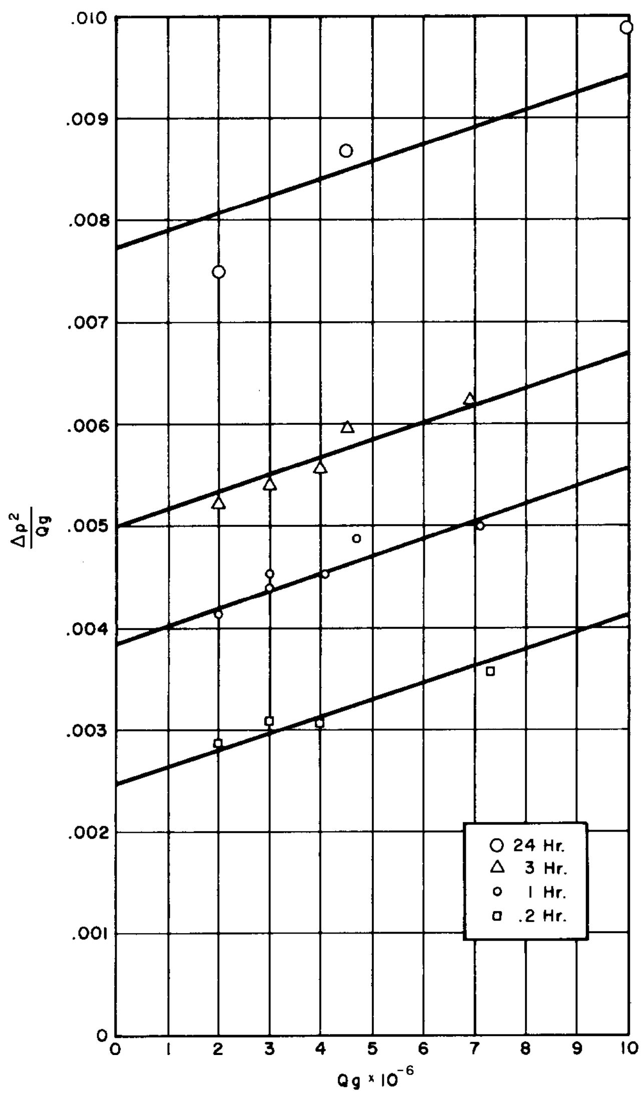

FIG. 2—ANALYSIS OF ISOCHRONAL FLOW TEST DATA, WELL NO. 1.

minute flow test is ordinarily not long; perhaps an hour or two. This provides the possibility for rapid and inexpensive determination of near-well flow restrictions, even in wells which will not stabilize rapidly. For example, for hydraulically fractured wells, almost all pressure drop from turbulence will be in the fractures and perforations. Flow restrictions in these locations can be pinpointed by analysis of short isochronal tests, even though the formation flow characteristics are not known and could only be obtained with a long-term buildup test.

Analysis of Back-Pressure Tests Data

Cullender also reported back-pressure test data for Well No. 1. It is useful to analyze these data so as to illustrate the potential pitfalls in using back-pressure test data on unstabilized wells. The data are given in Table 3. Each of the data points is for a 24-hr flow period. The first points of each back-pressure test sequence made up the 24-hr data of Fig. 2. The back-pressure test results are plotted on Fig. 3. As expected, the slope, D, is greater than the slope from isochronal tests when the back-pressure test sequence is run with the low rate first, and is lower when the back-pressure test sequence is run with the highest rate first. Both of these variations result from the increase in drainage radius with time in an unstabilized well.

TABLE 3—BACK-PRESSURE TEST DATA ON GAS WELL NO. 1.

Date	Q_g	Δp^2	$\Delta p^2/Q_g$
10-3-44	9900	97.70	.00987
	7091	70.73	.00997
	4360	46.16	.01059
10-24-44	4440	38.67	.0087
	6982	75.17	.0108
	8212	92.35	.0112
12-11-45	1947	14.56	.00748
	2841	25.07	.00882
	3941	38.82	.00985
	5165	50.53	.00978

Example 2 — Analysis of High-Rate Oil Well Flow Tests

A new oil discovery was tested at high production rates through a limited completion interval. Pressure drop from tubulence was evident because the productivity index (PI) declined as the rate

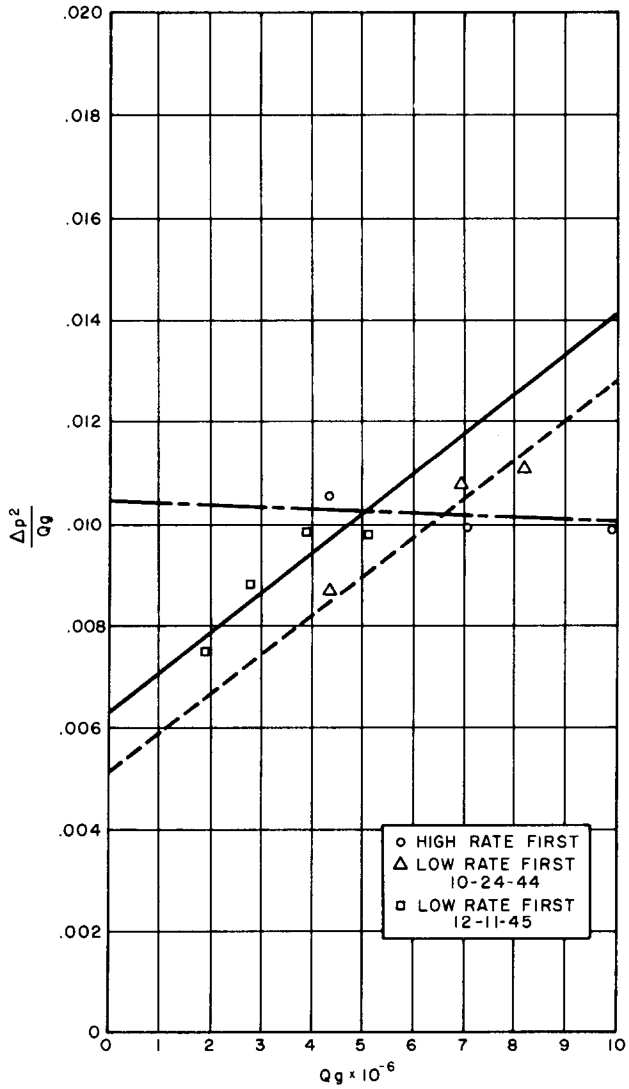

FIG. 3—ANALYSIS OF BACK PRESSURE TEST, WELL NO. 1

increased. The problem is to calculate pressure drawdown at higher rates and to estimate PI if the perforated interval is increased.

The well was tested at three different flow rates. Test data are given in Table 4. Since the producing zone is extremely permeable, the well stabilized so rapidly that no transient could be observed. All test data were taken above the bubble point pressure, which is less than 4000 psi. Initial shut-in pressure was 5948 psi.

The results are plotted on Fig. 4. The infinite PI at a rate of 1446 BPD merely indicates that the pressure drop was too small to read on a 10,000 psi gauge. From Fig. 4, the laminar flow coefficient, C, in Eq. (2) is nearly zero and the turbulence coefficient, D, is 9.8×10^{-7}. The laminar flow coefficient has to be greater than zero; but in this case, kh is apparently so great that pressure drop from laminar flow is negligible.

TABLE 4—PERFORMANCE DATA FOR HIGH RATE OIL WELL

Rate	Bottom Hole Pressure	Δp	$\frac{\Delta p}{q}$	PI
1446	5448	0	0	•
6199	5410	38	.00613	163
8115	5383	65	.008	125
20,000*	5056*	392*	.0196*	51*
30,000*	4566*	882*	.029*	34.0*

* Calculated Values

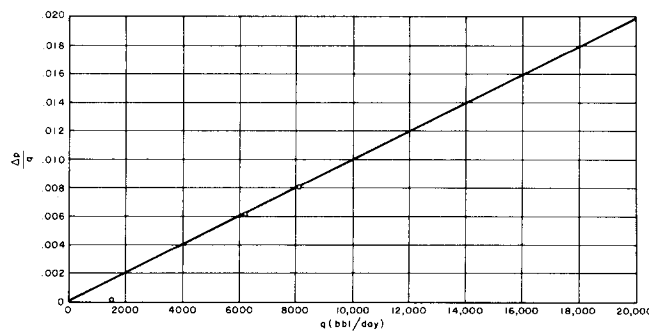

FIG. 4—ANALYSIS OF HIGH RATE OIL WELL FLOW TESTS

Pressure drawdown at higher rates can be calculated with Eq. (2) or by extrapolating the straight line of Fig. 4 to the desired rate to obtain a value for $(P_{ws} - P_{wf})/q$, which also equals $1/PI$. At a rate of 30,000 BPD:

$$\frac{P_{ws} - P_{wf}}{q} \cong Dq = 9.7 \times 10^{-7} \times 30{,}000 = 0.0294$$

and

$$PI \cong 1/0.0294 = 34.$$

Drawdown required to attain 30,000 BPD therefore is:

$$\Delta p = 30{,}000/34 = 882 \text{ psi}$$

At a rate of 20,000 BPD, PI would equal 51 and drawdown would be 392 psi.

Since $C \cong 0$, the adjustment in PI caused by extending the length of the perforated interval can be estimated as follows:

$$\frac{PI_2}{PI_1} \cong \frac{D_1}{D_2} \cong \frac{h_{P_2}{}^2}{h_{P_1}{}^2} \tag{7}$$

Therefore, PI in this well can possibly be quadrupled by doubling the length of the perforated interval.

The large D value in this case probably indicates inadequate perforations, in the sense that most of the effect of turbulence occurs at the perforations. Therefore, reperforating the same interval or increasing the perforation density to 8 shots per foot may also increase the PI by a factor of 4.

DISCUSSION

The analysis procedure presented here allows prediction of well performance and diagnosis of many well completion problems with inexpensive short-term tests. The procedure is particularly applicable to gas wells in "good" formations where completion problems are apt to occur. The results allow separation of mechanical problems, such as too few open perforations or too narrow fractures, from formation damage or low permeability. This permits the engineer to choose the type of stimulation or workover most likely to solve the existing problem. The ability to pinpoint reasons for low productivity should make the test procedure a valuable new engineering tool.

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APPENDIX

DEVELOPMENT AND DISCUSSION OF EQUATIONS

LINEAR FLOW - GAS

Linear gas flow is a major contributor to pressure drop for wells producing through inside casing gravel packs or fractures. For either case, pressure drop from turbulence is frequently the major factor in limiting production rate.

For linear gas flow through a porous medium of length L, (Ref. 4),

$$P_{ws}^2 - P_{wf}^2 = \frac{8.932 T \mu Q_g L}{kA} + \frac{1.247 \times 10^{-16} \beta_g Z T \gamma_g^2 L}{A^2} \qquad \text{(A-1)}$$

Where:

- Z = dimensionless gas compressibility coefficient
- T = reservoir temperature, °R
- μ = viscosity, cp
- L = flow path length, ft
- k = permeability, md
- A = cross-sectional area open to linear flow, ft^2
- β = turbulence factor, ft^{-1}
- γ_g = gas gravity (air = 1.0)

This can be written:

$$\frac{\Delta p^2}{Q_g} = C + DQ_g \qquad \text{(A-2)}$$

where the darcy or laminar flow term is given by:

$$C = \frac{8.93ZT\mu L}{kA} \qquad \text{(A-3)}$$

and the turbulence flow term is:

$$DQ_g = \frac{1.247 \times 10^{-16}\, \beta \gamma_g ZTL}{A^2}\, Q_g \qquad \text{(A-4)}$$

A plot of $\frac{\Delta p^2}{Q_g}$ versus Q_g has a slope of D, and

$$C = \lim_{Q_g \to 0} \frac{\Delta p^2}{Q_g}.$$

Thus, if most of the pressure drop occurs in a linear geometry, the darcy and turbulence flow terms can be determined so long as L is known.

The above analysis should be particularly useful for completions with inside casing gravel packs. Since L, β, and k are fairly well known for that case, the area, A, can be calculated. From A, the number of effective perforations can be determined. This will allow evaluation of various perforating procedures.

In calculating potential benefits to be obtained by changing the completion geometry, a new turbulence coefficient can be estimated from the previously measured value by taking a ratio. Thus:

$$\frac{D_1}{D_2} = \frac{\beta_1 L_1 A_2^2}{\beta_2 L_2 A_1^2} \qquad \text{(A-5)}$$

If, as often is the case, only the area is changed:

$$\frac{D_1}{D_2} = \frac{A_2^2}{A_1^2} \qquad \text{(A-6)}$$

This analysis also can be applied to fractured wells in relatively tight formations. In that case L and A^2 are more or less inseparable. Probably the best tests to perform would be short-term isochronal flow tests at substantially different rates. If the turbulence flow term is large, the obvious corrective procedure to use for obtaining higher flow rates is to widen the fracture since A^2 appears in the denominator of the turbulence flow term. We suspect that pressure drop from turbulence is a major factor in limiting flow rates from fractured wells when the fractured formation is relatively thin. Based upon evaluation of a number of fractured wells, the fracture designs could be altered to take advantage of optimum flow conditions. Also, in those cases where turbulence is obviously a major problem in limiting flow rate, restimulation jobs producing greater fracture widths might be considered.

RADIAL FLOW - GAS

The base case for determination of completion effectiveness for ordinary wells is radial flow. The equation describing radial semisteady-state gas flow is:[4]

$$P_{ws}^2 - P_{wf}^2 = \frac{1.424\mu ZTQ_g}{kh}\left(\ln 0.472 \frac{r_e}{r_w} + s\right) + \frac{3.16 \times 10^{-18} \beta \gamma_g^2 ZT}{h^2}\left(\frac{1}{r_w} - \frac{1}{r_e}\right), \qquad \text{(A-7)}$$

where:

- r_e = external drainage radius in ft
- r_w = well radius, ft
- h = producing formation thickness, ft
- s = skin effect excluding turbulence effects

Equation (A-7) can also be rewritten as:

$$\frac{\Delta p^2}{Q_g} = C + DQ_g$$

where now the darcy term is:

$$C = \frac{1.424\mu ZT}{kh}\left(\ln 0.472 \frac{r_e}{r_w} + s\right) \qquad \text{(A-8)}$$

and the turbulence term is:

$$DQ_g = \frac{3.16 \times 10^{-18} \beta \gamma_g ZT}{h^2} \frac{1}{r_w} Q_g \qquad \text{(A-9)}$$

A plot of $\frac{\Delta p^2}{Q_g}$ versus Q_g again has a slope of D, and

$$C = \lim_{Q_g \to 0} \frac{\Delta p^2}{Q_g}$$

The $\frac{\Delta p^2}{Q_g}$ versus Q_g plot allows differentiation between simple damage and problems involving the turbulence term for radial flow.

Correlations are available for expected values of β for given values of k.[4,8] Occasionally the DQ_g term will be orders of magnitude higher than expected for the known formation kh. In that case it is obvious that the completion is inefficient and that more and deeper perforations are needed or that the completion interval should be lengthened.

Partial Completion Effects

The turbulence term is written above as though the whole interval were completed. This is seldom the case and it is useful to consider what happens to the turbulence term in a partially completed well. In that case, flow converges from the entire producing interval into the completed interval. In most cases the last few feet of flow paths are essentially radial.[9,10] Also, most of the turbulence pressure drop takes place in this last few feet of the flow path since the area perpendicular to flow becomes very small in that zone. Therefore, for practical purposes, the value of h in the turbulence term can be replaced by the length of the completed zone, h_p. Then

$$DQ_g = \frac{3.16 \times 10^{-18}\beta\gamma_g ZT}{h_p^2} \frac{1}{r_w} Q_g \qquad \text{(A-10)}$$

and

$$D = \frac{3.16 \times 10^{-18}\beta\gamma_g ZT}{h_p^2 \; r_w} \qquad \text{(A-11)}$$

This formulation for D has considerable implications concerning well test results. The value of D suggested by Ramey[3] is very much a function of the completion geometry and will change each time something is done to the well which changes the flow patterns into the wellbore. The effects on the turbulence coefficient of changing completion length, or well radius, or altering the formation can be estimated by comparing before and after completion conditions. Thus,

$$\frac{D_1}{D_2} = \frac{\beta_1 h_{P_2}{}^2 r_{w_2}}{\beta_2 h_{P_1}{}^2 r_{w_1}} \qquad \text{(A-12)}$$

If only the completion length is altered, as often will be the case,

$$\frac{D_1}{D_2} = \frac{h_{P_2}{}^2}{h_{P_1}{}^2} \qquad \text{(A-13)}$$

LINEAR FLOW - OIL

Linear oil flow also has a darcy pressure drop and a turbulence pressure drop. In many cases the turbulence term is quite low and can be neglected. However, for high-rate wells in naturally fractured zones, or for inside casing gravel packed completions, the linear flow turbulence component of pressure drop can become the major factor in limiting flow capacity.

For liquid flow, Forchheimer's Eq. (5) can be written:

$$\frac{dp}{dl} = \frac{q\mu B}{1.127 \times 10^{-3} kA} + \frac{9.08 \times 10^{-13}\beta q^2 B^2 \rho}{A^2} \qquad \text{(A-14)}$$

Therefore, for linear flow,

$$P_2 - P_1 = \frac{q\mu BL}{1.127 \times 10^{-3} kA} + \frac{9.08 \times 10^{-13}\beta q^2 B^2 \rho \cdot L}{A^2} \qquad \text{(A-15)}$$

where:

- P_2 = pressure at the entrance of the linear flow path, psi
- P_1 = pressure at the exit of the flow path, psi
- L = length of the flow path, ft
- A = cross sectional area perpendicular to flow, ft^2
- q = flow rate, STB/D
- B = formation volume factor, RB/STB
- ρ = fluid density, lb/ft^3

This can be written as:

$$\frac{\Delta p}{q} = C + Dq \qquad \text{(A-16)}$$

where

$$C = \frac{\mu BL}{1.127 \times 10^{-3} kA} \qquad \text{(A-17)}$$

and

$$D = \frac{9.08 \times 10^{-13}\beta B^2 \rho L}{A^2} \qquad \text{(A-18)}$$

A plot of $\frac{\Delta p}{q}$ versus q gives the laminar flow term, C, as

$$C = \lim_{q \to 0} \frac{\Delta p}{q}$$

and the turbulence flow term, D, as the slope.

Except for oil wells with exceptionally high flow rates, the turbulence flow term should be negligible when compared to the laminar flow term. If this is not the case, the completion design should be carefully reviewed because the flow area is probably too small. Even though the turbulence term is not a large percentage of the total pressure drop for an oil well, its noticeable presence indicates that the laminar flow term can also be dropped considerably by providing more area for flow into the wellbore.

RADIAL FLOW - OIL

The turbulence term in radial oil flow should be negligible for most oil wells. If the term is measureable and if pressure drop is appreciable, the well probably should be stimulated or reperforated or the completed interval should be extended.

For radial flow,

$$\frac{dp}{dr} = \frac{q\mu B}{1.127 \times 10^{-3} kA} + \frac{9.08 \times 10^{-13} \beta q^2 B^2 \rho}{A^2} \qquad \text{(A-19)}$$

where $A = 2\pi rh$. This is easy to integrate and

$$P_2 - P_1 = \frac{q\mu B}{1.127 \times 10^{-3}\,(2\pi kh)} \ln \frac{r_2}{r_1} - \frac{9.08 \times 10^{-13} \beta q^2 B^2 \rho}{4\pi^2 h^2} \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \qquad \text{(A-20)}$$

where P_2 is the pressure at some outer radius, r_2; and P_1 is the pressure at the inner radius, r_1. If we let $P_2 = P_{ws}$, $r_2 = r_e$, $P_1 = P_w$ and $r_1 = r_w$, if we account for skin effect and pseudo-steady state flow in the laminar flow term, and if we assume r_e is fairly large, then

$$P_{ws} - P_{wf} = \frac{q\mu B}{1.127 \times 10^{-3}\,(2\pi kh)} \left(\ln 0.472 \frac{r_e}{r_w} + s \right) + \frac{9.08 \times 10^{-13} \beta q^2 B^2 \rho}{4\pi^2 h^2} \left(\frac{1}{r_w} \right) \qquad \text{(A-21)}$$

Once again we obtain the form:

$$\frac{\Delta p}{q} = C + Dq \qquad \text{(A-16)}$$

where C, the laminar flow coefficient, is given by

$$C = \frac{\mu B}{1.127 \times 10^{-3}\,(2\pi kh)} \times \left(\ln 0.472 \frac{r_e}{r_w} + s \right) \qquad \text{(A-22)}$$

and the turbulence coefficient is:

$$D = \frac{9.08 \times 10^{-13} \beta B^2 \rho}{4\pi^2 h^2 r_w} \qquad \text{(A-23)}$$

Once again, a plot of $\frac{\Delta p}{q}$ versus q has a slope of D, and

$$C = \lim_{q \to 0} \frac{\Delta p}{q}$$

Partial Completion Effects

It is worth noting here that D again is very much a function of the length of the completed interval. If only part of the interval is completed, h is replaced by h_p and

$$D = \frac{9.08 \times 10^{-13} \beta B^2 \rho}{4\pi^2 h_p^2 r_w} \qquad \text{(A-24)}$$

Again, a comparison of completions should include an adjustment for the length of the completion. Thus, assuming everything else is constant, for two different completion lengths:

$$\frac{D_1}{D_2} = \frac{h_{P_2}^2}{h_{P_1}^2} \qquad \text{(A-25)}$$