Tubing Movement and Tubing Anchor Payout in Pumping Wells

By J. L. LOGAN
Baker Oil Tools, Inc.

INTRODUCTION

In the United States, almost 90% of all producing crude oil wells are being artificially lifted; the other 10% are flowing. Of those being artificially lifted, the vast majority are rod pumped; the total number is probably in the neighborhood of 1/2 million. It is not surprising, therefore, that tubing anchors, which can decrease lifting costs in most of these wells, would be popular. The primary purpose of this paper is to review the necessity for, and advantages gained by, anchoring the bottom of the tubing string in a rod-pumped well. Theoretical considerations will be used to analyze the effects of changes in well conditions, as the well is pumped, on the tubing string. Actual field cases will be included to acquaint the reader with some of the practical aspects of the paper.

In order to make a prudent selection of the proper type anchor to use, and to be able to properly evaluate its performance, it is helpful to first consider the tubing to be unanchored and determine the direction and distance the bottom of the tubing would move if it were free to do so. Two recent papers [1,2] consider the problems which result in rod-pumped wells, with emphasis on the cause and cure of the tubing wrapping itself helically around the rods on the upstroke. Both papers show how tubing anchors prevent this movement and make available the equations and charts by which the initial setting force and final pumping force in the tubing and on the anchor can be calculated. These 2 publications are recommended if an anchor is to be used. Neither of these papers, however, presents a means for calculating the distance the tubing moves while the well is being pumped.

A third paper[3] considers the effects of pressure and temperature changes on a string of tubing sealed at its lower end in a packer. It includes equations for calculating total movement of the bottom of the tubing, if it is free to move, due to changes in well conditions acting on the entire string. Unanchored tubing in a pumping well reacts in exactly the same manner to changes in well conditions. For this reason, the equations in Ref. 3 can be used, with minor modifications, in this paper. It is not felt necessary, therefore, to derive any of the equations used here.

WHY STOP TUBING MOVEMENT

There are a few pumping oil wells where little advantage will be gained by anchoring the tubing to the casing. Past experience has shown, however, that this is not generally the case; in most wells, it is financially practical to keep the tubing from moving. Tubing anchors were developed to reduce the total lifting cost of production. The problems which can arise when pumping a well through a string of unanchored tubing vary from well to well, and depend upon well conditions. Some of the more costly consequences of tubing movement should be discussed here as justification for the theoretical analysis which is to follow.

Increased Production

Whenever the barrel of the pump is free to move up as the plunger does, effective stroke length will be decreased and production will not be equal to the maximum output of the pump. The loss in pump stroke can be eliminated by preventing any movement of the pump barrel. This is true in all wells except those where over-travel results in a pump efficiency in excess of 100%. In these wells, however, this unusually high production will be more than offset by wear, which will be discussed in the next section.

In some wells, production is limited by state regulatory bodies. The advantage of tubing anchors in these wells is not the increased production which would be possible, but is, instead, the decreased lifting costs per bbl. to produce the prorated volume.

Wear

Wear is not only dependent upon one part rubbing against another, but also upon the environment in which it takes place. In some wells, there is no apparent wear of the rods, tubing or pump. The production from these wells is usually sweet, clean crude. If the produced fluid contains many corrosive components or solids such as sand, the wear becomes more apparent. Even with a tubing anchor, the rods must move up and down inside the tubing, so some wear will occur. However, if the tubing is unanchored, it will start to wrap itself around the rods as soon as they start to move up; the rods will then have to complete their upstroke through this buckled tubing. The resultant wear of the inside of the tubing and the rods has long been a serious problem. Also, if the tubing is free to move up, wear of the couplings is visible and wear of the inside of the casing is conceivable. The cyclical cocking of the barrel of the pump with respect to the plunger may cause, under corrosive and erosive well conditions, excessive damage to the pump.

Tubing Parting

Tubing parting is dependent upon wear and the corrosiveness of the produced fluid. An unanchored string of tubing in a pumping well may part close to bottom where the rods rub against its inside surface and where the couplings rub against the casing. This wear, as was explained, is aggravated by buckling of the tubing around the rods. The tubing, however, does not buckle to the surface; there is some point in the string above which the tubing remains straight. Yet the tubing often parts in this straight portion. Failure in this area can be attributed to stress fatigue of the tubing as the cyclical load on the bottom of the tubing is transferred from the rods to the tubing and back again. This cyclical stress can be eliminated by a tubing anchor.

TUBING MOVEMENT IN A ROD-PUMPED WELL

The following study is based on an unanchored tubing string under these assumed pumping conditions: (1) the hole is straight and vertical; (2) tubing movement is not affected by rod-to-tubing friction; (3) initial and produced fluids are of the same density, and; (4) the pump is at the bottom of the tubing string.

Consider, first, a string of tubing and sucker rods freely suspended in a well to be pumped. The rods and pump have been spaced out at the surface, the surface pumping unit has been installed and pumping begins. On each upstroke of the sucker rods, the fluid in the tubing gets closer to the surface, and the fluid in the annulus drops toward the pumping fluid level. This continues until the well reaches pumping equilibrium, after which there will be no further changes in fluid levels or temperatures. How the tubing reacts to those changes in well conditions involves 2 separate, non-related analyses.

First, it may be important to know how far the bottom of the tubing would move down from its original freely hanging pre-pumped position to its lowermost point on the downstroke after pumping equilibrium is reached. This distance, although it does not enter into any of the economic payout calculations of a tubing anchor, does provide the information necessary to properly set a tension tubing anchor. It will be considered in the next section.

Second, it has been stated previously that the bottom of the tubing moves up on the upstroke of the sucker rod and down on the downstroke. This occurs both as the well is being pumped up and after it reaches pumping equilibrium. After equilibrium is reached, the bottom of the tubing always goes to the same point on the upstroke and back to the same point on the downstroke. It is quite helpful to know the amount of this cyclical pumping movement, which is commonly known as "breathing".

Buckling of the tubing has been previously mentioned in several statements in this paper; it will be discussed in more detail in another section. It should, however, be related to breathing at this point. Part of the force which moves the bottom of the tubing up on the upstroke is caused by the lower portion of the tubing string, which was originally hanging in a straight line, being buckled by internal pressure. This buckling generally moves the bottom of the tubing up by some small amount. However, because part of the movement called breathing is due to buckling, it should be considered in order to make a complete analysis of breathing. The most important aspect of buckling is the previously discussed rod, tubing, pump and casing wear as well as the increased rod load.

PUMPING THE WELL TO EQUILIBRIUM

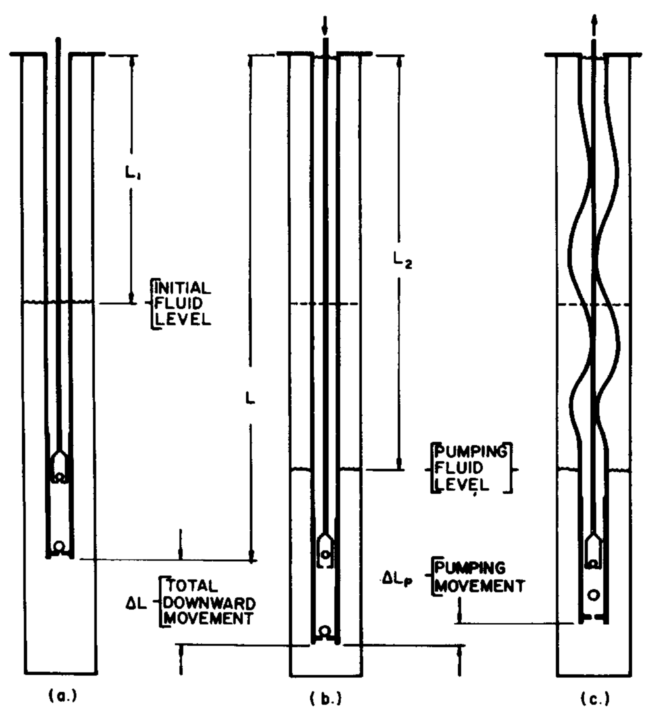
In a rod-pumped well, the static fluid level, at the time the tubing and rods are run, may be quite close to the surface or near the producing zone. Before pumping begins, the tubing is exposed to those initial pressures and temperatures which exist in the well.

As the well is pumped up, these conditions change: (1) the fluid level in the tubing rises to the surface; increasing the pressure inside the tubing; (2) the fluid level in the annulus drops to the pumping level, decreasing the external tubing pressure, and; (3) the temperature of the tubing string increases as warm fluid moves up the tubing.

As shown in Figs. 1-a and 1-b, changes in fluid levels and temperatures move the bottom of the tubing down by an amount denoted ΔL. This ΔL is the sum of the effects of pressure changes acting vertically on the bottom of the tubing, acting horizontally on the inside and outside walls of the tubing, and the temperature change of the tubing string. These 3 changes will be considered in the following sections. All nomenclature defined at end of article.

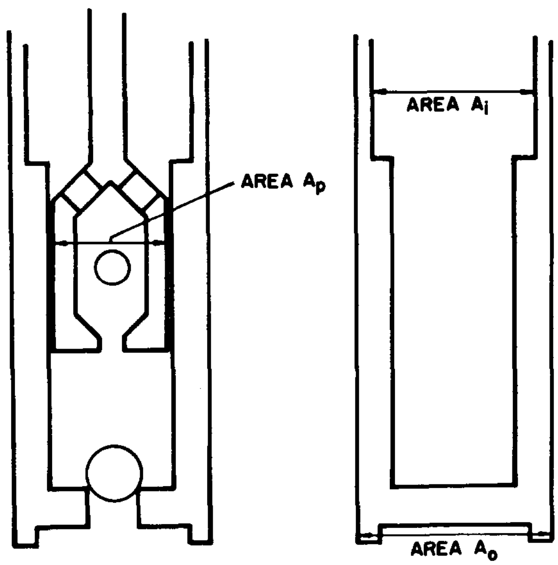
Pressure Changes Acting Vertically on the Bottom of the Tubing

The initial pressure above the closed standing valve in Fig. 1-a is equal to the density of the fluid inside the tubing times the height of the fluid column. Initial pressure below the standing valve will be considered to be the same as that above (same fluid density and level). The pumping pressure above the closed standing valve in Fig. 1-b is equal to the density of the produced fluid times the height of the fluid column (disregarding any surface gas pressure which may accumulate). Pressure below the standing valve decreases until the pumping fluid level reaches some equilibrium point. The pumping pressure below the closed standing valve in Fig. 1-b is equal to the density of the produced fluid times the height of the fluid column up to the pumping fluid level (again disregarding gas pressure). The areas on which these pressures are acting are shown in Fig. 2. Fig. 2-a shows the standing valve closed and the traveling valve open. For explanatory purposes, however, it can be imagined that the bottom of the tubing is closed, as shown in Fig. 2-b.



(a.) (b.) (c.)

FIG. 1



(a) (b.)

FIG. 2

That part of the total downward movement caused by pressure changes acting vertically on the bottom of the tubing and the standing valve is denoted ΔL_1:

$$\Delta L_1 = \frac{pL}{EA_s}(A_i L_1 + A_o L_2 - A_o L_1) \qquad (1)$$

A positive answer indicates this movement is downward.

Pressure Changes Acting
Horizontally on the Walls of the Tubing String

In Fig. 1-a, the hydrostatic pressure below the initial fluid level exerts itself laterally against the inside and outside walls of the tubing. In Fig. 1-b, these hydrostatic pressures are changed as the produced fluid is pumped to the surface inside the tubing and the fluid level drops in the annulus. The movement as a result of these changes will be denoted ΔL_2 and is:

$$\Delta L_2 = \frac{v\,p}{E(R^2 - 1)}\Big[L_1^2 - 2L_1 L + R^2(L_2^2 - L_1^2 - 2L_2 L + 2L_1 L)\Big] \qquad (2)$$

The solution of the above equation will yield a minus value, indicating a shortening of the tubing.

Temperature Change of the Tubing String

In Fig. 1-a, the tubing is at static formation temperature. In Fig. 1-b it is at the same temperature as the produced fluid. The initial average tubing temperature is the average of the mean yearly temperature for the area [1] and the static bottom-hole temperature. The final average tubing temperature is the average of the surface producing temperature and producing bottom-hole temperature. If the change in average tubing temperature is denoted Δt and is given a positive sign to indicate a temperature increase, the lengthening due to the temperature change of the tubing string will be denoted ΔL_3:

$$\Delta L_3 = L\,\beta\Delta t \qquad (3)$$

Total Downward Movement

The distance the bottom of the tubing moves down as the well goes from initial to pumping conditions is denoted ΔL:

$$\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3$$

In Eq. (4), ΔL_1 and ΔL_3 represent increases in tubing length, and ΔL_2 represents a decrease in tubing length. The overall movement ΔL, however, always represents an increase in tubing length. As previously stated, this distance is important only when a tubing anchor is used; it will be discussed again later in the paper.

TUBING MOVEMENT AFTER EQUILIBRIUM IS REACHED

Once pumping equilibrium is reached, there are no further changes in pressures and temperatures acting to move the bottom of the tubing string. Yet, it has been stated that the bottom of a string of tubing which is not anchored to the casing will move up on the upstroke of the sucker rods and down on the downstroke. As the bottom of the tubing rises on the upstroke, it buckles helically around the rods, even though it may be under tension from pressure. The tubing will straighten as the rods start back down.

The above described cyclical tubing movement is caused, not by pressure changes, but by a constant pressure acting on a different area on the upstroke than it does on the downstroke. On the downstroke, the standing valve closes and, again for analysis, the bottom of the tubing can be considered to be bull-plugged. The column of fluid inside the tubing, therefore, acts downward on the entire inside area of the tubing. The fluid in the annulus acts upward on the entire outside area. On the upstroke, that part of the fluid column in the tubing above the pump plunger is being lifted by the rods, so it cannot exert a downward pressure force against the entire tubing inside area. Annulus pressure, on the other hand, now acts upward on that area between the outside of the tubing and the pump plunger.

This shifting of a constant pressure load is the cause of cyclical pumping movement; it is this movement that will next be considered.

Movement on the Upstroke

In Fig. 1-b, the well has reached pumping equilibrium and on the downstroke, the bottom of the tubing is as far down as it will go. The hydrostatic columns of fluid in both the tubing and annulus exert forces on the bottom of the tubing and standing valve as shown in Fig. 2. Fig. 3 shows that, on the upstroke, those same pressures are acting on different areas. The upstroke, in other words, removes part of the tension which is in the tubing during the downstroke. This reduction in tension lets the bottom of the tubing move up by an amount denoted ΔL_H. It is the movement which is commonly referred to as breathing. This distance is an application of Hooke's Law and can be calculated by the equation:

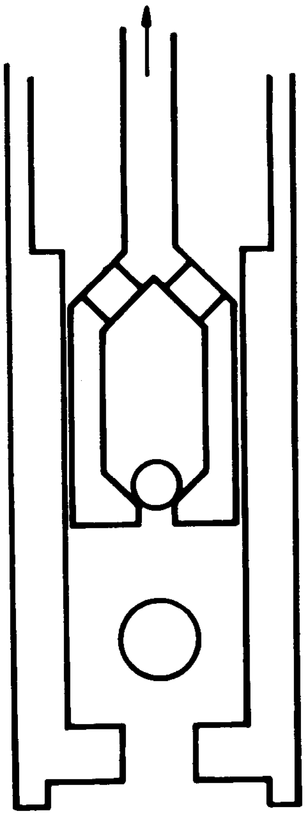


FIG. 3

$$\Delta L_H = -\rho \frac{A_p\ L\ L_2}{EA_s} \qquad (5)$$

The minus sign indicates this movement, as a result of decreased tension, is upward. This part of the pumping movement is the change in tubing tension between the upstroke and downstroke. It does not include the upward movement due to the buckling. It is proven in Ref. 1 that the lower portion of the tubing string is straight on the downstroke but buckles as soon as the rods and plunger start up and the standing valve comes off its seat. The general explanation of buckling will be given here, but Ref. 1 is suggested for a more detailed study.

In Fig. 1-b, hydrostatic pressure inside the tubing increases with depth. This pressure acts laterally on the inside walls of the tubing string to not only increase the inside diameter of the tubing (the previously discussed ΔL_2), but also to buckle the tubing. On the downstroke, as the standing valve closes and the traveling valve opens (removing the load on the rods), this buckling tendency is exactly counteracted by the force of the hydrostatic pressure acting downward on the entire inside area of the tubing string. On the upstroke, part of this force is transferred from the tubing to the rods. The horizontal pressure forces will then buckle the tubing around the rods, even though, as deduced from Fig. 3, the bottom of the tubing is in tension from the hydrostatic pressure acting downward on the area between the tubing ID and the pump plunger. Also, hydrostatic pressure in the annulus tends to overcome the buckling effect of the tubing pressure. The overall effect, however, is a buckling which moves the bottom of the tubing string up by an amount denoted ΔL_B:

$$\Delta L_B = - \frac{r^2\ A_p^{\ 2}\ \rho^2\ L_2^{\ 2}}{8EI\ (w_s + w_i - w_o)} \qquad (6)$$

Eq. (6) assumes that the top of the buckled section is below the pumping fluid level. Ref. 1 can be used to determine this distance. If the pumping fluid level is near the pump, w_o can be assumed to be zero; this assumption will introduce very little error into Eq. (6).

The total cyclical movement between the upstroke and downstroke is the sum of the change in fluid load and the buckling of the tubing. This pumping movement is denoted ΔL_p:

$$\Delta L_p = \Delta L_H + \Delta L_B \qquad (7)$$

This ΔL_p then, is the amount of lost pump stroke as the barrel of the pump moves part way up with the plunger. Preventing this tubing movement will, of course, increase the pump efficiency. The purpose of the next section is to determine the resultant production increase.

THE ECONOMIC PAYOUT OF A TUBING ANCHOR

To determine the number of pumping days required to pay for a tubing anchor, it is first necessary to calculate the amount of lost oil production per day as a result of the cyclical movement ΔL_p. This volume is found by calculating the area of the pump plunger A_p, multiplying it by the movement ΔL_p, converting the volume to some useful production value (such as barrels), and then multiplying this quantity by the pump strokes per day. The answer represents the lost pump displacement (and also lost production) per day due to cyclical tubing movement and can be shown by the equation:

$$\text{Lost Oil Production} = A_p \times \Delta L_p \times \text{Pump Strokes/Day} \qquad (8)$$

The total lost production from the above calculations is corrected for the oil content by multiplying the lost pump displacement by the amount of oil in the produced fluid (1 - per cent water cut/100). The daily volume of produced oil which would be gained by stopping tubing movement is then multiplied by the value of the oil per bbl. to obtain the monetary daily gain when using a tubing anchor. This value is then divided into the cost of the anchor to give the required answer of pay out time in days and can be calculated by the equation:

$$\text{TUBING ANCHOR PAYOUT} = \frac{\text{Cost of Anchor}}{\text{Units of Lost Oil Production} \times \text{Value/Unit}} \qquad (9)$$

Suppose, in a given example problem, that a 6,000 ft. well is being pumped at 12 strokes per minute through a 2 in. pump. The calculated cyclical movement ΔL_p would be 8.04 in., resulting in a lost pump displacement of 44.96 bbl. of fluid per day. If the produced fluid were 60% water cut and the oil had a value of \$2.50 per bbl., a standard anchor-catcher for 7 in. OD casing would, strictly from an increased production standpoint, pay for itself in 9.22 producing days. Additional saving through reduced wear would probably decrease this payout time.

In the example problem, the bottom of the tubing would move 16.08 inches per pump stroke, or a total of 23,155 ft. per day. This, in corrosive fluid with a high sand content, would soon cause excessive tubing coupling wear.

TYPES OF TUBING ANCHORS

The primary purpose of any tubing anchor, regardless of its design, is to prevent cyclical movement of of the pump and buckling of the lower portion of the tubing. Either or both of these can be accomplished by several different types of down-hole tubing anchoring devices.

Tension Anchor

Tension anchors are set by rotation and once set, the bottom of the tubing cannot move up but it can move down, although this downward movement is not desirable as the tubing anchor may become unset or it may be damaged. It is usually advantageous to set the anchor with just enough initial tension in the tubing to prevent anchor movement and tubing buckling. In Fig. 1-a, a string of tubing is hanging freely in a well which is to be pumped. In Fig. 1-b, the bottom of the tubing has moved down an amount ΔL and even though the tubing is full of produced fluid which tends to buckle the tubing, it remains straight. If, therefore, the bottom of the tubing had been positioned as shown in Fig. 1-a and firmly anchored to the casing, and then tension causes a ΔL elongation of the tubing and anchor, the bottom of the tubing would not move down as the well is pumped to equilibrium, nor would the lower portion of the tubing string buckle. The previously calculated distance ΔL, therefore, is critical when using a tubing anchor. If the tubing is unanchored, this value is not important.

Compensating Tubing Anchor

The compensating tubing anchor is a variation of the standard tension anchor in that it will not move up the hole, but will move down and set at the new position in the casing. In other words, as it moves down it will mechanically set itself at new downward positions until the tubing has reached maximum elongation. If a compensating tubing anchor is set as shown in Fig. 1-a, it will move down the hole and set at each downstroke until the tubing has reached pumping equilibrium and the tubing anchor has moved down a distance ΔL. At this point, as with a tension anchor, the tubing will remain straight on the upstroke. One of the major advantages of the compensating tubing anchor over the standard tension anchor is that no calculations are required in order to properly pre-strain the tubing prior to pumping; the compensating tubing anchor maintains just enough tension in the tubing at all times to keep the tubing from buckling. Another advantage is the ease of flanging up the tubing, as no initial pre-strain need be applied.

Hydraulic Anchor

The hydraulic anchor is actuated by the same hydrostatic pressure that moves the bottom of the tubing down. Depending upon the design of the hydraulic anchor, this pressure exerts a force on either a friction member pushing against the casing or a positive locking member between the anchor and the casing. With either design, as the well is pumped to equilibrium, the downward movement of the bottom of the tubing is limited by anchor-to-casing friction. This means that when pumping equilibrium is reached, the bottom of the tubing and the hydraulic anchor, on the downstroke of the rods, will be somewhere between the positions shown in Fig. 1-a and Fig. 1-b. The assembly will not move down the entire distance ΔL. As a result, the bottom of the tubing will not move up on the upstroke of the sucker rods (therefore, no loss in pump stroke) but the lower portion of the tubing will buckle around the rods.

As with the compensating tubing anchor, no initial pre-strain, and hence no calculations, are necessary to to operate the hydraulic tubing anchor. Its use also facilitates flanging up the tubing at the surface.

Compression Anchor

Compression anchors are set by rotation and once set, the bottom of the tubing can move up but it cannot move down. If a compression anchor is set as shown in Fig. 1-a, as the well is pumped to equilibrium, the bottom of the tubing will tend to move down a distance ΔL as shown in Fig. 1-b. It cannot move down at all, however, so the lower portion of the tubing string could be severely buckled on both the upstroke and the downstroke. The resultant rod, tubing and pump wear is usually serious. Although the compression anchor aggravates buckling, the compression in the bottom of the tubing does prevent the movement ΔL_p while the well is being pumped. Also, in the event the tubing parts, the compression anchor will keep the tubing from falling to bottom. The disadvantages of excessive wear in most wells, however, usually overcome the advantage of catching the tubing.

Anchor-Catcher

The anchor-catcher has the advantages of both the tension anchor and the compression anchor. It permits the tubing to be initially landed in just enough tension to keep the tubing straight after the well is pumped to equilibrium, yet it will keep the tubing from falling to bottom in the event of parting. The anchor-catcher also prevents the upward movement ΔL_p on the upstroke while the well is being pumped. There is, therefore, no lost pump displacement.

FIELD RESULTS OF TUBING ANCHORS

The most popular down-hole tubing anchor in use today is one with which the tubing can initially be put in sufficient tension (or one that will automatically maintain sufficient tension) to keep it straight after the well is pumped up. The added advantage of catching the tubing if it parts, eliminating costly fishing operations, has, in recent years, increased the attractiveness of the anchor-catcher. Because of the obvious advantages of anchoring tubing in enough tension to prevent buckling and pump movement, the following field examples cover only the compensating tubing anchor and the anchor-catcher.

In one West Texas well pumping from a depth of 4600 ft, pump efficiency was increased 20% by using an anchor-catcher. In several other wells in the same area, tubing parting had been a serious problem, resulting in a very expensive fishing operation. Anchor-catchers have reduced these tubing parting problems and have caught the tubing in those wells where they did part.

A well in Mississippi, pumping about 750 bbl. of fluid per day from a depth of 4500 ft, had such extreme tubing coupling wear that it was necessary to work over the well approximately every 90 days. An anchor-catcher was installed and production increased about 10% while reducing coupling wear. The well has pumped over 9 months with no need for any workover.

In another area in Mississippi, compensating tubing anchors and anchor-catchers have increased production approximately 15% while reducing the frequency of pulling the tubing by about 25%. These wells are producing from 5000 ft.

In one part of the Rocky Mountains, the tubing had parted due to badly worn tubing collars. An anchor-catcher, which was then run to 6800 ft, eliminated the worn collar problem. In another well in the same general area, tubing had parted on 2 occasions with resultant costly fishing jobs; an anchor-catcher eliminated the parting of the tubing.

A well in Oklahoma was pumping from a depth of 5000 ft. An anchor-catcher was installed and production increased 30%.

In one area in Kansas, production was increased in 21 wells through the use of compensating tubing anchors and anchor-catchers. In several of these wells, production was increased to the previously unattainable proration rate.

RECOMMENDATIONS

In those wells where increased production is the only desired result, any of the tubing anchors previously discussed will be satisfactory. In those wells where wear is a problem, an anchor which permits the tubing to be held in sufficient tension to prevent buckling should be used. This includes the tension anchor, compensating tubing anchor and the anchor-catcher. If corrosion is also a problem, resulting in frequent rod and/or tubing parting, the anchor-catcher should be used.

The compensating tubing anchor requires no pre-strain; if, however, a tension tubing anchor or an anchor-catcher is used, the tubing should initially be landed in just enough tension to keep the tubing straight while the well is being pumped. The tables in Ref. 2 provides this information with a limited amount of calculations.

Any type anchor will pay for itself by stopping cyclical pumping movement and increasing pump efficiency and production. The financial savings, through decreased rod, tubing and pump wear when using a tension type anchor, strengthen the theory of tubing anchor payout.

NOMENCLATURE

All equations in this paper pertain to a system of consistent units. For example: lengths in inches, densities in lb/cu in. (which is the same as psi/in.), and areas in sq. inches.

A_i = Area corresponding to tubing ID
A_o = Area corresponding to tubing OD
A_p = Area corresponding to pump plunger OD
A_s = Cross-sectional area of the tubing wall
E = Young's modulus (for steel, $E = 30 \times 10^6$ psi)
I = Moment of inertia of tubing cross-section with respect to its diameter:

$$I = \frac{\pi}{64}(D^4 - d^4) \text{ where D is OD and d is ID}$$

L = Length of freely-hanging tubing string
L_1 = Initial fluid level
L_2 = Pumping fluid level
r = Radial rod-to-tubing clearance or tubing-to-casing clearance, whichever is smaller
R = Radio OD/ID of the tubing
w_s = Average (i.e., including couplings) weight of tubing per unit length
w_i = Weight of produced fluid in the tubing per unit length
w_o = Weight of displaced produced fluid outside the tubing per unit length
β = Coefficient of thermal expansion of the tubing material (for steel, $= 6.9 \times 10^{-6} /1°$ F)
ΔL = Total downward movement as the well is pumped to equilibrium
ΔL_1 = Tubing movement caused by pressure changes acting vertically on the bottom of the tubing
ΔL_2 = Tubing movement caused by pressure changes acting horizontally on the walls of the tubing string
ΔL_3 = Tubing movement due to changes in tubing temperature
ΔL_B = Tubing movement due to buckling of the tubing on the upstroke
ΔL_H = Tubing movement due to reduction of tension in the tubing on the upstroke
ΔL_p = Total cyclical tubing movement between pump upstroke and downstroke
Δt = Change in average tubing temperature
ρ = Density of produced fluid
υ = Poisson's ratio of the material (for steel, $\upsilon = 0.3$)

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