A COMPUTERIZED MODEL FOR VISCOSITY CORRECTION OF CENTRIFUGAL PUMP PERFORMANCE CURVES

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ABSTRACT

The variation of centrifugal pump performance curves with liquid viscosity is a well-known fact and is usually properly accounted for when selecting such pumps for oilfield service like artificial lifting by ESP units, pipeline transport, etc. Thus, several theoretical and empirical methods are available for correcting performance curves (conventionally measured with water) to other liquids. Most of them utilize specially developed charts and tedious hand calculations and are viable for simple design calculations but find very limited use when a great number *of* cases is present or when computerized calculations are desired.

The aim of the present paper is the development of specific formulas instead of charts to be used for correcting the performance curves *of* centrifugal pumps for higher viscosities. The formulas are based on the widely used and generally accepted procedure of the Hydraulic Institute whose previously published charts, after digitalization, were fitted with analytical functions of the relevant parameters. The model introduced in the paper easily lends itself to computer applications and may be used as a subroutine in many kinds of program packages that involve selection or evaluation of centrifugal pumps.

The use of the model is illustrated by a detailed example problem. At the end of the paper, two additional examples are shown to prove the accuracy of the proposed model. Here, calculated performance curves are compared to actually measured ones for two pumps in pipeline service. The two examples prove that the proposed analytical model gives very good accuracies.

Introduction

The proper selection and evaluation of centrifugal pumps used in the petroleum industry relies on the knowledge of pump performance curves supplied by the manufacturer. These curves represent the variation of pumping head, pump efficiency, and pump power vs. pump capacity. If an ideal, frictionless liquid were pumped the performance curves would be straight lines and could be determined easily. Their shape, however, considerably changes if real liquids are pumped, due to the frictional and form-drag losses arising during the pumping action. Since these losses are affected by a host of design and manufacturing parameters (blade angle, gap width, surface roughness, etc.) it is impossible to determine a given pump's performance curves by calculations only. Therefore, the performance curves of centrifugal pumps are always established experimentally by actual measurements using water as a conventional test liquid.

For the majority of cases when the actual liquid has a viscosity close to that of water (1 cSt), the use of the performance curves established as given above is justified. In many cases, however, the liquid pumped (like heavier crudes, other production liquids, etc.) may exhibit higher viscosities. In such cases the performance of the pump considerably changes as compared to the case of pumping water. Because of higher hydraulic losses in the pump caused by a more viscous liquid, pumping head and pump efficiency decrease, whereas the required power increases as viscosity increases. It was found that the pumping head and pump efficiency curves valid for the more viscous liquids fall below the corresponding water performance curves, while the shut-off head point remains the same irrespectively of viscosity.

Ways to Correct Pump Performance Curves

Due to the wide application of centrifugal pumps in the petroleum industry and the predominance of oilfield liquids exhibiting higher viscosities, a common task both in production and pipeline operations is the correction of the manufacturer's performance curves for viscosities other than that of water. There are several models available to accomplish this as detailed below.

It follows from the theory of the centrifugal pump's hydraulic performance that at its best efficiency point (b.e.p.) i.e. at the pump capacity belonging to the highest pump efficiency, only frictional losses are present and form-drag losses can be neglected. Friction losses, however, analogously with pipe flow, are known to vary with Reynolds number and wall roughness. Since the Reynolds number contains the viscosity of the liquid pumped, this feature of the best efficiency point enables one to correct the head and efficiency values of water tests to other viscosities. **As** shown below, most of the available correction models make use of this feature.

Stepanoff's Model

Stepanoff [1] performed several experiments using conventional design centrifugal pumps made by Ingersoll-Rand and pumping water as well as eleven kinds of oils with viscosities between 1 and 2,020 cSt. Based on his experimental results he presented a diagram for head correction factor and efficiency valid at the b.e.p. defined above. The independent variable of the diagram is a Reynolds number-like parameter:

$$R_{Stepanoff} = 248387 \frac{Q_{Wbep}}{D \nu_o} \quad (1)$$

First, the pumping head at the b.e.p., H_{Wbep}, is corrected to get the new value H_{Obep} valid for the viscous liquid, then the latter value is plotted at a pump capacity Q_{Obep} found from the following equation suggested by the author:

$$\frac{Q_{Wbep}}{Q_{Obep}} = \left(\frac{H_{Wbep}}{H_{Obep}} \right)^{1.5} \quad (2)$$

The pump's new head curve was hand-plotted using the point just calculated and the original shut-off head value as a starting point. The pump power curve was drawn through the corrected power calculated for the b.e.p. by following the general slope of the original curve valid for water. Finally, the points of the corrected efficiency curve are calculated from the new power and head curves.

Stepanoff s correction diagram is applicable to a maximum Reynolds number of $R_{Stepanoff} = 4\ 10^5$.

Paciga's Model

The method suggested by Paciga [2] is also based on a slightly modified Reynolds number-like parameter:

$$R_{Paciga} = 10.753 \frac{nD^2}{\nu_o} \qquad (3)$$

Paciga presented correction factors for all performance curves and used two correlating parameters, the pump's specific speed, n_s :

$$n_s = 0.7067\, nQ_{Wbep}^{1/2}\, H_{Wbep}^{-3/4} \qquad (4)$$

and the ratio Q_W/Q_{Wbep} belonging to any arbitrary point on the performance curve for water. Using the author's chart, capacity-, head-, and pump power correction factors for any flow rate can be calculated. Applicability of his model is in the range of $R_{Paciga} = 4\ 10^7$ - $4\ 10^9$.

The Hydraulic Institute Model

The viscosity correction method suggested by the Hydraulic Institute [3] involves the use of two charts. The first chart employs the capacity (pumping rate) Q_{Wbep} belonging to the b.e.p. of the water performance curves as an independent variable rather than a Reynolds number-like value. The chart's parameters are the pumping head at b.e.p. H_{Wbep} and the kinematic viscosity ν_o of the liquid pumped. Based on these values, a rate Q^* (a correlation parameter only) is determined from the chart. This parameter serves as the independent variable in the second chart which allows the determination of factors CH, CQ, and $C\eta$, i.e. the correction factors for the head, capacity and efficiency curves. respectively.

In order to obtain several points on the most important performance curve, the pumping head vs. pump capacity curve, four different CH values are determined. These belong to four different capacities which are 0.6; 0.8; 1.0; and 1.2 times Q_{Wbep}, respectively. The correction of pump capacity and efficiency values, in contrary, is done by using correction factors independent of water rate. The corrected performance curves, valid for pumping the required liquid of a given viscosity, can easily be constructed by plotting the calculated heads and efficiencies in the function of the corrected pump capacities. The required accuracy is ensured by the fact that four different points on each performance curve are known. In addition, the head vs. capacity curve has one more set point that corresponds to the shut-off head value at zero pumping rate. This is because the shut-off head value of any centrifugal pump remains the same irrespectively of the viscosity of the liquid pumped.

The application ranges of the Hydraulic Institute charts are 100 – 10,000 GPM for pump capacity and 6 – 600 ft for pumping head. The pump must be of a conventional design and the kinematic viscosity of the liquid pumped may vary in the range of 4 – 3,000 cSt.

Development of the Proposed Model

Evaluation of Previous Methods

A common feature of the methods detailed above is the use of correction charts. Their application inevitably involves visual read-off and hand calculations, methods rather anachronistic in our era of high-speed computers. At the same time, petroleum engineers frequently face the problem of performance curve correction, e.g. when selecting ESP units or designing pipelines. This is why a numerical, easy-to-program calculation model, the objective of the present paper, will surely find wide application and will ease the life of practicing engineers.

Our goal set forth above could not be realized by running specific experiments because of the prohibitive costs likely to incur. Instead, we tried to find an existing model of sufficient accuracy and broad ranges of applicability to form the base of a numerical calculation. This is why our first task was a critical evaluation of all existing correction methods. Of those, Stepanoff's turned out to be usable around the b.e.p. only, because the determination of the pumping head curve between the shut-off head value and the b.e.p. is prone to human error. The Paciga method, on the other hand, seemingly enables one to establish complete performance curves but its viscosity range is not appropriate for the high viscosity oils and liquids commonly encountered in the petroleum industry. This is because higher liquid viscosities can easily reduce the actual Reynolds number below the lower limit ($4\ 10^7$) of Paciga's correlation.

On contrary to the above models, the Hydraulic Institute's correction method was promising and was finally chosen after considering its broad application ranges and its more detailed results. Thus the second phase of our work was to convert the original, time-consuming hand procedure to an easy-to-use numeric one. For this, **we** had to digitize the original charts and had to perform a regression analysis on the data bank thus formed. The individual curves on each chart were curve-fitted to find their most accurate approximating functions. The results of the regression analysis are given in the next section where a detailed procedure for their use is also presented.

Description of the New Model

In the following, the steps of the developed correction model are detailed along with the relevant equations received from the regression analysis performed. The essential curves and points required for performance curve correction are shown in a schematic diagram in **Fig. 1.** Original performance curves, i.e. those measured by using water are shown in bold line.

Our formulas, as presented later, of course, allow hand calculations but the preferred method is the use of a small computer program. This approach eliminates the need for visual reading-off of several points on the original performance curves and the inaccuracies thus introduced. Therefore, we suggest to use a digitizer to read several points on the original pumping head, and efficiency vs. pumping capacity curves and to fit those with power series of pumping capacity. Experience has shown that use of a third-, and a second-order series gives sufficient accuracies for the head vs. capacity and the efficiency vs. capacity curves, respectively. In the following, we assume that these functions have previously been established.

The first step involves the determination of the pumping head and efficiency values valid at the pump's b.e.p. i.e. the pump capacity at its highest efficiency. After the maximum of the efficiency vs. pump capacity curve is determined, the points corresponding to Q_{Wbep} thus determined must be found. This involves simple calculations if the performance curves were previously fitted with power series, as suggested above.

Since the viscosity of the liquid being pumped, as well as the pumping head and capacity at the b.e.p. are known, the correlation parameter Q^* of the first Hydraulic Institute chart can be calculated. Curve-fitting of that chart resulted in the following equation:

$$Q^* = \exp\left(\frac{39.5276 + 26.5605 \cdot \ln(\nu_o) - y}{51.6565}\right) \qquad (5)$$

where:

$$y = -7.5946 + 6.6504 . \ln(H_{Wbep}) + 12.8429 \cdot \ln(Q_{Wbep}) \qquad (6)$$

As shown before, all the required correction factors are the sole functions of the parameter Q^* given above, Thanks to this, all curves on the second chart of the original model could easily be fitted with functions of Q^*. The results of our regression analysis are given below in the order they are required in the correction process.

The next steps involve determinations of several points on the new pumping head and pump efficiency curves. **As** detailed before, these are calculated for several different pump capacities to facilitate creation of a wide range of corrected data. For this purpose, four points on the original performance curves are determined, those belonging to the pumping capacities of $0.6\,Q_{Wbep}$; $0.8\,Q_{Wbep}$; Q_{Wbep}; and $1.2\,Q_{Wbep}$. The corresponding corrected Q_o capacities, i.e. those valid for pumping the viscous liquid are found by simply multiplying the water capacity values by the capacity correction factor CQ. The formula best describing the capacity correction factor was found as:

$$CQ = 1.0 - 4.0327 \cdot 10^{-3} \cdot Q^* - 1.7240 \cdot 10^{-4} \cdot Q^{*2} \qquad (7)$$

The new pumping head values, H_{oi}, are calculated as products of the H_{wi} values of the water head curve and the appropriate CH_i head correction factors. **As** discussed before, head correction factors are different for the four capacities. Regression analysis resulted in the following formulas:

$$CH_1 = 1.0 - 3.6800 \cdot 10^{-3} Q^* - 4.3600 \cdot 10^{-5} \cdot Q^{*2} \tag{8}$$

$$CH_2 = 1.0 - 4.4723 \cdot 10^{-3} Q^* - 4.1800 \cdot 10^{-5} \cdot Q^{*2} \tag{9}$$

$$CH_3 = 1.0 - 7.0076 \cdot 10^{-3} Q^* - 1.4100 \cdot 10^{-5} \cdot Q^{*2} \tag{10}$$

$$CH_4 = 1.0 - 9.0100 \cdot 10^{-3} Q^* + 1.3100 \cdot 10^{-5} \cdot Q^{*2} \tag{11}$$

Finally, the corrected pump efficiency values η_{oi} are found by multiplying the original efficiencies η_{wi} by an efficiency correction factor, $C\eta$. This was fitted by the formula:

$$C\eta = 1.0 - 3.3075 \cdot 10^{-2} \cdot Q^* + 2.8875 \cdot 10^{-4} \cdot Q^{*2} \tag{10}$$

The above process gives four points on each of the new pump performance curves. In addition to these a fifth point can easily be found, based on the hydraulic theory of centrifugal pumps. According to those principles, the value of the shut-off head is independent of the viscosity of the liquid pumped, so the new head H_{O0} at zero pump capacity falls on H_{W0} of the original curve. Similarly, pump efficiency η at zero pumping capacity must be zero in both cases, so $\eta_{o0} = \eta_{w0}$.

At the end of the calculation process, there are five points available on each new curve and the corrected pump performance curves can be drawn with sufficient accuracy. However, a more preferred solution and the one suggested by the present authors fits the calculated data points with power series of the pump capacity. The functions thus determined enable one to find pumping head and efficiency values at any pump capacity.

As indicated before, the whole calculation model developed in this paper easily lends itself to computer programming and may be used as a subroutine in many kinds of program packages that involve selection or evaluation of centrifugal pumps. The use of such a subroutine will surely ease the practicing engineer's work when selecting ESP units, designing crude oil pipelines, etc.

An Example Problem

The performance curves of a centrifugal pump, as supplied by the manufacturer, are depicted in bold lines in **Fig. 2.** In the following, the correction model developed in this paper will be used to find the pump's performance curves when pumping a liquid with a viscosity of 55 cSt.

As the first step of solution, the original curves were fitted with power series of pumping capacity. The two functions are given below:

$$\eta_w = 2.7001 + 1.8114 \cdot Q_w - 9.8300 \cdot 10^{-3} \cdot Q_w^2$$

$$H_w = 893.2 + 3.6232 \cdot 10^{-1} \cdot Q_w - 8.5260 \cdot 10^{-3} \cdot Q_w^2 - 1.080 \cdot 10^{-4} \cdot Q_w^3$$

To find the b.e.p. of the given pump, the first differential of the efficiency function is set to zero. From this, the pump capacity at b.e.p. is: Q_{Wbep} = 9,212 GPM. Using this value, the original head at the b.e.p. is found from the above equation as H_{Wbep} = 770 ft.

In order to calculate the correction factors, first the correlation parameter, Q^*, is to be found from **Eq.** ***5:***

$y = 94.7$ $\qquad\qquad$ $Q^* = 2{,}698$ GPM

Now, head correction factors from **Eqs. 8 - 11** are:

$CH_1 = 0.9898$ $\qquad$ $CH_2 = 0.9870$ $\qquad$ $CH_3 = 0.9810$ $\qquad$ $CH_4 = 0.9758$

Capacity and efficiency correction factors are calculated using **Eqs. 7 and 10:**

$CQ = 0.9879$ $\qquad\qquad$ $C\eta = 0.9129$

The original pumping head and efficiency curves are read off at the four capacities of 0.6 Q_{Wbep}; 0.8 Q_{Wbep}; Q_{Wbep}; and 1.2 Q_{Wbep}. The results are given in **Table 1.** Since the correction factors are already known, the coordinates of the new performance curves are easily calculated (see **Table 1):**

new capacities	$Q_{oi} = Q_{wi}\, CQ$
new heads	$H_{oi} = H_{wi}\, CH_i$
new efficiencies	$\eta_{oi} = \eta_{wi}\, C\eta$

The corrected pump performance curves are plotted in normal lines in **Figure 2.** The figure displays the best-fit power series for the two curves, second order for efficiency and third order for pumping head.

Evaluation of the Proposed Model's Accuracy

For the evaluation of developed model's accuracy two cases were examined. Both involved actually measured performance curves of pumps in pipeline service pumping higher viscosity oils. **Case 1** is illustrated in **Fig. 3,** where measured and calculated performance curves are plotted for a liquid viscosity of *55* cSt. **Case 2** is shown in **Fig. 4,** the liquid pumped had a viscosity of 397 cSt.

Calculation accuracies for both cases are displayed in **Fig. 5** where absolute errors of head and efficiency calculations are plotted vs. relative capacities Q/Q_{Wbep}. The relative capacity range of 20% - 120% is displayed where the developed calculation model is supposed to be valid.

Errors in pumping head calculations are seen to be less the 2.5% up to a pumping capacity of 100%, above which value they tend to increase. It may be noted also that accuracy is slightly lower for **Case 2** with the higher viscosity liquid pumped. Calculation errors in pump efficiencies are greater but are still below 4% for **Case 1. Case 2** with the higher viscosity oil, as before, involves greater errors. **A** general observation for both parameters is that calculation accuracy slightly increase with an increase in pumping capacity.

Conclusions

The most important features of the centrifugal pump performance curve correction method developed in this paper are summarized below.

- The proposed method gives an easy-to-use calculation process and eliminates the errors associated with previous models involving visual read-off from correction charts,

- the model easily lends itself to computer programming,
- can be used in ESP selection, pipeline design, or any problem involving pumping by centrifugal pumps of liquids with a viscosity higher than that of water,
- the proposed model's application ranges (pump capacity, viscosity, etc.) are the same as those of the original Hydraulic Institute model,
- the accuracy of the suggested calculation model, as proved by two example cases, meets the requirements of general engineering work.

Symbols

D	pump impeller outside diameter, in
n	pump speed, 1/min
n_S	specific speed, -
H_W	pumping head for water, ft
H_O	corrected pumping head, ft
Q_W	water capacity, belonging to H_W, 100 GPM
Q_O	corrected capacity, belonging to H_O, 100 GPM
η_W	pump efficiency for water, %
η_O	corrected pump efficiency, %
H_{Wbep}	water head at b.e.p., ft
Q_{Wbep}	water capacity at b.e.p., 100 GPM
Q^*	corrected capacity to determine head, capacity and efficiency correction factors, 100 GPM
ν_O	kinematic viscosity of liquid pumped at pumping temperature, cSt
CH	head correction factor, -
CQ	capacity correction factor, -
$C\eta$	efficiency correction factor, -

References

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2. Paciga, **A.: Projektovanie zariadeni cerpacej techniky.** Slovenské vydaverelstvo technickej literatury, Bratislava, 1967.
3. **Determination of Pump Performance When Handling Viscous Liquid.** Hydraulic Institute Standards, 20th Edition. 1969.

Acknowledgment

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Table 1 - Calculated Points for an Example Case

Point	Q_w	H_w	η_w	$Q_o = Q_w x CQ$	CH	$H_o = H_w x CH$	$\eta_o = \eta_w x C\eta$
No.	GPM	ft	%	GPM	-	ft	%
1	5,527	870	72	5,160	0.9898	860	66
3	7.370	833	82	7,280	09870	819	75
3	9.212	774	86	9,100	0.9810	755	79
4	11,054	690	83	10,920	0.9758	666	76

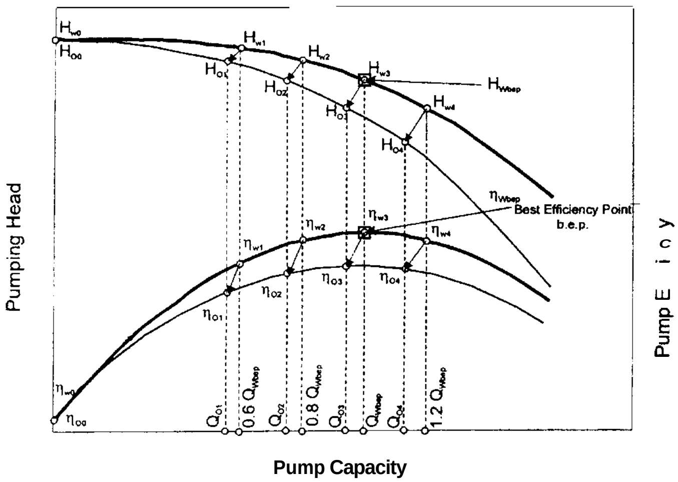

Figure 1 - Schematic Diagram Depicting the Essential Curves and Points Required for Performance Curve Correction

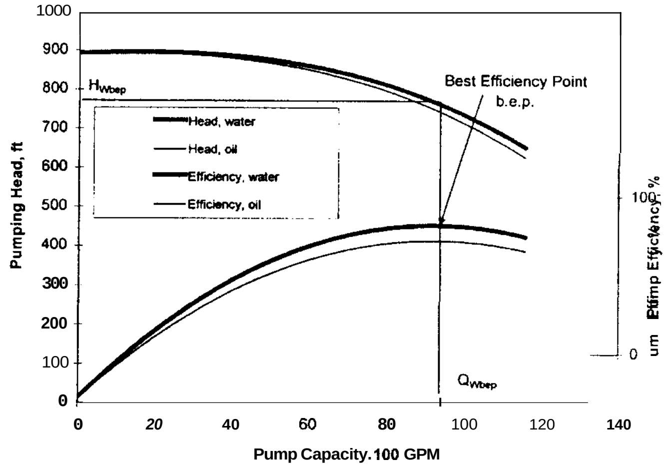

Figure 2 - Graphical Results of an Example Calculation

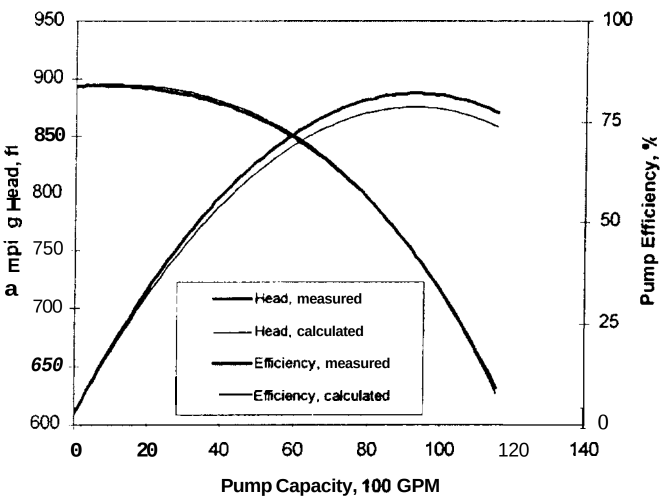

Figure 3 - Measured and Calculated Performance Curves for **Case 1,** Liquid Kinematic Viscosity *55* cSt

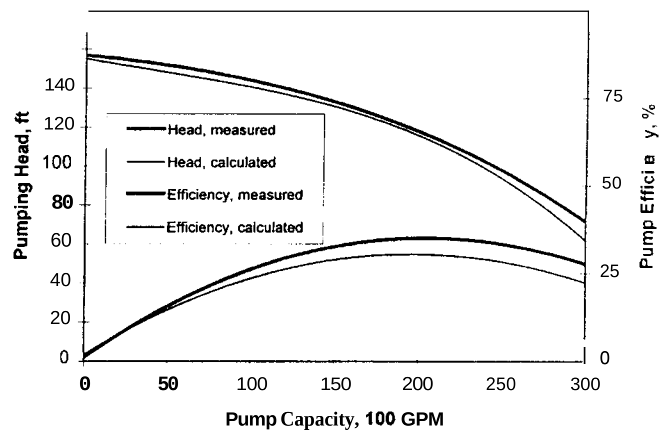

Figure 4 - Measured and Calculated Performance Curves for **Case 2,** Kinematic Viscosity 397 cSt

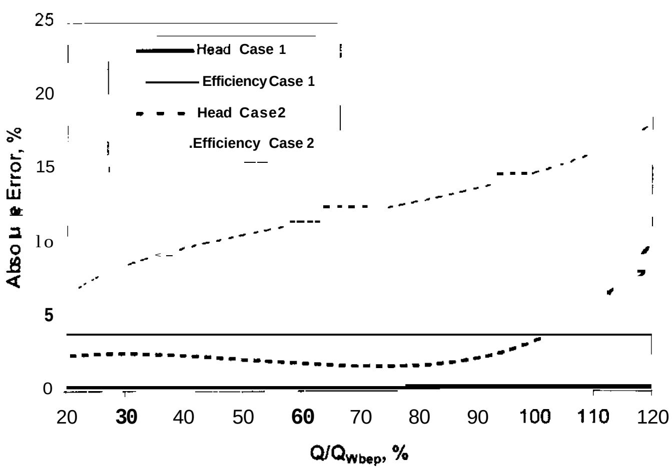

Figure 5 - Absolute Errors of Corrected Performance Curves