USING A MATLAB-BASED WELL TESTING SIMULATOR TO INVESTIGATE THE EFFECT OF ANISOTROPY ON BUILD-UP TEST DATA USING THE HORNER METHOD

Vegard Elverhaug and Anders Torland
Texas Tech University

ABSTRACT

This study presents how a MATLAB-based well testing simulator can be developed and used to investigate reservoir parameters and obtain results comparable to both theory and an industry standard simulator like Eclipse. The simulator solves the governing equation for slightly compressible flow in a reservoir using a fully implicit numerical scheme. The reservoir model used is 3D, anisotropic and homogeneous. In this study, the Horner theory is used to compare Horner-derived permeabilities with the true horizontal permeability to check the validity of the simulator. An investigation on possible effects of the vertical to horizontal permeability anisotropy-ratio and perforation height on build-up tests using the Horner method has also been conducted.

Buildup tests have been simulated successfully and the estimation of the permeability using Horner analysis showed that the results are consistent with Eclipse and the Horner theory. It was proved that for fully perforated reservoirs the Horner permeability was not affected by the anisotropy-ratio. However, for a partially perforated reservoir the effect of both anisotropy and height of perforation is evident. It is therefore important that the height of perforation relative to the reservoir thickness is considered when conducting a build-up test, and analyzing the results.

INTRODUCTION

Reservoir simulation is a widely-used tool in the petroleum industry. In this paper a well testing simulator, written in MATLAB, is presented. The simulator is 3D and solves the slightly compressible diffusivity equation implicitly. It discretizes the reservoir using finite difference and considers vertical to horizontal permeability anisotropy. The pressures for each grid block are calculated and stored every time step, and Horner analysis is automatically applied on the stored results. To check the validity of the well testing simulator, its results have been compared to theory and the industry standard simulator Eclipse.

Build-up testing is a simple method to obtain an approximate value for the reservoir permeability. By applying Horner analysis to the build-up data, the matrix permeability can readily be found from the slope of the Horner plot. The Horner theory assumes a homogeneous and isotropic reservoir, and it is not well documented how anisotropy will affect the results. The well testing simulator has therefore been used to investigate how the anisotropy ratio and height of perforation will affect the Horner-derived permeability.

HORNER THEORY

The Horner theory describes a method to analyze the data obtained by preforming a buildup test. By plotting shut-in pressure versus Horner time on a semi-log plot the permeability can be found from the slope of a straight line fit to the build-up results. The straight line is found in the transient region, where there is no effect of the boundaries or the early time region. The initial pressure can be estimated by extrapolation of the result to Horner time = 1. The Horner time is defined as:

$$\frac{t_s + \Delta t}{\Delta t} \qquad (1)$$

where t_s is the time of shut in and Δt is the time after shut in, i.e. $t - t_s$.

Fluid flow in purpose media can be described by the diffusivity equation. When assuming:

- Single phase flow.
- Homogeneous and isotropic matrix permeability.
- Slightly compressible fluid.
- Isothermal condition.
- Horizontal flow.
- Radial symmetry.

The flow equation can be written in oil field units as:

$$P(r_w,t) = P_i + \frac{162.6B\mu q}{kh}\log\left(\frac{t_s+\Delta t}{t_s}\right) \tag{2}$$

Where

- $P(r_w,t)$ is the pressure at any time at the well.
- P_i is the initial pressure.
- B is the formation volume factor.
- μ is the viscosity.
- q is the flow rate.
- k is the permeability.
- h is the thickness of the pay zone.
- t is time.
- r_w is the well bore radius.

For practical purposes the Horner time has been slightly modified in equation (2). When $P(r_w,t)$ is plotted versus $\log\left(\frac{\Delta t}{t_s+\Delta t}\right)$ the slope, m, of the straight line will be equal to:

$$m = \frac{162.6B\mu q}{kh} \tag{3}$$

The slope, m, is found graphically and by rearranging equation (3) the permeability can be calculated:

$$k = \frac{162.6B\mu q}{mh} \tag{4}$$

In this paper this permeability is referred to as the Horner permeability.

Based on Horners theory, and the assumption of horizontal flow, the Horner permeability will be equal to the horizontal matrix permeability. To fulfill the assumption of horizontal flow the reservoir needs to be fully perforated.

DERIVATION OF THE FLOW EQUATION

The following is a summary of the derivation of the flow equation. By writing a mass-balance equation over a control volume through which the fluid is flowing the continuity equation can be obtained. When considering the fluid to be slightly compressible and neglecting gravity the continuity equation can be written as:

$$\frac{\partial}{\partial x}\left[\beta_c \frac{A_x k_x}{\mu B}\frac{\partial p}{\partial x}\right]\Delta x + \frac{\partial}{\partial y}\left[\beta_c \frac{A_y k_y}{\mu B}\frac{\partial p}{\partial y}\right]\Delta y + \frac{\partial}{\partial z}\left[\beta_c \frac{A_z k_z}{\mu B}\frac{\partial p}{\partial z}\right]\Delta z + q_{sc} = \frac{V_b \varphi c_l}{a_c B_i}\frac{\partial p}{\partial t} \tag{5}$$

Where

- β_c is the transmissibility conversion factor.
- A_α is the cross sectional area in the α-direction.

- k_α is the permeability in the α-direction.
- B_i is the initial formation volume factor.
- $\Delta x, \Delta y, \Delta z$ is the length, width and thickness of each grid block.
- $V_b = \Delta x \Delta y \Delta z$ is the volume of a grid block.
- φ is the matrix porosity.
- c_l is the compressibility of phase l.
- a_c is the volume conversion factor.

By:

1. Defining the transmissibility as $T_{x_{i,j,k}} = \left[\beta_c \frac{A_x k_x}{\mu B} \frac{\partial p}{\partial x}\right]_{i,j,k}$, and similar for $T_{y_{i,j,k}}$ and $T_{z_{i,j,k}}$.
2. Applying spatial central difference derivative approximation for $T_{x_{i,j,k}}, T_{y_{i,j,k}}$ and $T_{z_{i,j,k}}$ around (i,j,k)
3. Applying central difference approximation of $\left(\frac{\partial p}{\partial x}\right)$, $\left(\frac{\partial p}{\partial y}\right)$ and $\left(\frac{\partial p}{\partial z}\right)$.
4. Applying backward difference approximation in time.

The following equation is then obtained:

$$\left[T^n_{i+\frac{1}{2},j,k} P^{n+1}_{i+1,j,k}\right] + \left[T^n_{i-\frac{1}{2},j,k} P^{n+1}_{i-1,j,k}\right] + \left[T^n_{i,j+\frac{1}{2},k} P^{n+1}_{i,j+1,k}\right] + \left[T^n_{i,j-\frac{1}{2},k} P^{n+1}_{i,j-1,k}\right] + \left[T^n_{i,j,k-\frac{1}{2}} P^{n+1}_{i,j,k-1}\right] + \left[T^n_{i,j,k+\frac{1}{2}} P^{n+1}_{i,j,k+1}\right] - \left(\left[T^n_{i+\frac{1}{2},j,k} + T^n_{i-\frac{1}{2},j,k} + T^n_{i,j+\frac{1}{2},k} + T^n_{i,j-\frac{1}{2},k} + T^n_{i,j,k-\frac{1}{2}} + T^n_{i,j,k+\frac{1}{2}}\right] + \left[\frac{V_b \varphi c}{\alpha_c B \Delta t}\right]_{i,j,k}\right) P^{n+1}_{i,j,k} = -\left[\frac{V_b \varphi c_l}{\alpha_c B \Delta t}\right]_{i,j,k} P^n_{i,j,k} - q_{sc_{i,j,k}} \tag{6}$$

Defining a global grid block index as:

$$M = i + (j-1)N_x + (k-1)N_x N_y \tag{7}$$

where N_x & N_y is the number of grid blocks in the x- and y-direction respectively.

Equation (6) can now be written as:

$$\left[T^n_{M+1,M} P^{n+1}_{M+1}\right] + \left[T^n_{M-1,M} P^{n+1}_{M-1}\right] + \left[T^n_{M+NX,M} P^{n+1}_{M+NX}\right] + \left[T^n_{M-NX,M} P^{n+1}_{M-NX}\right] + \left[T^n_{M-NX*NY,M} P^{n+1}_{M-NX*NY}\right] + \left[T^n_{M+NX*NY,M} P^{n+1}_{M+NX*NY}\right] - \left(\left[T^n_{M+1,M} + T^n_{M-1,M} + T^n_{M+NX,M} + T^n_{M-NX,M} + T^n_{M-NX*NY,M} + T^n_{M+NX*NY,M}\right] + \left[\frac{V_b \varphi c_l}{\alpha_c B \Delta t}\right]_M\right) P^{n+1}_M = -\left[\frac{V_b \varphi c}{\alpha_c B \Delta t}\right]_M P^n_M - q_{sc_M} \tag{8}$$

Where $T_{\alpha,\beta}$ is the average transmissibility between block α and block β.

This is the equation used in the well testing simulator.

INVERSE MATRIX MULTIPLICATION

A system of implicit equations in matrix form can be written as:

$$[A][x] = [b] \tag{9}$$

This can be solved by multiplying by the inverse of matrix A on both sides of equation (9):

$$[A]^{-1}[A][x] = [A]^{-1}[b] \tag{10}$$

Equation (10) is then reduced to:

$$[x] = [A]^{-1}[b] \quad (11)$$

ANISOTROPY-RATIO AND HEIGHT OF PERFORATION

The anisotropy-ratio is defined as:

$$AR = \sqrt{\frac{k_{horizontal}}{k_{vertical}}} \quad (12)$$

and quantifies the relative magnitude of the permeabilities in the horizontal and vertical direction.

The perforation ratio (PR) is defined as:

$$PR = \frac{h_{perforation}}{h_{pay}} \quad (13)$$

Where $h_{perforation}$ is the height of the perforated section of the well and h_{pay} is the entire height of the pay zone.

MATLAB IMPLEMENTATION AND HORNER ANALYSIS

The well testing simulator implements the slightly compressible diffusivity equation to calculate pressures in every grid block in a 3D reservoir model. This equation was discretized using finite difference, and then implemented in the simulator. Applying equation (8) to every grid block in the reservoir model yields a set of implicit linear equations, which is represented in matrix form. The transmissibilities and the volumetric term produces seven diagonals in the coefficient matrix. In the simulator, this set of equations is solved by inverse matrix multiplication. To increase the memory efficiency, the simulator utilizes sparse matrices and a built-in MATLAB function to perform matrix multiplication. For every time step the pressures in each grid block is stored and Horner analysis is automatically applied.

When the Horner analysis is applied, the simulator performs linear regression to estimate the slope of the straight line. The straight line occurred within the same timespan for all the simulations. Because of this, the same data points were used in the linear regression for every simulation to give a basis for comparison. To verify that these points indeed were in the transient region, the simulator uses the linear regression to estimate the initial pressure and compare it to the true value.

SIMULATIONS

To validate that the results obtained with the well testing simulator coincided with Horner theory and Eclipse, as well as considering the effect of anisotropy-ratio and perforation height, several simulations were conducted. All input parameters were kept constant except for anisotropy-ratio, perforation ratio and horizontal permeability. Only one of these parameters were changed between each run, as summarized in Tables 1 and 3. The same simulation scheme was used in the Eclipse simulations. To analyze the results, the horizontal permeability was then compared to the Horner permeability after each run.

BOUNDARY AND WELL MODELS

The reservoir is homogeneous and anisotropic, and all the reservoir model parameters are shown in Table 2. The simulator uses Neuman boundary condition with zero flux, i.e. no-flow boundaries. The well is automatically placed in the middle of the reservoir, independent of the size of the reservoir model. In the MATLAB simulator a well model has not been implemented, which means that the well is simulated as a grid block or as a string of grid blocks. Because of this there will be some errors in computation of the shut-in pressure, but those errors are found to acceptable.

RESULTS AND DISCUSSION

The theory states that the Horner permeability should be equal to the input horizontal permeability for a fully perforated reservoir. All simulations performed in both MATLAB and Eclipse proved that for a fully perforated reservoir, the anisotropy ratio had no effect on the Horner permeability. This can be seen in Figure 1. In all the simulations conducted the well testing simulator obtained good results, as shown in

Figure 2. There is great consistency between the results obtained in MATLAB and Eclipse, and both show comparable results to the Horner theory. The pressure data for the build-up test from both MATLAB and Eclipse are shown in Figure 3. The simulations in Eclipse generally yielded a slightly higher Horner permeability, as shown in Table 1 and Figure 2. On average the Horner permeability from the well testing simulator was 12% lower than the horizontal permeability, while Eclipse was on average 9% lower. The maximum relative difference between Eclipse and the well testing simulator was 4%. This is shown in Table 1. The reasons for the slight deviation between MATLAB and Eclipse was not determined, but it could be caused by gravity and the well model.

As stated above, both the well testing simulator and Eclipse obtained results lower than expected. To determine the reason for this deviation, a simulation with a higher number of grid blocks was conducted in Eclipse. This was not performed in MATLAB as the current simulator can not handle the memory requirements. By increasing the number of grid blocks, the Horner permeability seemed to converge to the input horizontal permeability. The relative error from Horner theory went from 14% to 3%. This can be seen in Figure 4, where the number of grid block where increased with a factor of approximately 33 ($N_x \times N_y \times N_z$=201x201x10), while maintaining the size of the reservoir model. In this simulation the early time effect in the Horner plot is significantly less dominating. The early time effect seen in the Horner plot with fewer grid blocks (35x35x10) in Figure 4 is larger, and could therefore be regarded as a grid block truncation error effect. This is because the grid is too coarse to accurately resolve the near wellbore radial flow. Based on this, it is reasonable to believe that the well testing simulator could achieve better results with a finer reservoir model.

PARTIALLY PERFORATED RESERVOIRS

To examine the effect of the perforation ratio on Horner permeability, simulations with partially perforated reservoirs were conducted. The results are presented in Table 3. In this case, the effect of anisotropy-ratio is evident. When the anisotropy-ratio is low, the Horner permeability will be approximately equal to the input horizontal permeability. As the anisotropy-ratio increases the Horner permeability will decrease to a constant value of approximately:

$$k_{ML} = PF * k_{sim-ML} \quad (14)$$

where

- k_{sim-ML} is the Horner permeability for a fully perforated reservoir with the same input horizontal permeability.
- k_{ML} is the Horner permeability from MATLAB.

The same relation is also valid for Eclipse data. k_{sim-ML} will converge toward the horizontal permeability with decreasing grid block size. Horner permeability as a function of anisotropy-ratio is shown in Figure 5, and the associated data is presented in Table 4. The values of Horner permeabilities obtained between the lowest and highest values of anisotropy-ratio seems to follow the same trend for different perforation ratios, but a correlation in this range has not been determined. In this range, the maximum relative error between MATLAB and Eclipse was significantly higher, while the start and end-point permeabilities still yielded low errors. The reason for this increased error was not determined.

Even though the perforation ratio is not an input in equation (14), it is shown here that it will have a great impact on the Horner permeability depending, on the degree of anisotropy in the reservoir. It is therefore important that the height of perforation is considered when conducting a build-up test, and analyzing the results.

CONCLUSION

The results of this study are:

- A three-dimensional MATLAB-based simulator was created by solving the slightly compressible diffusivity equation using an implicit numerical scheme. The simulator implements an anisotropic reservoir model, and obtained similar results to an industry standard simulator like Eclipse and the Horner theory.

- The well testing simulator showed that for a fully perforated reservoir the anisotropy-ratio does not affect the Horner permeability, which coincides with the Horner theory.
- For a partially perforated reservoir both the anisotropy-ratio and the perforation ratio will affect the Horner permeability. It is therefore important to be aware of this relation when conducting a build-up test.

REFERENCES

1. Ertekin, T., Abou-Kassem, J.H., and King, G.R. 2001. Solution of Linear Difference Equations. In *Basic Applied Reservoir Simulation*, Vol. 7 Chapt. 7, 133—134. Richardson, Texas: Textbook Series, SPE.
2. Horner, D.R. pe.tamu.edu. *Pressure Build-up in Wells*, http://www.pe.tamu.edu/blasingame/data/z_zcourse_archive/P620_12C/P620_12C_Final_Exam/P620_12C_Prob_01_ref_3rd_World_Pet_Cong_(Horner)_(pdf).pdf (accessed 10 November 2016).
3. Jelmert, T.Å. www1.uis.no. *Pressure Buildup*, http://www1.uis.no/Fag/Learningspace_kurs/PetBachelor/webpage/tech%5CReservoir%5CPressureTestAnalysis%5Cpressbui.pdf (accessed 13 November 2016).

ACKNOWLEDGEMENTS

The authors would like to gratefully acknowledge advice and technical support from Dr. Sheldon Gorell and Dr. Habib Menouar.

Table 1: The results for fully perforated reservoirs.

Run #	AR	k_{iH} $[md]$	k_{ML} $[md]$	k_{ECL} $[md]$	δ_{M-E}	δ_{M-HT}	δ_{E-HT}
1	1	20	16.52	17.15	4 %	17 %	14 %
2	5	20	16.52	17.15	4 %	17 %	14 %
3	10	20	16.52	17.15	4 %	17 %	14 %
4	25	20	16.52	17.16	4 %	17 %	14 %
5	100	20	16.52	17.15	4 %	17 %	14 %
6	1	40	35.28	36.12	2 %	12 %	10 %
7	5	40	35.28	36.14	2 %	12 %	10 %
8	10	40	35.28	36.14	2 %	12 %	10 %
9	25	40	35.28	36.16	2 %	12 %	10 %
10	100	40	35.28	36.14	2 %	12 %	10 %
11	1	60	54.05	55.21	2 %	10 %	8 %
12	5	60	54.05	55.24	2 %	10 %	8 %
13	10	60	54.05	55.25	2 %	10 %	8 %
14	25	60	54.05	55.31	2 %	10 %	8 %
15	100	60	54.05	55.24	2 %	10 %	8 %
16	1	100	91.77	94.65	3 %	8 %	5 %
17	5	100	91.77	94.78	3 %	8 %	5 %
18	10	100	91.77	94.78	3 %	8 %	5 %
19	25	100	91.77	95.02	3 %	8 %	5 %
20	100	100	91.77	94.75	3 %	8 %	5 %
				Average	**3 %**	**12 %**	**9 %**

Where:

AR	Anisotropy-ratio.
k_{iH}	Input horizontal permeability.
k_{ML}	Horner permeability from the well testing simulator data.
k_{ECL}	Horner permeability from Eclipse data.
δ_{M-E}	Realtive error between Matlab and Eclipse.
δ_{M-HT}	Realtive error between Matlab and Horner Theory.
δ_{E-HT}	Realtive error between Eclipse and Horner Theory.

Table 2: The reservoir model parameters.

Parameter	Symbol	Value
Total number of grid blocks, x-direction	N_x	35
Total number of grid blocks, y-direction	N_y	35
Total number of grid blocks, z-direction	N_z	10
Total number of grid blocks		12250
Length of reservoir, x-direction		5000 ft
Width of reservoir, y-direction		5000 ft
Depth of reservoir		100 ft
Length of grid block, x-direction	Δx	142.9 ft
Width of grid block, y-direction	Δy	142.9 ft
Thickness of grid block, z-direction	Δz	10 ft
Volume conversion factor	α_c	5.615
Formation volume factor	B	$1\ RB/STB$
Porosity	φ	0.18
Effective compressibility	c_e	$2 * 10^{-5}\ 1/psi$
Oil viscosity	μ	0.4 cp
Transmissibility conversion factor	β_c	1.127
Oil surface rate	q_{sc}	1000 STB/day
Total duration of simulation	t_{end}	3 days
Time after shut-in	Δt	1.2 hours
Shut-in time	t_{shutin}	12 hours
Input horizontal permeability	k_{iH}	*Varying*
Anisotropy-ratio	AR	*Varying*
Perforation ratio	PR	*Varying*
Initial pressure	P_i	6000 psi

Table 3: The results for partially perforated reservoirs. The bold numbers highlight the relation obtained in equation (14).

Run #	AR	PR	k_{sim-ML} $[md]$	k_{ML} $[md]$	k_{ML}/k_{sim-ML}	$k_{sim-ECL}$ $[md]$	k_{ECL} $[md]$	$k_{ECL}/k_{sim-ECL}$
1	1	0.7	16.5	16.5	1.00	17.1	17.1	1.00
2	5	0.7	16.5	13.6	0.82	17.1	16.8	0.98
3	100	0.7	16.5	11.6	**0.70**	17.1	12.0	**0.70**
4	1	0.7	54.0	54.0	1.00	55.2	55.2	1.00
5	5	0.7	54.0	52.1	0.96	55.2	55.2	1.00
6	100	0.7	54.0	37.8	**0.70**	55.2	39.0	**0.71**
7	1	0.7	72.8	72.8	1.00	74.6	74.6	1.00
8	5	0.7	72.8	71.4	0.98	74.6	74.6	1.00
9	100	0.7	72.8	51.0	**0.70**	74.6	52.9	**0.71**
10	1	0.5	16.5	16.5	1.00	17.1	17.1	1.00
11	5	0.5	16.5	11.8	0.72	17.1	15.4	0.90
12	100	0.5	16.5	8.3	**0.50**	17.1	8.6	**0.50**
13	1	0.5	54.0	54.0	1.00	55.2	55.2	1.00
14	5	0.5	54.0	50.7	0.94	55.2	54.7	0.99
15	100	0.5	54.0	27.0	**0.50**	55.2	27.9	**0.51**
16	1	0.5	72.8	72.8	1.00	74.6	74.6	1.00
17	5	0.5	72.8	70.4	0.97	74.6	74.4	1.00
18	100	0.5	72.8	36.4	**0.50**	74.6	37.9	**0.51**
19	1	0.3	16.5	16.5	1.00	17.1	17.1	1.00
20	5	0.3	16.5	10.5	0.64	17.1	13.4	0.78
21	100	0.3	16.5	5.0	**0.30**	17.1	5.2	**0.30**
22	1	0.3	54.0	54.0	1.00	55.2	55.2	1.00
23	5	0.3	54.0	49.5	0.92	55.2	53.8	0.97
24	100	0.3	54.0	16.2	**0.30**	55.2	16.9	**0.31**
25	1	0.3	72.8	72.8	1.00	74.7	74.7	1.00
26	5	0.3	72.8	69.5	0.95	74.7	73.9	0.99
27	100	0.3	72.8	21.7	**0.30**	74.7	23.0	**0.31**

Where:

PR — Perforation ratio.

k_{sim-ML} — The Horner permeability from MATLAB data for a fully perforated reservoir with the same input horizontal permeability.

k_{sim-E} — The Horner permeability from Eclipse data for a fully perforated reservoir with the same input horizontal permeability.

Table 4: The results for 70% and 30% partially perforated reservoirs.

Run #	AR	PR	k_{iH}	k_{ML}	k_{ECL}	δ_{M-E}
1	1	0.3	50	44.7	45.7	2 %
2	2	0.3	50	44.6	45.7	2 %
3	3	0.3	50	44.1	45.6	3 %
4	4	0.3	50	42.4	45.1	6 %
5	5	0.3	50	39.4	43.7	10 %
6	7	0.3	50	31.7	38.6	18 %
7	10	0.3	50	23.2	---	---
8	15	0.3	50	17.1	22.3	23 %
9	30	0.3	50	13.2	16.4	20 %
10	50	0.3	50	13.1	14.8	12 %
11	100	0.3	50	13.4	13.9	4 %
12	150	0.3	50	13.4	13.8	3 %
13	1	0.7	50	44.7	45.6	2 %
14	2	0.7	50	44.6	45.7	2 %
15	3	0.7	50	44.4	45.7	3 %
16	4	0.7	50	43.7	45.6	4 %
17	5	0.7	50	42.4	45.5	7 %
18	7	0.7	50	38.5	44.9	14 %
19	15	0.7	50	30.6	39.4	22 %
20	30	0.7	50	31.0	34.6	10 %
21	50	0.7	50	31.3	33.0	5 %
22	100	0.7	50	31.3	32.2	3 %
23	150	0.7	50	31.3	32.0	2 %

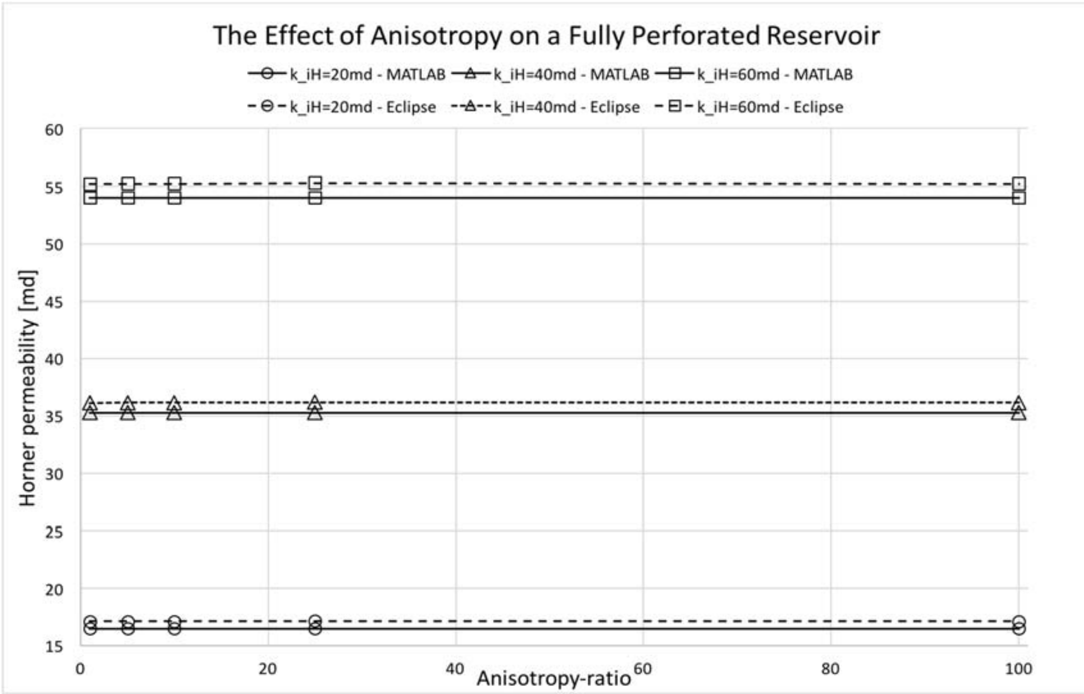

Figure 1 – The effect of anisotropy on a fully perforated reservoir for different horizontal permeabilities.

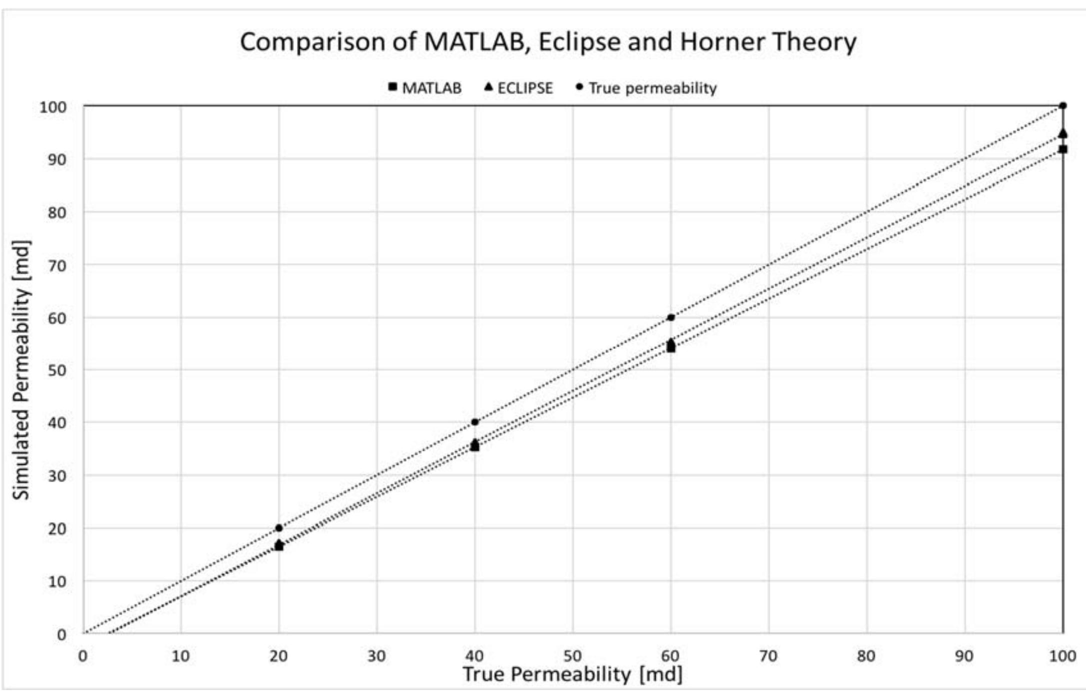

Figure 2 - Comparison of the Horner theory and data obtained in MATLAB and Eclipse.

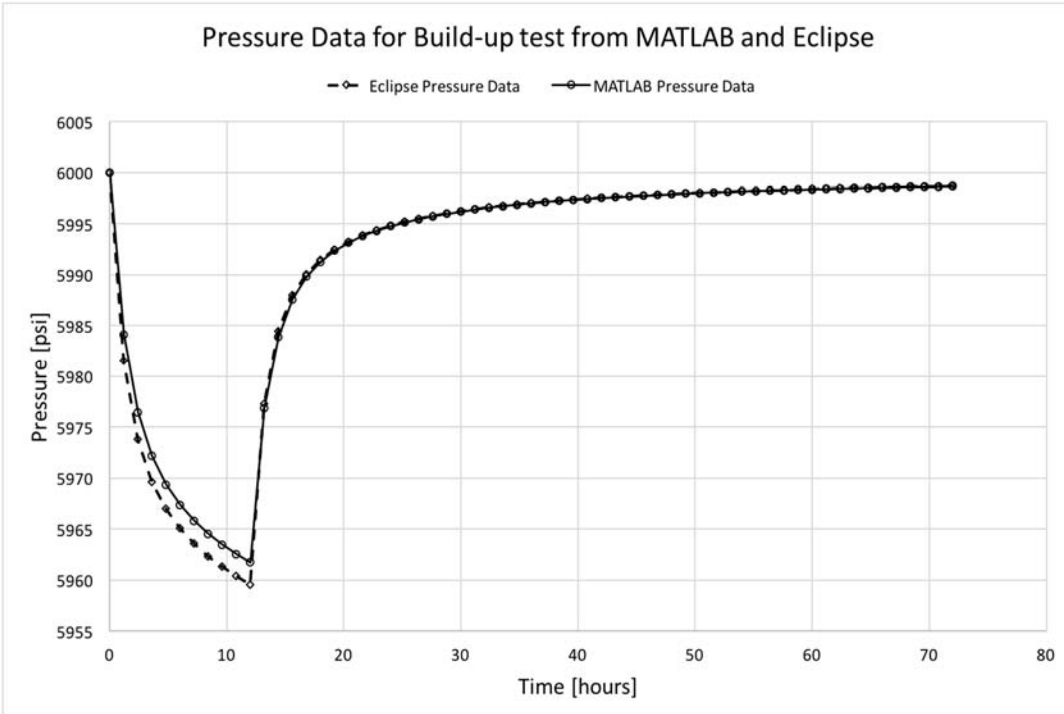

Figure 3 – Pressure data for a build-up test from MATLAB and Eclipse. The matrix permeability in the reservoir model is 40 md.

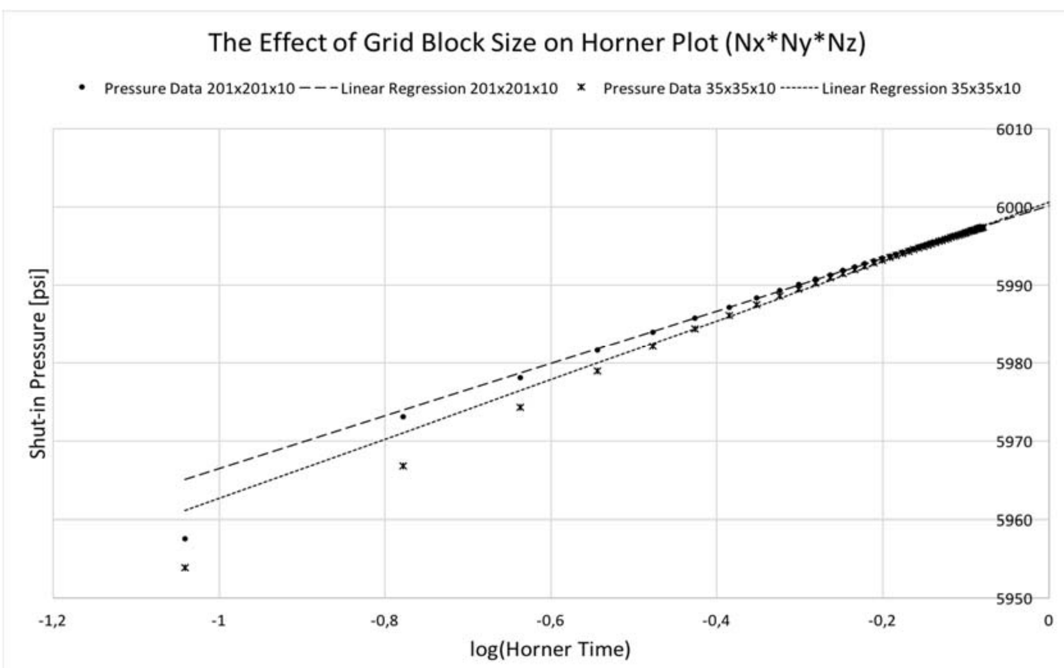

Figure 4 - The effect of grid block size on Horner Plot. Both reservoirs are the same size.

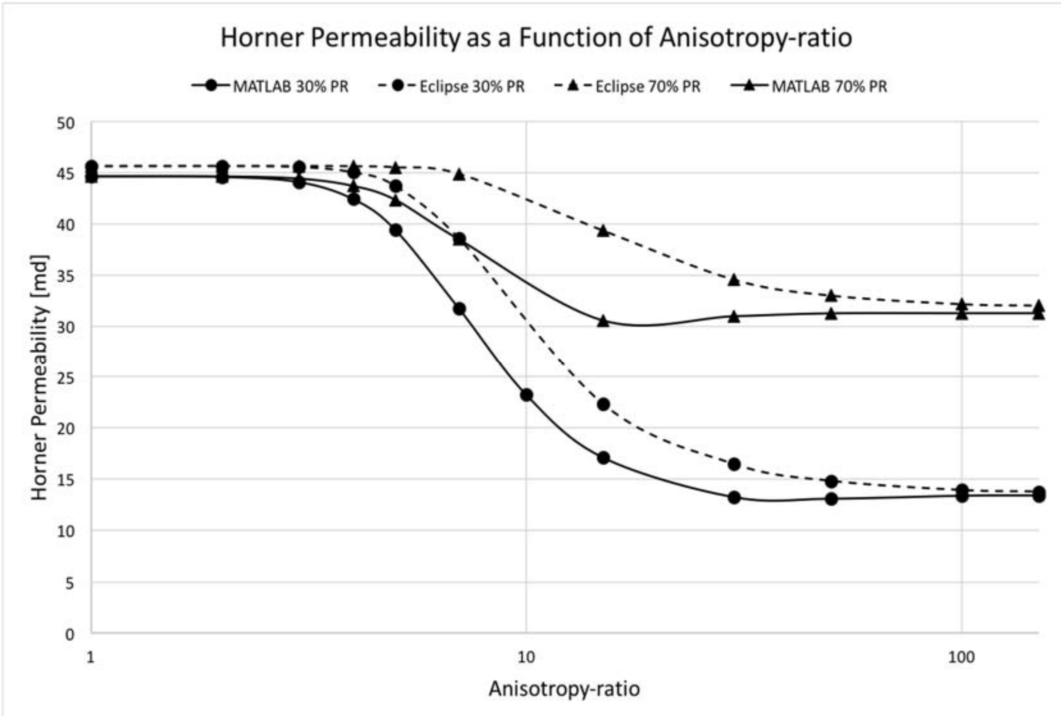

Figure 5 - This graph show the effect of anisotropy on partially perforated reservoirs.