A SIMULATION STUDY ON COMPARISON BETWEEN SEMI-ANALYTICAL AND NUMERICAL SOLUTIONS FOR HORIZONTAL WELL PRODUCTIVITY INDEX

Nur Wijaya
Texas Tech University

ABSTRACT

Because the drainage geometry and flow regime of horizontal well are different from those of vertical well, there is a need to develop a distinct analytical Productivity Index (PI) solution for horizontal well. Productivity Index is an important parameter used for several objectives, such as monitoring individual well performance over time and selecting well candidates to be stimulated. There have been several researches in the early 1990's developing the solutions; however, each solution generates different PI values. The degree of disparity depends on how conservative or optimistic the input values are, which possibly results in the reduction in confidence level when applying these semi-analytical solutions. This paper presents a review on some of the proposed semi-analytical solutions for horizontal well PI. In addition, this paper attempts to conduct a comparative study between the semi-analytical solutions and numerical simulation approach to investigate which semi-analytical solution is the most applicable for a wider range of reservoir parameters, on the assumption that the numerical simulation approach generates more accurate PI values. There is no history match applied on the model, hence the presented discussion on the extra level of treatments taken to characterize the simulation model. The paper concludes with a sensitivity analysis on the input parameters of the PI solution to suggest which parameters should be handled more carefully.

INTRODUCTION

As opposed to vertical wells in which the drainage volume moves radially around the well and is assumed to form a cylindrical boundary in a homogeneous isotropic reservoir, the drainage volume in horizontal wells is not easily defined, because the pressure wave from the horizontal perforation moves in a certain order, such as from early-time pseudo-radial flow, intermediate-time linear flow, until the late-time pseudo-radial flow [1]. This encourages researchers to develop a separate PI model for horizontal wells. Therefore, since 1990's, there have been some semi-analytical PI models developed for horizontal wells, which include Borisov, Giger, Joshi, and Renard. They are called semi-analytical instead of analytical since the inclusion of drainage geometry utilizes approximation to mimic the actual drainage geometry for an easier calculation. The following section briefly reviews the assumptions made in each of the PI models and the corresponding solution.

SEMI-ANALYTICAL SOLUTIONS FOR HORIZONTAL WELL PI

The analytical PI formula for a vertical well exhibiting a radial single-phase pseudo-steady state flow in a homogeneous isotropic reservoir is presented as follows:

$$\mathrm{PI} = \frac{0.00708\, k_o h}{\mu_o B_o \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \tag{1}$$

Due to the difference in drainage volume between vertical and horizontal wells, horizontal well PI can be measured from a more practical equation, which requires field production data but less information on reservoir parameters as follows:

$$\mathrm{PI} = \frac{q}{P_e - P_{\mathrm{wf}}} \tag{2}$$

However, if a more technical method to calculate horizontal well PI is needed, there have been semi-analytical models developed, some of which are presented as follows in a chronological order of publication.

1) Borisov (1984)

Borisov proposed the first solution for horizontal well PI. The researcher assumed steady-state flow and infinite conductivity along the horizontal wellbore (i.e. the friction loss along the horizontal well is not accounted). He proposed the following solution for isotropic formation:

$$\mathrm{PI} = \frac{0.00708\, k_h h}{\mu_o B_o \left[\ln\left(\frac{4\, r_{\mathrm{eh}}}{L}\right) + \left(\frac{h}{L}\right) \ln\left(\frac{h}{2\pi\, r_w}\right) \right]} \tag{3}$$

Where:

h = thickness, ft
k_h = horizontal permeability, md
k_v = vertical permeability, md
L = length of the horizontal well, ft
r_{eh} = drainage radius of the horizontal well, ft
r_w = wellbore radius, ft
PI = productivity index, STB/day/psi

2) Giger *et al.* (1984)

Giger then extended Borisov's solution by incorporating the effect of permeability anisotropy ratio [2]. The solution is presented below:

$$\mathrm{PI} = \frac{0.00708\, k_h}{\mu_o B_o \left[\left(\frac{1}{h}\right) \ln(X) + \left(\frac{\beta^2}{L}\right) \ln\left(\frac{h}{2\pi\, r_w}\right) \right]} \tag{4}$$

$$\beta = \sqrt{\frac{k_h}{k_v}} \tag{5}$$

$$X = \frac{1 + \sqrt{1 - \left(\frac{L}{2\, r_{\mathrm{eh}}}\right)^2}}{\frac{L}{2\, r_{\mathrm{eh}}}} \tag{6}$$

Where:

β = anisotropy ratio

3) Joshi (1990)

Later, Joshi modified the solution by assuming that a horizontal well drainage volume is constructed by two vertical wells at each heal and toe [3]. Figure 1 illustrates the drainage volume assumption made by Joshi.

$$\mathrm{PI} = \frac{0.00708\, k_h h}{\mu_o B_o \left[\ln(R) + \left(\frac{\beta^2 h}{L}\right) \ln\left(\frac{h}{2 r_w}\right) \right]} \tag{7}$$

$$R = \frac{a + \sqrt{a^2 - \left(\frac{L}{2}\right)^2}}{\frac{L}{2}} \tag{8}$$

$$a = \frac{L}{2} \sqrt{0.5 + \sqrt{0.25 + \left(\frac{2 r_{\mathrm{eh}}}{L}\right)^4}} \tag{9}$$

Based on his assumption on the drainage volume geometry, the equation below also applies to estimate the equivalent horizontal radius (r_{eh}).

$$A = \frac{L(2b) + \pi b^2}{43560} \tag{10}$$

Where:

A = drainage area, acre
b = drainage radius from either heel or toe (as illustrated in Figure 1)

4) Renard and Dupuy (1990)

The solution proposed by Renard and Dupuy is presented as follows [4]:

$$\mathrm{PI} = \frac{0.00708\, k_h h}{\mu_o B_o \left[\cosh^{-1}\left(\frac{2a}{L}\right) + \left(\frac{\beta h}{L}\right) \ln\left(\frac{h}{2\pi r_w'}\right) \right]} \tag{11}$$

$$r_w' = \frac{(1 + \beta) r_w}{2\beta}$$

(12)

Due to the different approaches by which each solution incorporates the drainage volume geometry, each of them results in different PI value, given the same set of input parameters. The degree of disparity depends on how conservative or optimistic the input values are, which possibly results in the reduction in confidence level when applying these analytical solutions. Therefore, to compare the accuracy and repeatability of the above PI solutions, this paper utilizes numerical simulation approach to investigate which of those semi-analytical solutions is the most applicable for a wider range of reservoir parameters, on the assumption that the numerical simulation approach generates more accurate PI values. This assumption is partly based on the fact that numerical simulation is able to produce an estimate solution out of an exact problem (i.e. less assumptions made). After deciding which solution is the most applicable, engineers are encouraged to utilize this particular solution more confidently among the other solutions. The following section discusses the steps taken to characterize the reservoir model.

MODEL CHARACTERIZATION

The base parameters to build the model are presented in Table 1. The grid dimension is presented in Table 2, followed by the simple visualization of the grid refinement around the wellbore compared to the global grid system. The number of grids in each direction is maintained at an odd number to ensure the eccentricity of the wellbore trajectory in the model. Furthermore, since Productivity Index oftentimes changes with time as a result of the depleting reservoir pressure, the grids around the wellbore are refined, such that the numerical calculation is more accurate during the early time of the production. Table 2 also presents the magnitude of the grid refinement. The effect of introducing this Local Grid Refinement (LGR) is quite significant, because the initial production rate changes from 1400 bopd before the grid refinement to 1200 bopd after the grid refinement. Figure 2 illustrates the Local Grid Refinement introduced around the well.

The initial reservoir pressure is created by a combination of top depth, fluid density, and rock compressibility. It is important to note that the semi-analytical PI solutions do not explicitly include some of the reservoir parameters required to build the simulation model, such as the porosity, rock and fluid compressibility, density, and most importantly Fluids-In-Place (FIP) Volume. Therefore, in order to assign the initial reservoir pressure, the model assumes those missing parameter values such that the combination results in 3000 psi initial reservoir pressure.

MODEL INITIALIZATION

Due to the lack of field data, history match on production is not applied on this model. However, the model is built on an alternative care, in which:

1. the model is bounded (proven by zero transmissibility in all of the six boundary faces),
2. the horizontal well trajectory lies exactly on the mid-height of the model and eccentrically relative to the grid,
3. connate water has been forced to show minimum relative permeability so the production is single-phase oil,
4. and the production measured at the surface includes a low GOR value, which is possibly caused by gas liberation as pressure along the tubing reaches below the oil bubble point.

An important contribution from this paper is the inclusion of friction calculation in calculating PI. None of the semi-analytical PI solutions incorporate friction along the tubing. Since what most engineers are interested in is the PI observed at the surface, ignoring the friction may overestimate the PI, because friction along such a long horizontal pipe could cause a significant pressure drop.

As an alternative to the missing field production history match, a match on the initial production rate between the semi-analytical and the simulation model is conducted. This attempt is possible because given a certain PI value and pressure drawdown, all of the semi-analytical PI models can be used to calculate the production rate. Since each semi-analytical PI model yields different initial production rate, an average value is obtained to match the value generated by the simulation model. As a result of these attempts, the initial production rate is matched confidently, with a percent difference of less than 5%.

SENSITIVITY ANALYSIS

After the model is successfully initialized, several runs are carried out to obtain PI values from the simulation model. In order to compare the PI values between the semi-analytical solutions and the numerical simulation model, the author applies the Mean Squared Error (MSE) approach with the following equation:

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2$$

where n represents the number of PI values compared, Y_i represents the PI value from the semi-analytical model, while $\hat{Y}_i$ represents the PI value from the simulation model with friction inclusion. This equation shows that the lower the MSE value, the more accurate the semi-analytical is to the simulation model. The following discussion presents the results of the comparison study categorized into sensitivity analysis of each PI input parameter. In each parameter discussed, several key observations will also be elaborated with the attached figures.

Permeability (Isotropic)

Since permeability is directly proportional to PI, PI will keep increasing indefinitely as permeability increases (Figure 3). However, this is physically quite immaterial because the subject in interest is the production rate observed at the surface. As the high permeability allows greater production rate, with the given wellbore radius and the horizontal length, the flow will likely turn turbulent which induces much higher friction drop and less production rate at the surface. As indicated by the simulation model (Eclipse) that includes the friction, PI will reach an optimum value at some point beyond which friction drop will start to take over the incremental PI. In this example, the optimum permeability seems to occur between 300 and 400 md. A different set of base input parameters will result in a different optimum value. Solely based on the comparison on isotropic permeability, it seems that Joshi's solution is the most accurate out of the four presented solutions. As the discussion proceeds, it should become apparent that Joshi's consistently results in PI value closest to the simulation model PI value.

Anisotropy Ratio

Since the anisotropy ratio is defined as the root ratio of the horizontal over the vertical permeability, PI will increase as anisotropy ratio decreases (Figure 4). This makes sense because in horizontal well, vertical permeability is crucial in making sure that the fluid can flow perpendicularly (up and down) into the horizontal wellbore. Generally, horizontal permeability is higher than the vertical due to the deposition mechanism in which overburden stress tends to create a horizontal bedding. Renard's shows a higher accuracy than Joshi's in this case; however, the author believes that it happens because the range of anisotropy ratio sensitized mostly lies on the lower side, in which Renard shows higher accuracy. However, when the range lies on the higher side, Joshi's still shows higher accuracy than Renard's.

Oil Viscosity

All of the semi-analytical solutions match excellently with the simulation model in term of oil viscosity (Figure 5). Higher viscosity fluid is more difficult to flow on its own, so PI is higher in low viscosity. It is important to note that oil viscosity is a fluid property, which is not related to the drainage volume geometry at all. This suggests that the semi-analytical solution which shows the most accurate PI in term of oil viscosity, which in this case is Joshi's, should become the most reliable solution on its own, because even before incorporating the assumption on the drainage volume geometry, the solution already generates the most accurate PI.

Oil Formation Volume Factor

Higher volume factor implies that the fluid is more volatile which causes more loss of liquid production observed at the surface (i.e. low PI at surface), as shown in Figure 6. Similar to the sensitivity on oil viscosity, since oil formation volume factor is a fluid property which is not interfered by the drainage volume geometry, the comparison on oil formation volume factor should emphasize that Joshi's serves as the most reliable solution.

Wellbore Radius

Larger wellbore radius theoretically allows larger production (assuming significant drawdown). The author would like to draw the attention to how the simulation model (Eclipse) with friction inclusion results in much lower PI value in smaller wellbore radius (Figure 7). This is caused by the fact that smaller radius more easily turns the flow to turbulent in which the friction drop will take up the drawdown, causing production loss. In fact, the inclusion of friction into PI calculation results in a 42% drop in PI at 0.1-ft wellbore radius. This shows the importance of incorporating the friction term when calculating the PI in horizontal wells.

Thickness

Thicker reservoir results in a higher PI because of the larger amount of fluids in place (Figure 8). Due to the lack of dimensionless scaling parameter to proportion the change in size and shape of the simulation model, the simulation model cannot well generate the sensitivity on thickness. Without the scaling parameter, this simulation work becomes partly useless because the change in thickness is not yet accompanied by the change in the drainage area. However, a minimum comparison can be made at the base value of the thickness (i.e. 300 ft), and Joshi's still shows the highest accuracy.

Drainage Area (Horizontal-Equivalent)

Higher drainage area yields a lower PI because the wellbore is farther apart from the boundary which requires a longer time for the pressure drawdown to propagate (Figure 9). A similar explanation applies to why the model sensitivity on drainage area is not entirely useful. To reiterate, a typical simulation sensitivity work requires a constant fluids in place volume, while a change in either thickness or drainage area will violate this workflow. Thus, this becomes a future work, namely how to account the change in reservoir model size for a sensitivity analysis.

Horizontal Length

As demonstrated in Figure 10, the match is quite excellent when the horizontal length is less than around 3000 ft. However, as the horizontal length increases, both Borisov's and Giger's generate erroneous PI values, which is caused by the inability of their models to automatically enlarge the drainage area as the horizontal length approaches the boundary of the drainage. A longer horizontal wellbore generates higher PI because there is a longer interval from which the wellbore generates the drawdown. The author would like to draw the attention on the point at horizontal length of around 3000 ft beyond which the simulation model without friction inclusion starts to deviate from that with friction inclusion. This indicates the optimum horizontal length from the PI standpoint. This could be a useful approach to design the optimum horizontal length.

Finally, the error in each parameter is normalized and summed up for each semi-analytical model. The result is presented in Figure 11 bar chart. Since Borisov's bar does not account the anisotropy ratio error, the bar should be higher, while the other three bars should be lower (i.e. due to a more even error distribution). As Joshi's consistently shows the most accurate PI, Joshi's should ultimately be utilized more often since it works in a wider range of parameters proven by the lowest error, compared to the other three solutions. Figure 12 concludes the discussion with a tornado sensitivity analysis which shows that oil viscosity plays as the most sensitive parameter because it determines the Reynold's number and affects the degree of drawdown loss due to friction. Anisotropy ratio and horizontal length comes next as the second and third most sensitive parameter, respectively.

CONCLUSION

Joshi's semi-analytical solution is shown as the most applicable PI solution because it consistently results in the least error even when various input parameters are sensitized from the low to the high side of the range. In addition, since friction could dominate in such a long horizontal pipe, friction loss should be accounted in PI calculation; otherwise, PI may be overestimated. Lastly, oil viscosity, anisotropy ratio, and the horizontal length become the three most sensitive parameters respectively. Therefore, any method to measure the viscosity and the horizontal and vertical permeability (such as PVT and core lab analysis) should be conducted more carefully to come up with a reliable PI.

[1] Oilfield Review July 1990 Volume 2 Issue 3

[2] Giger, F. M., Reiss, L. H., & Jourdan, A. P. (1984, January 1). The Reservoir Engineering Aspects of Horizontal Drilling. Society of Petroleum Engineers. doi:10.2118/13024-MS

[3] Joshi, S. D. (1986, January 1). Augmentation of Well Productivity Using Slant and Horizontal Wells. Society of Petroleum Engineers. doi:10.2118/15375-MS

[4] Renard, G., & Dupuy, J. M. (1991, July 1). Formation Damage Effects on Horizontal-Well Flow Efficiency (includes associated papers 23526 and 23833 and 23839). Society of Petroleum Engineers. doi:10.2118/19414-PA

Figure 1 – Illustration of Drainage Volume Geometry by Joshi (1990)

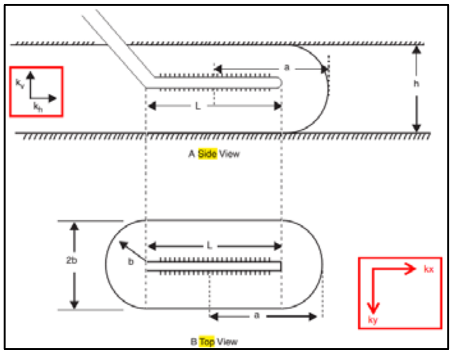

Figure 4 – Anisotropy Ratio Sensitivity

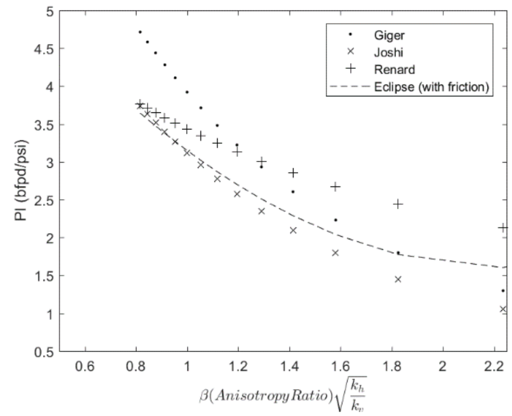

Figure 2 – Local Grid Refinement

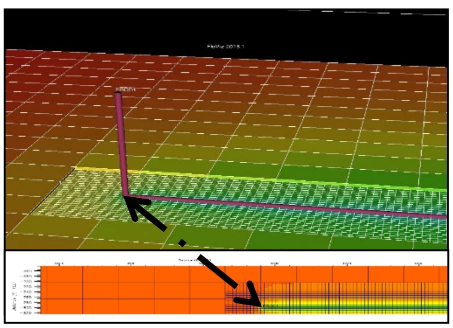

Figure 5 – Oil Viscosity Sensitivity

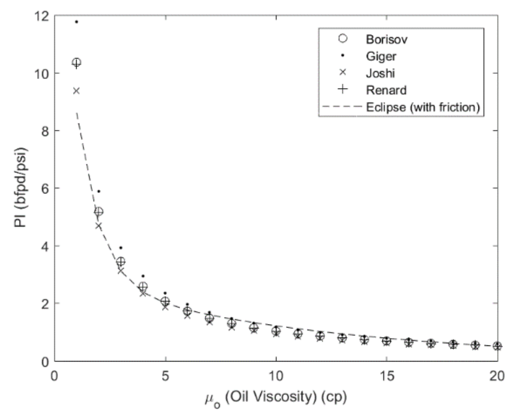

Figure 3 – Permeability Sensitivity

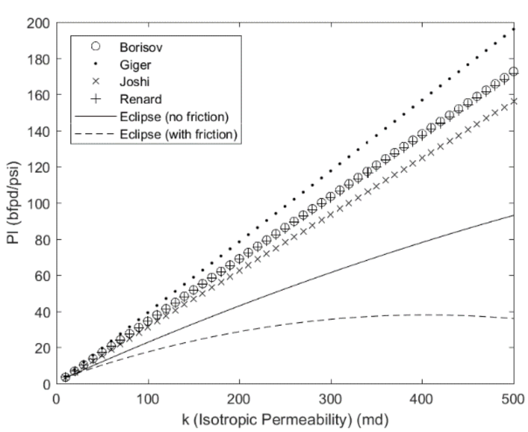

Figure 6 – Formation Volume Factor Sensitivity

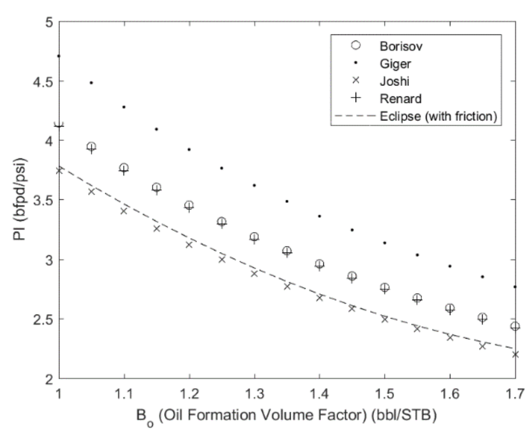

Figure 7 – Wellbore Radius Sensitivity

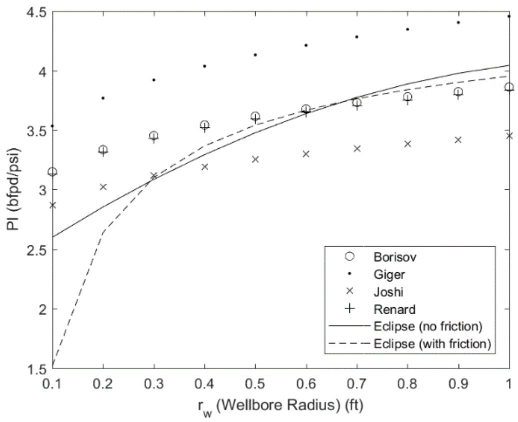

Figure 8 – Thickness Sensitivity

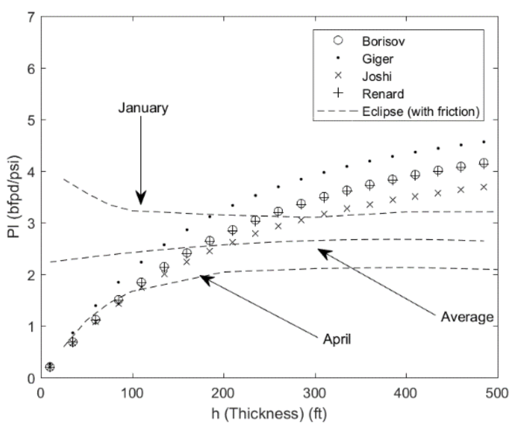

Figure 9 – Drainage Area Sensitivity

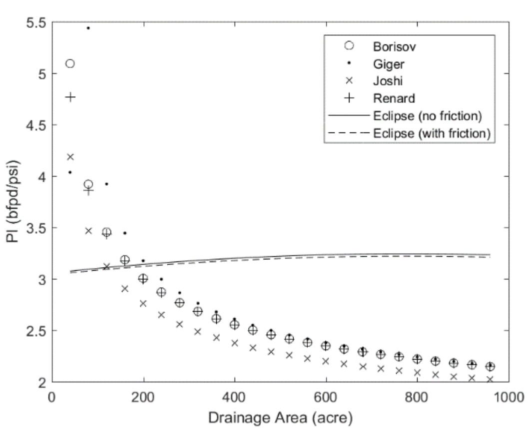

Figure 10 – Horizontal Length Sensitivity

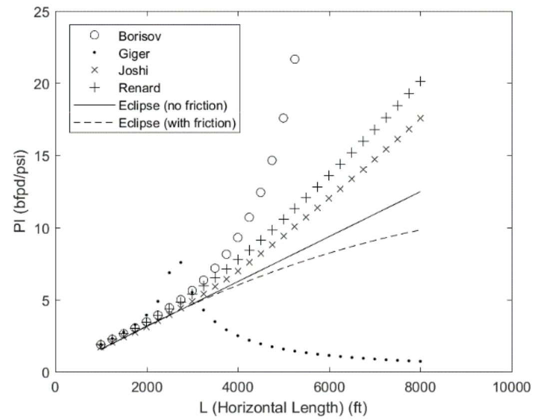

Figure 11 – Sum of Errors (MSE) among Models

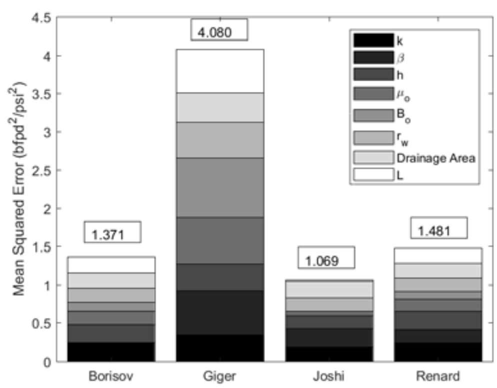

Figure 12 – Sensitivity Tornado Chart

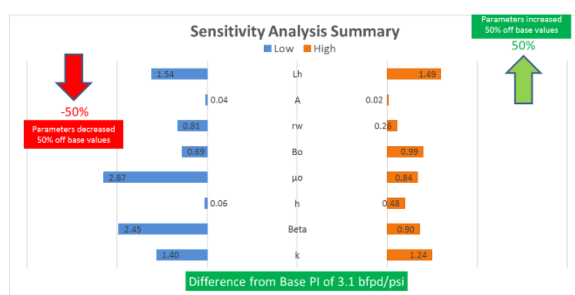

Table 1 – Base Input Parameters

Input Parameters	Value	Unit
Horizontal Length	2000	ft
A_drainage	120	acres
k_v	10	md
k_h	10	md
B_o	1.2	rb/STB
P_e	3000	psi
r_w	0.3	ft
h	300	ft
μ_o	3	cP
P_{wf}	2500	psi
B (Anisotropy Ratio)	1	

Table 2 – Grid Dimension

Direction	Total Length	Number of Grids	Grid Length (ft)	Number of Grids Refined	Refinement Scale	Grid Length after Refinement (ft)
X	10,000	101	~100	40	10x	10
Y	1,600	25	64	12	9x	7
Z	300	25	12	12	9x	1.3