Torque Factors for Pumping Units—How They Are Calculated from Rig Geometry—How They Are Put to Practical Use

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INTRODUCTION

Manufacturers are constantly being asked to furnish special combinations of stroke length, beam sizes and other structural modifications to meet the particular requirements of those responsible for the application of pumping units. These special demands are often made without due consideration of their effect on the loading of the unit gear reducer and prime mover.

Torque factors which were made available through the workings of API committees have not been used as much as they should be; if they were, some of the special arrangements would not be requested. The proper use of torque factors would make it evident that such changes could easily overload the gear reducers.

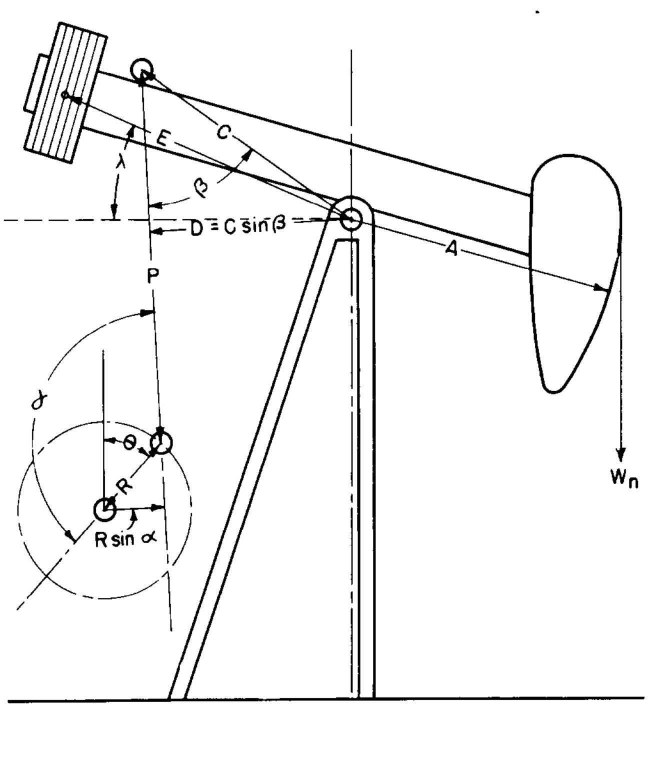
Although the use of torque factors was started approximately 15 years ago, their general adoption has been slow. Producers have not demanded and manufacturers have not furnished them because of the trouble in calculating them.

DEFINITION OF TORQUE FACTORS

A torque factor is that factor which converts load at the polished rod to torque at the crank. From the standpoint of how it is used, we might define it as follows: The torque factor at a given crank position multiplied by the net well load at that position gives the instantaneous torque due to the net well load.

The net well load is defined as the well load at any crank position minus the structural unbalance of the pumping unit and minus the beam counterbalance effect at the polished rod at that crank position.

The standard viewing position in determining the direction of rotation is to face the unit with the well to the viewer's right. Also, the reference for crank position gives the zero position with the crank at 12 o'clock. Torque Factors are then given for each 15 degrees of crank rotation in the clockwise direction.



HOW ARE TORQUE FACTORS DETERMINED

In order to show how torque factors are determined, we must adopt a system of nomenclature and symbols as shown on Fig. 1 and described below:

A = distance from saddle bearing to horsehead of beam.

C = distance from saddle bearing to tailbaord bearing.

P = length of pitman.

R = radius of crank.

β = angle between center line of saddle bearing and tailbaord bearing and center line of pitman.

α = angle between center line of pitman and center line of crank.

ϕ = angle of rotation of crank with zero degrees at 12 o'clock.

W = well load at a specific crank angle.

W_n = net well load ($W-B-W_{be}$).

T_{wn} = torque due to net well load.

T_r = torque due to rotary counterweights.

$\overline{TF}$ = torque factor.

$\overline{TF}$ = torque factor at a given value of O.

B = structural unbalance or beam off balance.

W_b = weight of beam counterbalance.

W_{be} = beam weight counterbalance effect at polished rod.

Torque Due To Net Well Load

By definition:

$$T_{wn} = \overline{TF}\,(W-B-W_{be}) = \overline{(TF)}\,W_n$$

When there is no beam balance, W_{be} is of course omitted. How is the torque factor $\overline{TF}$ determined for any given unit?

1. Determine the magnitude of the force along the pitman which will balance the net force at the polished rod, which is the net well load, W_n. Let F_p be the force along the pitman.

   Equating equal moments around the saddle bearing:

   $$W_n \times A = F_p \times D$$

Wherein: D is the distance along a line from the saddle bearing drawn perpendicular to the center line of the pitman.

So:

$$D = C \sin\beta$$

$$F_p = W_n \times \frac{A}{C \sin\beta}$$

To convert this force to torque, we must find its effective moment arm around the crank center which is $R\sin\alpha$.

$$T_{wn} = F_p R\sin\alpha = \left(W_n \frac{A}{C\sin\beta}\right) R\sin\alpha = Wn\,\overline{(TF)}$$

Then

$$\overline{TF} = \frac{A}{C\sin\beta} \times R\sin\alpha = \frac{AR}{C} \times \frac{\sin\alpha}{\sin\beta}$$

This means that torque factor is a function of $\frac{\sin\alpha}{\sin\beta}$, since the other factors are constants.

Torque Due To Rotary Counterweights

The torque due to rotary counterweights is simply $M\sin\Theta$ where M is the maximum moment of crank and counterweights about the crankshaft.

Net Torque On The Speed Reducer

In determining the net torque on the speed reducer, it is necessary to consider that the net well load torque and the torque due to the rotary counterweights are acting in opposite directions. If the unit is running in a clockwise direction, the well load torque will be acting in a counter-clockwise direction on the upstroke; and, at the same time, the counterbalance torque will tend to rotate the crank in a clockwise direction and hence must be considered negative. On the downstroke the well load torque is acting in a clockwise direction and the counterbalance torque is acting in a counter-clockwise direction.

The net torque on the speed reducer is the algebraic sum of the two torques.

On the upstroke the torque factor has a positive sign and the sine of Θ is positive, so the formula for net torque on the speed reducer is then as follows:

$$T = \overline{TF}\,(W-B-W_{be}) - M\sin\Theta$$

On the downstroke the torque factor is negative in the torque factor tables and the sine of Θ is also negative since it falls between 180 and 360 degrees. Refer to Fig. 3B where the well load torque, counterbalance torque, and resultant torque are shown graphically.

MATHEMATICAL DETERMINATION OF TORQUE FACTORS AND POLISHED-ROD POSITION

Referring to Fig. 2, the following is a procedure which may be followed in calculating torque factors and polished rod position. The four parts to Fig. 2 show the crank in each quadrant of rotation.

From the law of cosines:

(1) $J^2 = C^2 + P^2 - 2CP\cos\beta$

β = angle between C & P
P = angle between K & R = $\Theta - \Phi$

$J^2 = K^2 + R^2 - 2KR\cos P$

for crank positions $\Theta = \Phi$ to $\Theta = \Phi + 180°$

$J^2 = K^2 + R^2 - 2KR\cos(360° - P)$

for crank positions $\Theta = \Phi + 180°$ to $\Theta = \Phi + 360°$

but $\cos(360° - P) = \cos P$

So for all positions of the crank, the expression for J^2 is the same and substituting

$P = \Theta - \Phi$ is written

(2) $J^2 = K^2 + R^2 - 2KR\cos(\Theta - \Phi)$

Equating (1) (2)

$C^2 + P^2 - 2CP\cos\beta = K^2 + R^2 - 2KR\cos(\Theta - \Phi)$

(3) $\cos\beta = C^2 + P^2 - R^2 + 2KR\cos(\Theta - \Phi)$

Since Φ is a constant angle

$\Phi = \tan^{-1}\left(\frac{I}{H-G}\right)$ (See Fig. 2D)

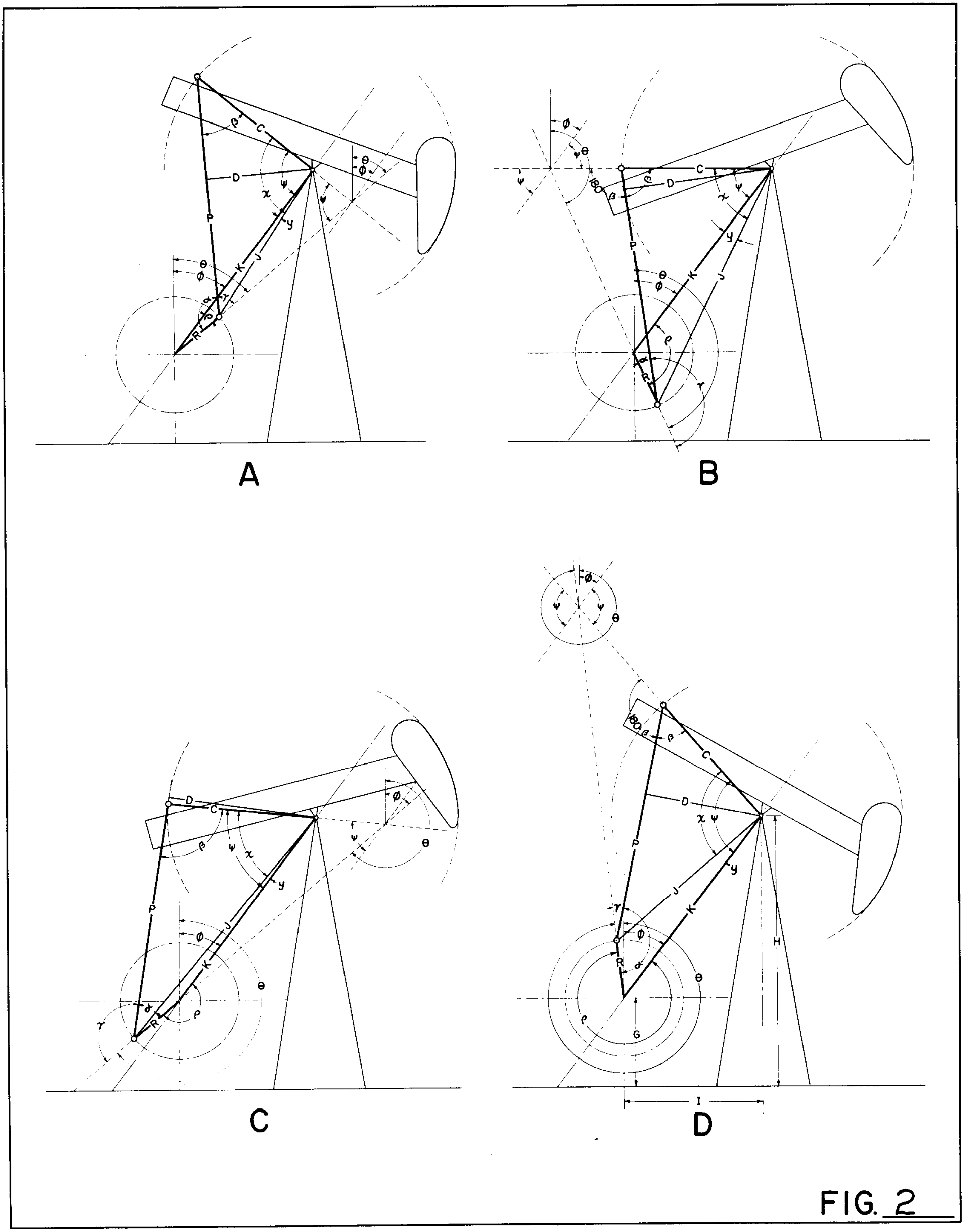
$\cos\beta$ may be readily calculated for each 15 degrees of crank rotation. That is, $\theta = 15°$, $\theta = 30°$, etc., also β and $\sin\beta$ may be calculated. Substituting values of $\cos\beta$ in (1), values of J^2 and J may be calculated. Let α and γ be the two angles that the pitman P makes with the axis of the crank R. α and γ are supplementary angles,

So: $\sin\alpha = \sin\gamma$.

Refer to Fig. 2 showing one position of the crank in each quarter of the rotation.

1st Quarter

$$\begin{cases} \gamma + \beta + [\psi - (\theta - \phi)] = 180° \\ \gamma = 180° - \beta - [\psi - (\theta - \phi)] \\ \alpha = 180° - \gamma \\ \alpha = \beta + \psi - (\theta - \phi) \end{cases}$$


A
B
C
D
FIG. 2

2nd Quarter

$$\alpha + (180° - \beta) + (\theta - \phi) - \psi = 180°$$
$$\alpha = 180° - (180° - \beta) - (\theta - \phi) + \psi$$
$$\alpha = \beta + \psi - (\theta - \phi)$$

3rd Quarter

$$\alpha + \beta + [\psi - (\theta - 180° - \phi)] = 180°$$
$$\alpha = 180° - \beta - [\psi - (\theta - 180° - \phi)]$$
$$\alpha = 180° - \beta - \psi + (\theta - 180° - \phi)$$
$$\alpha = -\beta - \psi + (\theta - \phi)$$

4th Quarter

$$\gamma + (180° - \beta) + (\theta - 180° - \phi - \psi) = 180°$$
$$\gamma = 180° - (180° - \beta) - (\theta - \phi - 180° - \psi)$$
$$\gamma = \beta + 180° + \psi - (\theta - \phi)$$
$$\alpha = 180° - \gamma$$
$$\alpha = -\beta - \psi + (\theta - \phi)$$

In order to solve for α, we must be able to compute ψ for any given value of θ. Let X and Y be the angles between (C and J) and (J and K) respectively.

For Crank Positions	For Crank Positions
$\theta =$ to $\theta = +180$	$\theta = +180$ to $+360$
$= X - Y$	$= X + Y$

From law of sines

$$\frac{\text{Sinx}}{P} = \frac{\text{Sin}\beta}{J} \quad \text{Then} \quad \text{Sinx} = \frac{P\sin\beta}{J}$$

For all crank positions.

For crank positions $\theta = \phi$ to $\theta = \phi + 180°$

$$\frac{\text{SinY}}{R} = \frac{\text{SinP}}{J} = \frac{\text{Sin}(\theta - \phi)}{J}, \quad \text{Then} \quad \text{SinY} = \frac{R\sin(\theta - \phi)}{J}$$

For crank positions $\theta = \phi + 180°$ to $\theta = \phi + 360°$

$$\frac{\text{Siny}}{R} = \frac{\text{Sin}(360° - P)}{J} = \frac{-\text{sinP}}{J} = \frac{-\sin(\theta - \phi)}{J}$$

$$\text{siny} = \frac{-\sin(\theta - \phi)}{J}$$

J may be computed from Equation (1) and the rest of the factors are known for any value of θ.

The angle ψ has special significance in that it is a measure of the stroke and polished rod position.

The polished rod position expressed as a fraction of stroke above lowermost position is:

$$\frac{\psi_b - \psi}{\psi_b - \psi_t}$$

Wherein: subscripts b and t denote bottom and top of stroke, respectively. At the bottom of the stroke and at the top P & R are in a straight line. By the law of cosines

$$\text{Cos}\,\psi_b = \frac{C^2 + K^2 - (P+R)^2}{2\,CK}$$

$$\text{Cos}\,\psi_t = \frac{C^2 + K^2 - (P-R)^2}{2\,CK}$$

DETERMINING THE INSTANTANEOUS TORQUE CURVE FROM THE DYNAMOMETER CARD BY THE USE OF TORQUE FACTORS

A dynamometer card taken on a 4,000 foot well is shown in Fig. 3A, together with pertinent well data.

The first step is to divide the dynamometer card so that the load may be determined for each 15 degrees of crank angle θ. Lines are projected down from the ends of the card as shown to determine its length, which is proportional to the actual length of the stroke.

The length of the base line or zero line is then divided into 10 equal parts and these parts are subdivided. This may easily be done with a suitable scale along a suitable diagonal line as shown.

To illustrate, a sample calculation will be made considering the point where the crank angle θ equals 75 degrees.

Fig. 4 is a specific example of pumping unit stroke and torque factor data presented in a standard form adopted by the API. This is for a specific unit of one manufacturer which was in use on the well in this example.

Fig. 5 is the API rating form data for the crank counterbalance for the same unit.

Before we refer to Fig. 4, we must note that the rotation of the crank is clockwise. The API standard viewing position is to face the unit with the well head to the right. The chart is read from top to bottom for clockwise rotation, and from bottom to top for counter-clockwise rotation.

From Fig. 4 we find that the position of the rods at 75 degrees for the 64 inch stroke is 0.397, and that the torque factor $\overline{TF}$ is 34.38. A vertical line is now drawn from the 0.397 position on the scale up to where it intersects the load on the upstroke (Fig. 3A).

The dynamometer deflection at this point is read to be 1.16 inches which, with a scale constant of 7,450 pounds per inch, makes a load (W) at that point 8, 650 pounds.

In a similar manner the polished rod load may be obtained for each 15 degree angle of crank rotation. The dynamometer card has been marked to show the load and position involved for each 15 degrees of crank angle.

The structural unbalance (B) or, as it has been called, the beam-off-balance for the unit equals 650 pounds.

Net well load $(W_n) = W - B = 8{,}650 - 650 = 8{,}000$ pounds.

Torque due to net well load $T_{wn} = \overline{TF} \times W_n = 34.38 \times 8{,}000 = 275{,}000$ inches pounds.

To find the torque due to the crank counterbalance, maximum moment (M) must be determined.

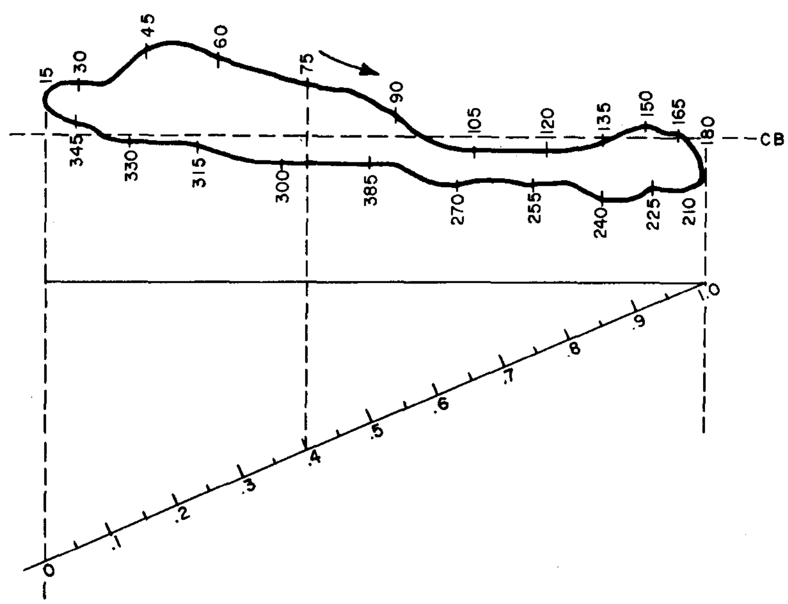
Observation of the counterweight settings on the unit showed one weight on each crank to be set at position 4 and one weight to be at position 5-1/2. It was also noted that the well is equipped with a standard counterweight assembly.

From Fig. 5 we find the following data:

Position of Counterweights	Maximum Moment
4	159,420
5	192,200
6	224,980

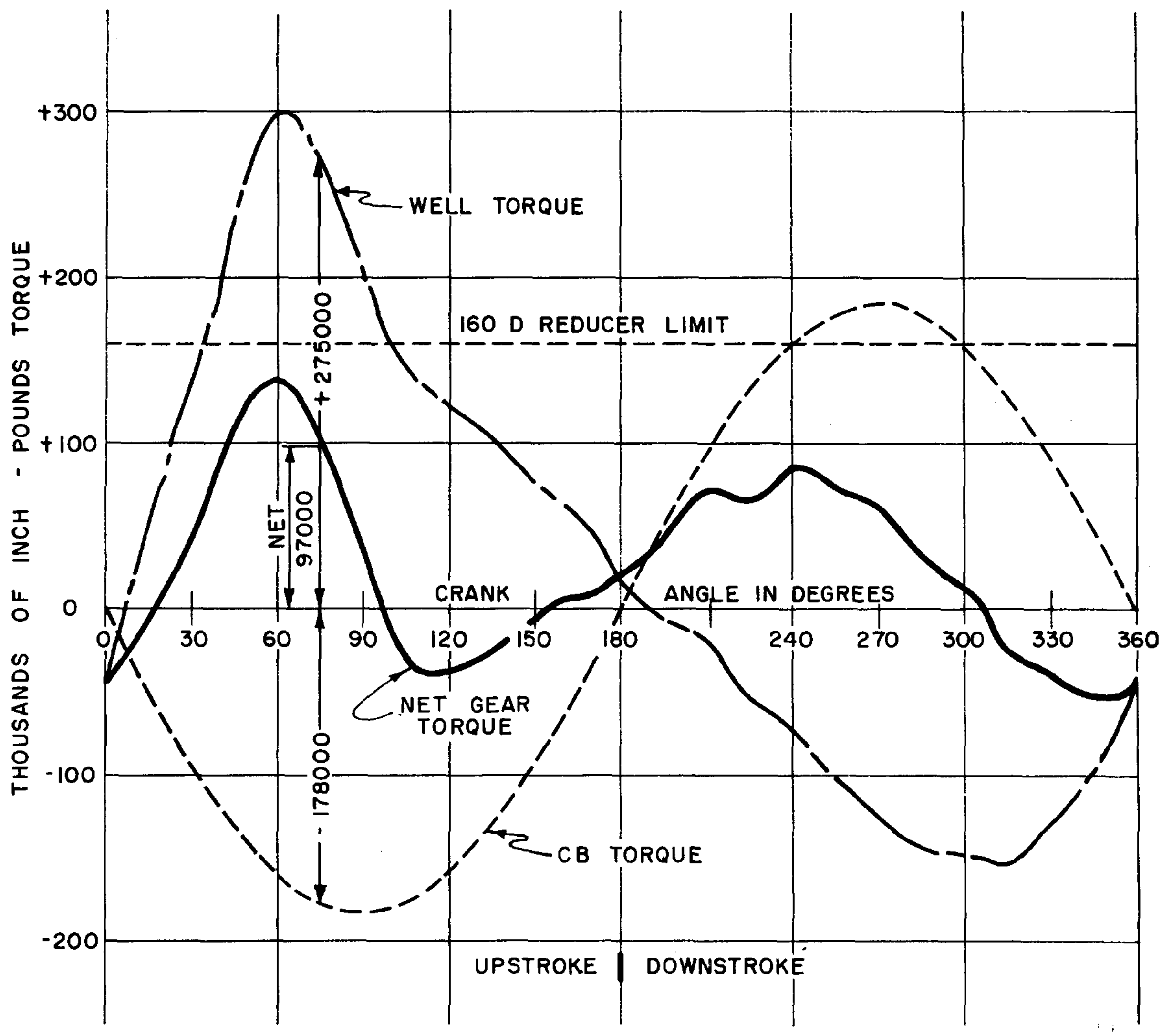
At 5-1/2 position the moment would be halfway between 5 and 6 or 208,590 inches-pounds.

The figures given are for all four weights (two on each crank) in the same position. To get the net result in this



DIVISION OF DYNAMOMETER CARD BY CRANK ANGLE USING API POLISHED ROD POSITION DATA

FIG. 3A



TORQUE CURVES USING API TORQUE FACTORS

FIG. 3B

API PUMPING UNIT STROKE AND TORQUE FACTORS

1	2	3	4	5	6	7	8	9	10	11
POSITION OF CRANK DEGREES (1)	POSITION OF RODS (2) LENGTH OF STROKE - INCHES					TORQUE FACTOR (3)(4) LENGTH OF STROKE INCHES				
	64	54	44	34	24	64	54	44	34	24
0	.004	.002	.002	.002	.001	-5.88	-4.20	-3.05	-2.13	-1.31
15	.005	.004	.004	.003	.003	5.83	5.11	4.12	3.26	2.22
30	.049	.053	.053	.053	.046	17.45	14.09	11.12	8.41	5.57
45	.141	.153	.138	.135	.127	27.05	21.60	17.08	12.54	8.41
60	.262	.257	.248	.244	.226	32.79	26.53	21.00	15.61	10.44
75	.397	.389	.376	.355	.350	34.38	28.22	22.60	17.12	11.73
90	.534	.524	.508	.499	.486	32.76	27.42	22.57	17.15	12.10
105	.655	.648	.640	.633	.612	28.83	24.92	20.66	16.32	11.57
120	.764	.756	.753	.743	.733	24.37	21.30	18.87	14.37	10.32
135	.849	.851	.854	.844	.835	19.62	17.10	14.55	11.70	8.50
150	.919	.924	925	.922	.919	14.56	12.69	10.74	8.62	6.26
165	966	.969	.973	.975	.971	9.54	7.99	6.52	5.24	3.74
180	.994	.998	.997	.997	.998	4.08	3.04	2.16	1.52	1.00
195	.999	.999	.998	.993	.991	-2.09	-2.47	-2.51	-2.15	-1.77
210	.977	.972	.972	.967	.960	-8.87	-8.13	-7.21	-6.00	-4.43
225	.924	.916	.913	.905	.895	-15.96	-14.09	-11.95	-9.55	-6.98
240	.845	.837	.828	.819	.804	-22.77	-19.59	-16.18	-12.74	-9.26
255	.743	.733	.721	.718	.697	-28.35	-23.97	-19.61	-15.15	-10.91
270	.622	.611	.602	.588	.571	-32.04	-26.87	-21.72	-16.78	-11.82
285	.489	.481	.470	.456	.439	-33.36	-27.78	-22.42	-17.21	-12.05
300	.358	.346	.339	.326	.312	-32.40	-27.00	-21.70	-16.38	-11.35
315	.232	.227	.216	.208	.191	-29.25	-24.00	-19.26	-14.31	-9.90
330	.128	.122	.131	.110	.104	-23.75	-19.25	-15.21	-11.24	-7.64
345	.049	.046	.044	.047	.037	-15.77	-12.61	-9.56	-7.32	-4.71

(1) Position of crank is the angular displacement measured clockwise from the 12 o'clock position, viewed with the well head to the right.

(2) Position is expressed as a fraction (percentage) of stroke above lowermost position.

(3) Torque factor $= \frac{T}{W}$ where T = torque on pumping-unit reducer due to polished rod load W.

(4) Negative signs on torque factor indicate a clockwise torque on crankshaft.

NET REDUCER TORQUE $= \overline{TF}\,(W-B) - M \sin\theta$

Where θ = Position of Crank Degrees (See Col. 1 above)

M = Maximum Moment of Counterbalance

W = Measured Polish Rod Load (Lbs.) At Position Of Rods Corresponding to θ

B = Structural Unbalance = 650 Lbs.

$\overline{TF}$ = Torque Factor Corresponding to θ.

FIG. 4

API RATING FORM FOR CRANK COUNTERBALANCE

DESCRIPTION	TOTAL WEIGHT LBS.	POSITION (1) OF COUNTER-WEIGHTS	MAXIMUM (2) MOMENT ABOUT CRANKSHAFT INCH POUNDS
Heavy Counterweight Assembly - "With" Filler Weights Includes the Following: 2 - Cranks - 17337 2 - Right Face Counterweights - 17339-A 2 - Left Face Counterweights - 17338-A 2 - Left Rear Counterweights - 17340 2 - Right Rear Counterweights - 17341 8 - Filler Weights - 17342 Plus Necessary Bolts Etc.	13,200	0	-12,800
		1	36,500
		2	85,800
		3	135,100
		4	184,400
		5	233,700
		6	283,000
		7	332,300
		8	381,600
		9	430,900
		10	480,200
Standard Counterweight Assembly - "Without" Filler Weights Includes the Following: 2 - Cranks - 17337 2 - Right Face Counterweights - 17339-A 2 - Left Face Counterweights - 17338-A 2 - Left Rear Counterweights - 17340 2 - Right Rear Counterweights - 17341 Plus Necessary Bolts Etc.	10,170	0	28,300
		1	61,080
		2	93,860
		3	126,640
		4	159,420
		5	192,200
		6	224,980
		7	257,760
		8	290,540
		9	323,320
		10	356,100
Light Counterweight Assembly With "Face Weights" and "Rear Clamps" Includes the Following: 2 - Cranks - 17337 2 - Right Face Counterweights - 17339-A 2 - Left Face Counterweights - 17338-A 2 - Counterweight Clamps - 18247 2 - Counterweight Clamps - 18264 Plus Necessary Bolts Etc.	8,170	0	48,840
		1	70,640
		2	92,440
		3	114,240
		4	136,040
		5	157,840
		6	179,640
		7	201,440
		8	223,240
		9	245,040
		10	266,840

(1) Pointers on all Counterweight Assemblies must be set to indicated number on crank to give Moment shown.

(2) Moments are given for horizontal position (90 and 270 degrees) only.

FIG. 5

CHART SHOWING POSITION OF PEAK LOAD WITH REFERENCE TO LOWER MOST POSITION OF POLISHED ROD FOR VARIOUS HARMONIC ORDER CARDS

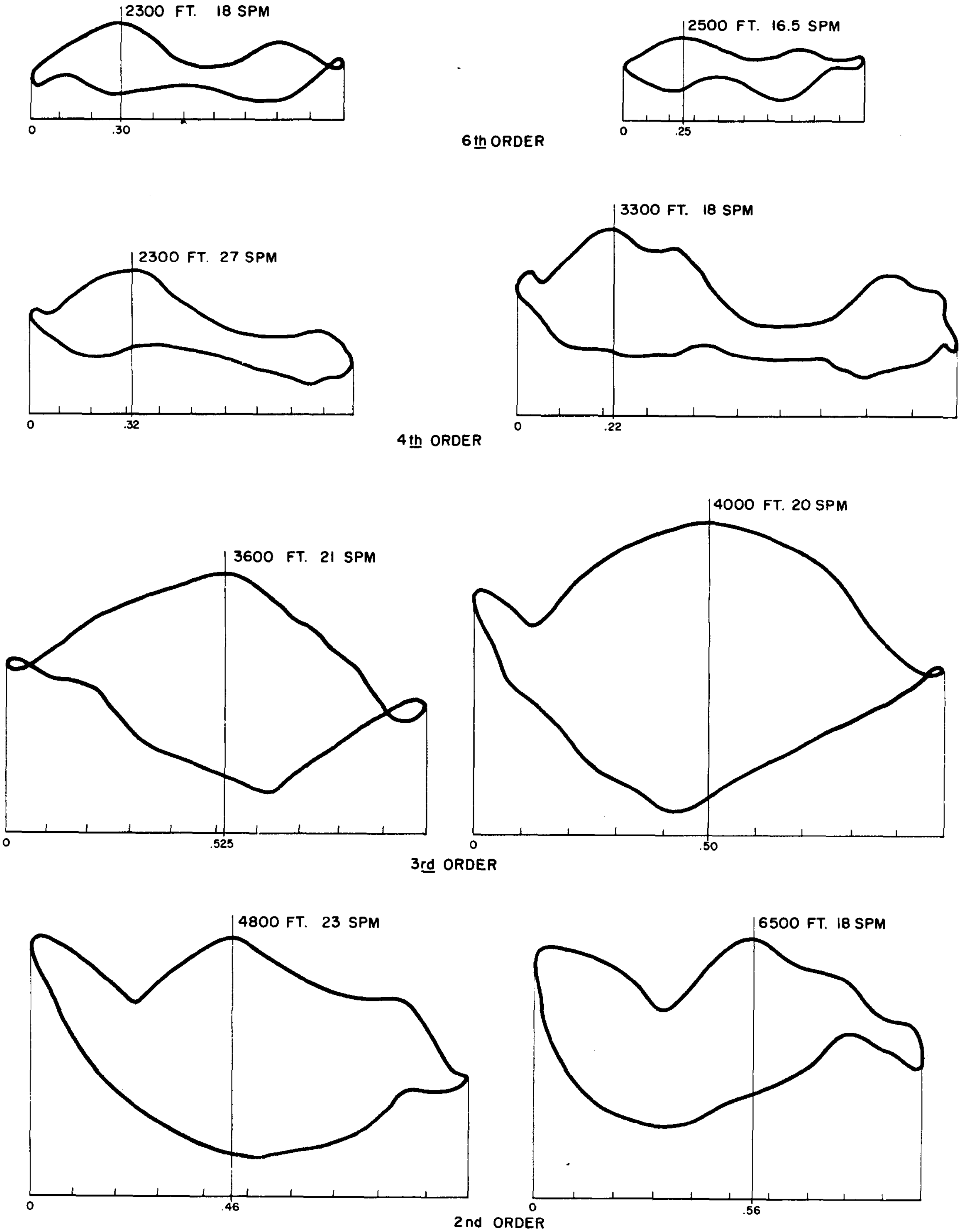


FIG. 6

case, we must consider two weights in each position, taking one-half of the value for the No. 4 position and one-half for the No. 5-1/2, and adding them.

$$\frac{159{,}420}{2} = 79{,}710$$

$$\frac{208{,}590}{2} = 104{,}295$$

$$M = \text{Sum} = 184{,}005$$

This is the maximum moment at crank angle $\theta = 90^{o}$.

At 75^{o} the moment would be 184,000 x the sine of 75^{o} = 184,000 x .966 = 178,000.

The net gear torque at 75 degrees referring to the formula, is then calculated as follows:

$$T = \overline{TF}\ (W-B) - M \sin \theta$$

$$= 34.38 \times (8{,}650 - 650) - 184{,}000 \times 0.966$$

$$= 275{,}000 - 178{,}000 = 97{,}000$$

These values are shown on the three curves in Fig. 3B for $\theta = 75^{o}$.

The dotted line drawn across the dynamometer card in Fig. 3A is the counterbalance effect measured with the dynamometer and is 6,340 pounds. This measured peak counterbalance effect is the sum of the peak crank counterbalance effect and structural unbalance.

The mathematical expression is:

$$\text{Peak counterbalance effect } \frac{M}{\overline{TF}} + B$$

Substituting in the proper values for this case:

$$\text{Peak counterbalance effect} = \frac{184{,}000}{32.76} + 650 = 6270\#$$

(The torque factor of 32.76 is the value for 90 degree crank position.)

This is a very good check on the measured valve.

Consideration Of Beam Counterbalance

When beam counterbalance is to be considered, it is the effect of this counterbalance at the polished rod that must be used in determining the net polished rod load.

Referring to Fig. 1, the net static effect of the beam counterweights at the polished rod is the actual weight of the beam weights multiplied by the ratio of the moment arms around the saddle bearing, or:

$$W_{be} = W_b \times \frac{E}{A} \cos \lambda$$

There is no simple way to compute the inertia effect because it depends upon the instantaneous acceleration of the beam which, in turn, depends upon rig geometry and prime mover speed variation. When computations are made it has been the practice to consider that the beam is moving with simple harmonic motion.

The formula for beam weight counterbalance effect at the polished rod, assuming simple harmonic motion, is as follows:

$$W_{be} = \left[W_b \pm \left(\frac{W^b}{g} \times \omega^2 R \cos \theta \right) \frac{E}{C} \right] \frac{E \cos \lambda}{A}$$

Wherein: g = acceleration of gravity = 32.2 feet per second,^{2}

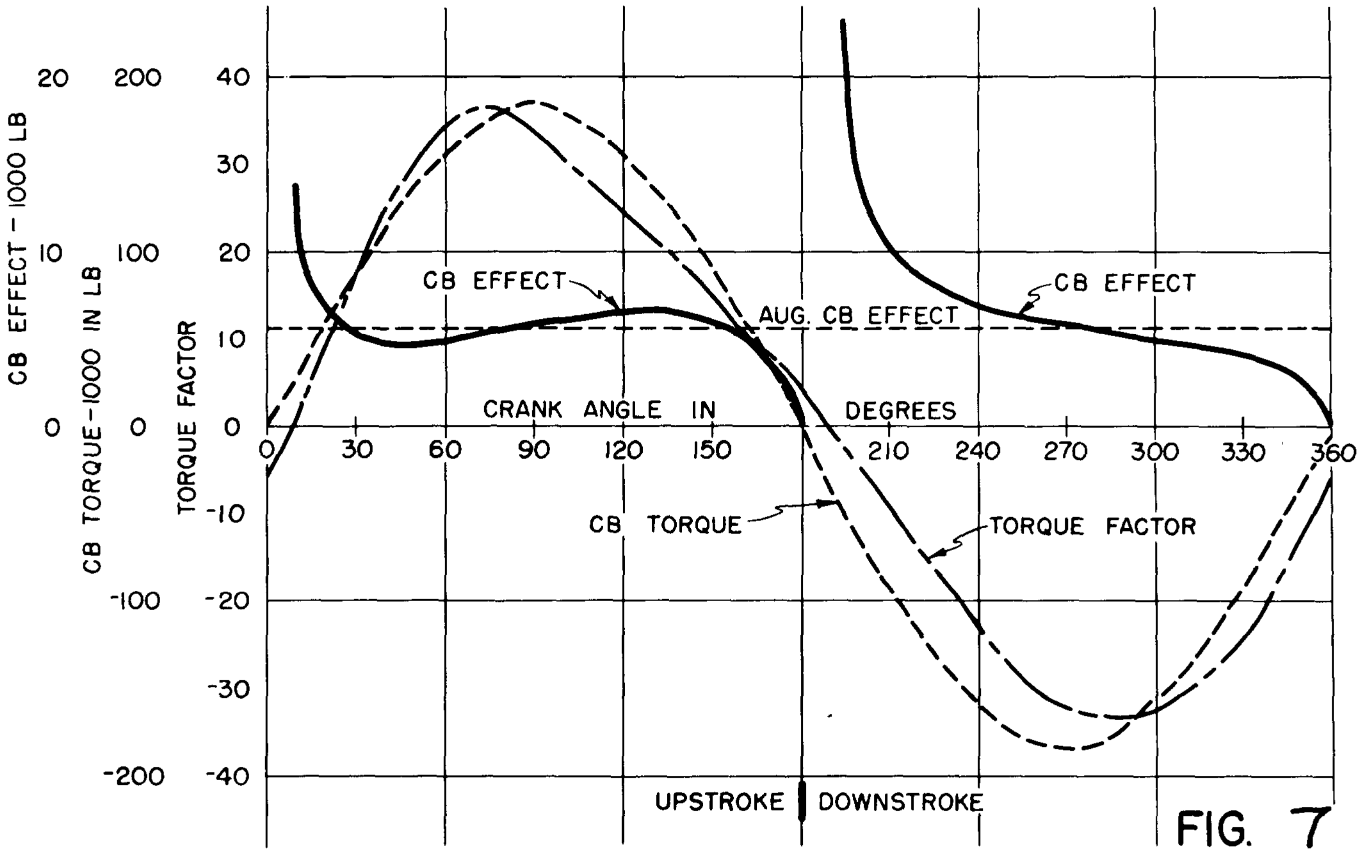


FIG. 7

ω = angular velocity = $\frac{2\pi SPM}{60}$

λ = angle that the centerline through the center of the beam weights and the saddle bearing makes with the horizontal. (Refer to Fig. 1.)

The expression $\omega^2 R \cos\theta$ represents the acceleration of the tailboard bearing.

The expression $(\frac{W_b}{g} \times \omega^2 R \cos\theta) \frac{E}{C}$ is the inertia force applied to the beam counterweights.

This subtracts from the static weight during the first part of the upstroke and last part of the downstroke and adds to the static weight during the first part of the downstroke and last part of the upstroke.

The expression $E \cos\lambda$ is the effective moment arm for the force exerted by the beam weights around the saddle bearing.

Counterbalance Effect At Polished Rod.

When counterbalance effect is measured at the polished rod with a dynamometer, the crank position should be at 90° or 270°. These are the positions of maximum moment of the rotary counterweights. The measured load includes the beam unbalance, so this must be considered.

The counterbalance effect is the maximum moment divided by the Torque Factor at 90 and 270 degrees respectively. These values represent the counterbalance effect for upstroke and downstroke and may be quite different depending on rig geometry. When this is true the direction of rotation of the crank becomes important in getting the best counterbalance.

The counterbalance effect at the polished rod due to rotary weights may be plotted for each crank angle from the following relationship:

$$\text{CB effect} = \frac{M \sin\theta}{TF}$$

Figure 7 shows that this is not a straight line, but the straight line is an average value.

Use Of Torque Factor In Calculations For New Pumping-Unit Applications

Torque factors may be used to good advantage in calculating a new pumping unit application. By assuming that the peak load and peak torque factor may occur at the same crank angle, the worst condition may be considered.

From the depth and speed the general shape of the card may be anticipated. Fig. 6 shows how the position of the peak load may vary.

In considering minimum load it is evident from examination of many dynamometer cards that it occurs approximately 180 crank degrees from the maximum load. This knowledge may then be used to calculate the anticipated torque on the downstroke using the proper torque factor for the angle provided.

SUMMARY AND CONCLUSIONS

Torque factors and polished rod position data are valuable tools in determining the torque characteristic of gear reducers.

There is nothing controversial about them, as they are based on the rig geometry and solved by layout or trigonometry.

Direction of rotation is very important for some units in keeping peak torque and peak horsepower to a minimum. The importance of torque factors is empahsized when considering the peak torque calculations on comparatively shallow high volume wells, such as are encountered in water flood operations.

Very often the peak torque factor and peak polished rod load occur at the same point of the stroke. This point comes early in the stroke, before the maximum counterbalance effect, so the net torque on the gear unit may be much higher than estimated.

ACKNOWLEDGEMENT

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