

THE THEORY, HARDWARE, AND APPLICATION
OF THE CURRENT GENERATION OF OIL WELL
JET PUMPS

Hal Petrie, National Production Systems
Phil Wilson, Kobe, Inc.
Eddie E. Smart, Guiberson Division,
Dresser Industries, Inc.

ABSTRACT

This paper will present a summary of jet pump theory and performance, and an algorithm for applying jet pumps to the wide range of conditions encountered in oil wells. Nozzle and throat sizes available from National Production Systems, Kobe, Inc., and Guiberson Hydraulic Pumping Systems, the three major suppliers of oilfield jet pumps, will be presented. Appendices will include hand-held calculator application programs for a TI-59 and an HP-41C.

Comparisons between the predictions from the calculator programs and the sophisticated programs used by the pump manufacturers as well as actual well data, will demonstrate the applicability and limitations of the techniques presented in this paper. The discussion will include potential sources of error and the sensitivity of the calculations to uncertainties in the well data.

INTRODUCTION

Hydraulic pumping systems for artificial lift consist of a fluid reservoir on the surface, a high pressure surface pump that transmits the fluid downhole, and a downhole pump driven by the high pressure fluid. The power fluid and the produced fluid both flow to the surface after passing through the downhole unit. Conventional downhole pumps have been of the positive displacement type, employing reciprocating pistons. In the last 10 years, however, hydraulic pumping systems using downhole jet pumps have been widely employed.

A typical downhole jet pump is shown in Fig. 1. Having no moving parts, jet pumps are rugged and tolerant of corrosive and abrasive well fluids. They are compact and adaptable to all existing hydraulic pump bottom hole assemblies. Jet pumps have high volume capabilities and handle free gas well, but they typically require higher pump intake pressures than conventional pumps to avoid cavitation. Also, their efficiency is lower than that of positive displacement equipment, leading to higher surface horsepower requirements. It has been observed, however, that in some wells with substantial gas production, they may actually require less power.

APPLICATION CONSIDERATIONS

As a type of dynamic pump, jet pumps have characteristic performance curves similar to electric submersible pumps. An example is shown in Fig. 2. Note that an infinite family of curves is possible, depending on the nozzle pressure. Different sizes of throats used in conjunction with a given nozzle give different performance curves. The curves are generally fairly flat, especially with the larger throats, making the jet pump sensitive to changes in intake or discharge pressure. Since fluid densities, gas, and viscosities affect the pressures the pump sees, the calculations to simulate performance are complex and iterative in nature, lending themselves to a computer solution.

Since the actual sizes of components and the associated performance curves have historically been proprietary with the pump manufacturers, it has been difficult for the production engineer to evaluate jet pumping systems in designing artificial lift systems for his wells. However, reasonable accuracy in application calculations can be obtained on hand held calculators if the commercially available sizes and their performance characteristics are available. This paper addresses that need.

THEORY OF JET PUMPS

In the following discussion, reference to Fig. 4 will be helpful. The jet pump is a hydrodynamic rather than hydrostatic type of pump and operates principally through momentum transfer between two adjacent fluid streams. High pressure power fluid passing through the nozzle has its potential energy (pressure energy) converted to kinetic energy in a jet of fluid at high velocity. Well fluid mixes with the power fluid in a constant area throat or mixing tube and momentum is transferred to the well fluid, causing an energy rise in it. As the mixed fluids exit the throat, they are still at a high velocity and thus contain substantial kinetic energy. The fluids are slowed in an expanding area diffuser which converts the remaining kinetic energy to static pressure sufficient to lift fluids to the surface. Design variables include the sizes of the nozzle and throat and the ratio of their flow areas as well as component shapes, angles, lengths, spacing, finishes, and materials. Assuming that the components have been optimized through testing, then the flow areas and their ratios are of interest for application purposes. Through selection of appropriate flow areas and ratios, the configuration of the pump can be optimized to match the well conditions.

The physical sizes of the nozzles and throats determine the flow rates while the ratio of their flow areas determines the trade off between produced head and flow rate. If, for example, a throat is selected such that the area of the nozzle is 60% of the throat area, a relatively high head, low flow pump will result. There is a comparatively small area around the jet for well fluids to enter, leading to low production rates, and with the energy of the nozzle being transferred to a small amount of production, high heads will be developed. Such a pump is suited to deep wells with high lifts.

If, on the other hand, a throat is selected such that the area of the nozzle is only 20% of the throat area, more production flow is possible, but, since the nozzle energy is being transferred to a large amount of production, lower heads will be developed. Shallow wells with low lifts are candidates for such a pump.

Any number of such area combinations are possible to match different flow and lift combinations. Attempting to produce small amounts of production with a nozzle-throat ratio of 20% will be inefficient due to high turbulent mixing losses between the high velocity jet and the slow moving production. Conversely, attempting to produce high production rates with a nozzle-throat ratio of 60% will be inefficient due to high friction losses as the produced fluid moves rapidly through the relatively small throat. Optimal ratio selection involves a tradeoff between these mixing and friction losses.

The cavitation characteristics of the pumps must also be considered. Cavitation will occur when the velocity of the produced fluid entering the throat around the power fluid jet is high enough that the static pressure in the fluid falls to its vapor pressure. This is a choked flow condition and cavitation damage to the throat is likely.

MATHEMATICAL REPRESENTATION

Since each manufacturer of jet pumps offers a large number of nozzle sizes and five or more throats for each nozzle, and because there is no standardization of these sizes between suppliers, the number of possible performance curves is very large. This is further complicated by the variability in the performance curves possible with different nozzle pressures. To simplify this situation, a unifying mathematical representation is needed.

As first presented by Gosline and O'Brien (1) and expanded on by Cunningham (2), it is possible to write a set of equations describing the performance of geometrically similar pumps. If the equations are written nondimensionally, they will apply to all sizes of pumps so long as the operating Reynolds' Numbers are close or sufficiently high that viscosity effects are negligible.

By considering the energy and momentum equations for the nozzle, suction passage, throat (mixing tube), and diffuser, the following equations can be derived (Refer to Fig. 4 and the glossary for nomenclature):

Nozzle Flow (BPD)

$$QN = 832 AN \sqrt{\frac{PN - PS}{GN}} \quad (1)$$

Dimensionless Area Ratio

$$R = \frac{AN}{AT} \quad (2)$$

Dimensionless Mass Flow Ratio

$$M = \frac{QS \times GS}{QN \times GN} \quad (3)$$

Dimensionless Pressure Ratio

$$N = \frac{PD - PS}{PN - PD} \quad (4)$$

$$N = \frac{2R + (1-2R) \frac{M^2 R^2}{(1-R)^2} - (1+KTD) R^2 (1+M)^2}{(1+KN) - \text{Numerator}} \quad (5)$$

Efficiency

$$EFF = N \times M = \frac{(PD - PS)}{(PN - PD)} \frac{QS \times GS}{QN \times GN} \quad (6)$$

Cavitation Area (in²)

$$ASM = (AT - AN) = \frac{QS}{691 \sqrt{\frac{PS}{GS}}} \quad (7)$$

Equation (1), the nozzle flow rate, can be recognized as the expression for flow through an orifice. Equation (2) defines R as the ratio of the nozzle area to that of the throat. Equation (3) defines a dimensionless mass flow coefficient equal to the ratio of the production or suction flow rate to the nozzle flow rate times the ratio of the suction gradient to the nozzle fluid gradient. Equation (4) defines a dimensionless pressure ratio. Physically, it is the ratio of the pressure rise imparted to the produced fluid to the pressure lost by the power fluid in the

pump. Equation (5), due to Cunningham (2), is a formulation for the dimensionless pressure of Equation (4) in terms of the area ratio, R, the mass flow ratio, M, and two loss coefficients, KN and KTD. These loss coefficients are experimentally determined and are similar to orifice and pipe friction loss coefficients. Equations (4) and (3) can be combined to give the efficiency, expressed in Equation (6). Since hydraulic power is the product of pressure differential times flow rate, Equation (6) is interpreted as the ratio of the power added to the produced fluid to the power lost from the power fluid. Equation (7) derived from the orifice flow equation for the annular production flow area at the entrance of the throat, defines the minimum flow area required to avoid cavitation if the suction flow rate is QS and is at a pressure of PS. This equation includes the assumption that the pressure at the entrance of the throat is zero at cavitation.

A representative set of dimensionless performance curves is shown in Fig. 3 for typical nozzle to throat area ratios of .50, .40, .30, .25, .20, and .15. The power fluid and produced fluid are of the same density. A nozzle loss coefficient of .03 was used, typical of a well shaped and smoothed design. A throat-diffuser loss coefficient of .20 was used. Lower values can be obtained in laboratory tests, but this conservative value compensates for average losses in routing the fluids through the rest of the pump and bottom hole assembly. The peak efficiencies of about 33% shown in Fig. 3 can be achieved with commercially available pumps producing typical well fluids at around a 700 BPD rate. Much larger or smaller pumps, or fluids of very low or high viscosity can result in pumps having somewhat higher or lower efficiencies respectively. Note that each area ratio curve has an associated efficiency curve, and that there is a most efficient ratio for a given value of the dimensionless mass flow ratio "M". These curves represent the type of non-cavitating performance obtainable from the jet pumps currently available from National, Kobe, and Guiberson. Operating under cavitating conditions will result in deviations from these curves.

APPROXIMATIONS FOR HANDLING GAS

The equations presented above are for liquids. Gas is present in many wells and affects pump performance. A rigorous treatment of the pumping of multiphase and compressible fluids is outside the scope of this paper. However, it has been found that simple but useful approximations can be made. Cunningham (2) found that if the free gas volume is added to the liquid volume as if it were liquid, pump performance follows the standard curves reasonably well. Equation (3) then becomes

$$M = \frac{QS + QG}{QN} \frac{GS}{GN} \quad (8)$$

A review of Standing's work (3) for a variety of bottom hole conditions results in an empirical correlation for the gas plus liquid phase volumes which, when substituted in Equation (8), gives

$$M = QS \left[\left(1 + 2.8 \left(\frac{GOR}{PS} \right)^{1.2} \right) (1 - WC) + WC \right] \frac{GS}{QN \times GN} \quad (9)$$

A cavitation correction for gas is also required. Assuming choked flow into the throat annulus around the power fluid jet, the additional area required to pass the gas is

$$AG = \frac{QS(1-WC)GOR}{24650 PS} \quad (10)$$

Equation (7) considering gas then becomes

$$ASM = QS \left[\frac{1}{691} \sqrt{\frac{GS}{PS}} + \frac{(1-WC)GOR}{24650 PS} \right] \quad (11)$$

NOZZLE AND THROAT SIZES

Kobe, National, and Guiberson have different sizes and combinations of nozzles and throats. Kobe and National Production Systems increase the areas of nozzles and throats in a geometric progression. The Kobe factor is $10^{1/9} = 1.29155$ and the National factor is $4/\pi = 1.27324$. The progression of sizes from Guiberson is not geometric. The sizes offered by Guiberson cover a slightly larger range than those of Kobe and National. The sizes from each manufacturer are listed in Table 1.

The strict progression employed by National and Kobe establishes fixed area ratios between the nozzles and throats. A given nozzle matched with the same number throat will always give the same area ratio - .383 in the National system and .400 in the Kobe system. This is called the "A" ratio. Successively larger throats matched with a given nozzle give the "B", "C", "D" and "E" ratios. In the Kobe and National systems the size of a pump is designated by the nozzle size and ratio. Examples are 11-B or 6-A.

The various combinations of nozzles and throats in the Guiberson system cover the same basic range as the Kobe and National combinations. The actual ratios are listed in Table 2. In the Guiberson system, the nozzle and mixing tube (throat) sizes designate the size of a pump. An example is C-5. The annular areas of the Guiberson Jet Pumps used in cavitation calculations are also included in Table 2. The annular areas for the Kobe and National Jet Pumps are listed in Tables 3 and 4.

The most commonly employed area ratios fall between .400 and .235. Area ratios greater than .400 are sometimes used in very deep wells with high lifts, or when only very low operating pressures are available. Area ratios less than .235 are used in shallow wells or when very low bottom hole pressures require a large annular flow passage to avoid cavitation. Referring to Fig. 3, it can be seen that the performance curves for the higher area ratios show higher values of the dimensionless parameter "N" within their regions of maximum efficiency. Since "N" is a measure of the pressure rise in the produced fluid, the higher area ratios are suited for high net lifts, but this is achieved only with production rates substantially less than the power fluid rate ($M < 1.0$). The smaller area ratios develop less head, but may produce more fluid than is used for power fluid ($M > 1.0$).

CALCULATION SEQUENCE AND ADDITIONAL EQUATIONS

The calculator programs in the appendices use the following algorithm:

1. From the well data for production, QS, and pump intake pressure, PS, and the GOR, calculate the minimum annular area needed to avoid cavitation from Eqn (11).
2. From the tables of annular areas, select a nozzle and throat combination which has an annular area greater than ASM.
3. If it is desired, by calculating operating pressures, to evaluate different sizes to select an optimum, set a flag. If it is desired to plot curves at a constant operating pressure for a given size pump, do not set the flag.
4. Pick a surface operating pressure, usually between 2000 and 4000 psi, with higher pressures required at greater depths and with lower values of "R". Known available surface pump limitations may affect this choice.

5. Calculate the pressure at the nozzle. PN is the sum of the operating pressure plus the hydrostatic pressure in the tubing minus friction losses in the tubing.

$$PN = PT + GN \times D - PFN \quad (12)$$

The friction in annular or circular (tubing) sections can be determined from the following equation (4):

$$PF = \left[\frac{202 \times 10^{-8} L}{(D1 - D2)(D1^2 - D2^2)(D1 / (D1 - D2))^{.1} \left(\frac{D1^2 - D2^2}{D1 - D2} \right)^{.21}} \right] \left[\left(\frac{\mu}{G} \right)^{.21} G \right] Q^{1.79}$$

	Annular Flow	Tubing Flow
D1	Casing I.D.	Tubing I.D.
D2	Tubing O.D.	0

(13)

The expression within the first set of brackets is a constant for a given tubing string or annular flow passage. The expression in the second set of brackets is a constant for the power fluid losses, but not for the production return conduit since it will contain a variable mix of power fluid and production.

6. Determine the power fluid rate from Equation (1) and the selected nozzle size.
7. Determine the return flow rate.

$$QD = QN + QS \quad (14)$$

8. Determine the production (pump suction) gradient.

$$GS = (GW \times WC) + (1 - WC)GO \quad (15)$$

9. Determine the return flow fluid gradient.

$$GD = \frac{(GS \times QS) + (GN \times QN)}{QD} \quad (16)$$

10. Calculate the return flow water cut.

$$WCD = \frac{QS \times WC}{QD} \quad \text{Oil Power Fluid} \quad (17)$$

$$WCD = \frac{QN + (QS \times WC)}{QD} \quad \text{Water Power Fluid} \quad (18)$$

11. Calculate the return flow gas liquid ratio.

$$GLR = \frac{QS(1 - WC)GOR}{QD} \quad (19)$$

12. If the GLR is greater than 10, it is recommended to use a vertical multiphase flow pressure gradient correlation to determine the pump discharge pressure using values from Equations (14) through (19). Gas lift charts can be used. Then go to Step 15.

13. If the GLR is less than 10, determine the return flow liquid viscosity for calculating friction losses. (20)

$$\mu_D = WCD \times \mu_w + (1 - WCD) \mu_o \quad (20)$$

This weighted average equation assumes that if oil is used for power fluid, it has the same viscosity as the produced oil and that no emulsions are formed.

14. Determine the pump discharge pressure. PD is the sum of the hydrostatic pressure in the return conduit, the friction losses, and the wellhead back pressure.

$$PD = (GD \times D) + PFD + PWH \quad (21)$$

15. Calculate "M" from Equation (9).

16. Calculate "N" from Equation (5) for the value of "R" selected, using the value of "M" from Step 15.

17. Compare the current value of N with the previous value. If the difference is less than 1/2%, iteration is complete and go to Step 20 (flag set) or Step 21 (flag not set). Otherwise, go to Step 18. Three to 10 iterations are normally required. The 1/2% convergence criterion leads to answers that are reproducible to within about ± 15 psi.

18. If the flag is set, calculate a new nozzle pressure from equation (4).

$$PN = \frac{PD - PS}{N} + PD \quad (22)$$

Then go to Step 6.

19. If the flag is not set, calculate a new pump intake pressure from equation (4).

$$PS = PD - N(PN - PD) \quad (23)$$

Then go to Step 5.

20. Determine the new surface operating (triplex) pressure.

$$PT = PN - (GN \times D) + PNF \quad (24)$$

21. Calculate the maximum non-cavitating flow.

$$QSC = QS \left(\frac{AT - AN}{ASM} \right) \quad (25)$$

22. Calculate the triplex horsepower, assuming 90% efficiency.

$$HP = \frac{QN \times PT}{52910} \quad (26)$$

23. Display
 Power Fluid Pressure, PT
 Power Fluid Rate, QN
 Triplex Horsepower, HP
 Cavitation Flow Rate, QSC
 Production Rate, QS
 Pump Intake Pressure, PS

24. For a new size pump, go to Step 1, inputting the new size at Step 2, and set the flag at Step 3. To calculate a performance chart for a chosen size, go to Step 1, inputting a new production rate, and do not set the flag (or remove it if it is already set) at Step 3. With this option, a number of points can be calculated for a given triplex pressure.

PERFORMANCE CHARTS

Kobe created the format shown in Fig. 6, and has the chart drawn by a computer driven plotter. Guiberson employs a similar procedure (Fig. 8). Referring to Fig. 6, enter the chart on the horizontal axis at a particular production rate (2040 BPD). Read upward to the appropriate pump intake pressure (1500 PSI) identified by the curves rising to the right. Read to the left to the vertical axis to obtain the power fluid rate (2695 BPD). The triplex pressure is obtained by interpolating between the power fluid pressure curves falling to the right (3230 PSI). The IPR curve can be superimposed on the chart.

National Production Systems presents pump performance data in the format shown in Fig. 10. Here a normal IPR curve is drawn and the pump performance curves are superimposed on it. Since this format is simpler for hand plotting and since there is a tendency in the current literature to present artificial lift performance curves in this manner, it is suggested for use with the programs in the appendices.

To interpret Fig. 10 enter the chart on the horizontal axis at a particular production rate (205 BPD). Read upward to the appropriate pump intake pressure (1550 PSI). The power fluid pressure can be read by interpolating between the curves (3200 PSI). Power fluid rates are obtained by interpolating between the values listed (approximately 803 BPD).

USE OF THE CALCULATOR PROGRAMS

Instructions for the use of the calculator programs are given in Appendix 1 (TI-59) and Appendix 2 (HP-41C). Program listings are also included.

As an example, consider the following data for a hypothetical well.

DEPTH: 5000 ft	PROD OIL GRAD: .353 psi/ft
TBG LENGTH: 6000 ft	WATER GRAD: .446 psi/ft
TBG OD: 2.375 in	OIL VISC: 2.5 cp
TBG ID: 1.995 in	GOR: 0 scf/BBL
RETURN ID: 4.892 in	WATER CUT: .30
WELL HEAD PRESS: 100 psi	PRODUCTION: 500 B/D
PWR GRAD: .353 psi/ft	BOTTOM HOLE PRESS: 1000 psi

The calculator program indicates a minimum suction area (throat annulus area) of .0141 in² (sample TI-59 output tapes are included in Table 5). Referring to Tables 2, 3, and 4, note that above a certain size nozzle, all combinations have adequate area. Below this size, only certain ratios have adequate area. Generally, the smallest pumps that will satisfy the throat annulus area requirement are candidates to consider. Usually, considering three different ratios will clarify the tradeoffs between power fluid rate, operating pressure, and required horsepower, and enable selection of an optimum size.

Likely candidates from the Guiberson options (Table 2) are A-3, B-3, and B-4. Possibilities from Kobe include 4-C, 5-B, and 6-B (Table 3) and the same size design-

nations from National (Table 4). For this example, the Guiberson sizes will be evaluated. The calculator program was run with the Flag Set. The following results were obtained.

Size	R	PT	QN	HP	QSC	QS	PS
A-3	.23	3284	489	30.4	659	500	1000
B-3	.40	2517	761	36.2	517	500	1000
B-4	.30	2278	733	31.6	775	500	1000

In this case, the A-3 combination would produce the desired 500 BPD with the least horsepower, but at the highest operating pressure. The calculator program was the run for the A-3 combination with the Flag Not Set and the following results were obtained.

Size	R	PT	QN	HP	QSC	QS	PS
A-3	.23	3000	462	26.2	703	500	1139
		3000	449	25.5	763	600	1340
		3000	475	27.0	636	400	934
		3500	507	33.6	627	500	907
		3500	494	32.7	698	600	1124
		3500	519	34.4	547	400	689

These data are plotted in Fig. 5. A line indicating a PI of 1.0 is also plotted through the target production point. The operating pressure curves are plotted over a range of intake pressures which includes the target flowing bottom hole pressure. This allows for some uncertainty in the input data and shows the pump performance over a range of conditions. Note that the maximum production obtainable with this pump is approximately 620 BPD at the intersection of the PI line with the cavitation line. Extrapolation indicates this would occur at an operating pressure of about 4100 psi. Power fluid rates are indicated at several points near the desired production rate.

FIELD RESULTS

Well data supplied by National Production Systems is listed in Table 6. This data was input into a jet pump simulation computer program for a 6-A nozzle and throat combination, and the results are plotted in Fig. 10. The actual well production varied between 195 BPD and 205 BPD during test. A bottom hole pressure recorder showed pump intake pressures (PS) starting at about 1805 psi and declining to a stable 1550 psi during the test. The surface operating pressure varied between 3150 and 3200 psi. The computer plot indicates that 206 BPD should be produced at a surface operating pressure (PT) of 3200 psi.

The calculator program in the appendices was run for the same data. The calculator generated 3200 psi line is shown as a dotted line with circled points in Fig. 10. Since gas was present, gas lift charts were used to estimate the pump discharge pressure between 3150 and 3300 psi over the range plotted.

Both the computer program and the calculator program agree very well with the test data, with the calculator routine being slightly optimistic in predicting 215 BPD production at 3200 psi operating pressure. The calculator routine also predicts a larger flow rate possible at cavitation. Both these effects are due to the approximations made in handling the gas volumes at downhole conditions. Gas effects are the most frequent cause of discrepancies between actual and predicted performance. Discrepancies of $\pm 10\%$ are not uncommon.

Well data supplied by Kobe, Inc. is listed at the top of Fig. 6, which is the computer plot of the calculations for an 11-B nozzle and throat combination. Recent bottom hole pressure information is not available, but the well is producing 2040 BFPD at a surface operating pressure of 3300 psi. The power fluid rate is 2695 BPD, indicating a pump intake pressure of about 1500 psi. Note that the computer chart

predicts an operating pressure of 3230 psi, agreeing very closely with the actual value of 3300 psi. In 1979, the well's PI was calculated at 2.65. Such a PI line has been constructed on Fig. 6.

A similar PI line has been drawn on Fig. 7, and the calculator program has been used to calculate operating pressure lines and a cavitation line. The field data point is circled. Note that the calculator program predicts an operating pressure (PT) of 3800 psi compared to the actual and computer predicted 3300 psi. At 3300 psi, the calculator program predicts about 1825 BPD production instead of the observed 2040 BPD. The calculator cavitation curve closely matches the upper boundary of the Kobe cavitation zone. The lower "potential damage" cavitation curve on the Kobe computer plot is not modelled in the calculator program.

Again, gas effects are not modeled in as sophisticated a manner in the calculator program as in the computer program. Also, this case involves a large pump, an 11-B nozzle-throat combination in a 3" pump, and better efficiencies can be expected. This is particularly true with low viscosity fluids, as is the case here, because of the resulting high Reynolds' Numbers. This effect is not included in the model used in the calculator program. This same nozzle and throat in a 2-1/2" pump would probably perform closer to the calculator predictions due to greater friction losses within the pump passages.

Guiberson supplied well data are presented in Table 7. These data were used to generate the map of pump performance shown in Fig. 8. The calculations were for a C-5 nozzle and mixing tube (throat). Well production tests varied from 690 BPD to 780 BPD with an average of 700 BPD. The static fluid level was measured 2000 ft from the surface and the well was reported to have a 1700 psi producing bottom hole pressure at 500 BPD. The IPR curve superimposed on the map indicates an operating pressure of 2800 psi at a 700 BPD production rate and 1566 psi producing bottom hole pressure. The actual operating pressure started at 2700 psi and is currently at 2800 psi. The exact volume of power fluid is unknown since some of the Triplex output is being bypassed. The maximum Triplex output is 1190 BPD, so the power fluid volume of 955 BPD indicated by the computer plot is a good prediction of the amount actually used.

The calculator program was run for this well and the results are presented in Fig. 9. Since gas was produced, flowing gradient curves were used to input a pump discharge pressure of 2485 psi. At the current conditions, the calculator program predicts an operating pressure of 2850 psi and a power fluid volume of 996 BPD. This closely matches both the actual well data and the computer calculations.

The cavitation line on the computer plot is generated without gas effects in this case, since the gas volumes present will greatly reduce any cavitation damage.

DISCUSSION AND CONCLUSIONS

The procedures presented above can be used to make reasonable predictions about the performance of jet pumps in oil wells. Employing the calculator programs in the mode which calculates a surface operating pressure and power fluid rate to produce a targeted production rate (Flag set) allows a comparison of different sizes

of nozzles and throats to minimize the horsepower requirements or to match existing power fluid supply limitations. Using the calculator programs in the constant operating pressure mode (Flag not set) enables calculation of operating pressure curves which can be drawn on an IPR plot. Such a plot allows a quick determination of operational characteristics under different bottom hole conditions. This includes the prediction of the intersection of the well's IPR curve with the pump's cavitation limit.

Simplifying assumptions in the performance equations and in correlations for liquid and gas properties have been made to fit the algorithm into the memory limitations of hand held programmable calculators. The well's IPR curve is often not well known, and the gas production of an individual well may be uncertain. Jet Pump performance is strongly affected by the pump intake pressure (determined by the IPR curve) and by the pump discharge pressure (significantly determined by the gas oil ratio). For every psi error in the pump intake pressure or pump discharge pressure, the effect on the surface operating pressure will be from three to five psi if the same production rate is to be obtained. Higher discharge pressures or lower pump intake pressures require higher operating pressures. This multiplier effect is greatest with the larger throats (low values of R). Fluid friction losses through the passages of the particular downhole completion hardware can affect these pressures as well.

Even when accurate well data are available, the performance predictions may not always match field performance. When the volume of free gas at producing bottom hole conditions is very large, performance will suffer beyond the degree predicted by the equations. Current jet pump designs have been optimized for liquid production, not for pumping gas. Accuracy of prediction begins to suffer at around five parts of gas to one part liquid, and at 90% gas the predictions are very questionable.

The performance of jet pumps with fluids of high viscosity is not modelled in the calculator programs. Heavy crudes with viscosities above about 1000 cp will cause significant deviations from predictions unless produced water is the dominant phase. Oil power fluids of less than about 22 API will also introduce losses that are not properly modelled.

Generally, the calculator algorithm presented here should tend to produce predictions in which any deviations are small or are on the conservative side. Using these techniques, production engineers can evaluate jet pumps for their artificial lift needs. If jet pumping looks reasonable, as it will in many wells, the suppliers can be contacted for confirming calculations and the actual surface and downhole pump hardware recommended for the application.

REFERENCES

1. Gosline, James E. and O'Brien, Morrourgh P., "The Water Jet Pump", Univ. of Calif. Pub. in Eng. (1933).
2. Cunningham, R.G., "The Jet Pump As a Lubrication Oil Scavenge Pump for Aircraft Engines", Wright Air Development Center Technical Report 55-143 (1955).
3. Standing, M.B., "Volumetric and Phase Behavior of Oil Field Hydrocarbon Systems", New York. Reinhold Publishing Corp., 1952.
4. Coberly, C.J., "Theory and Application of Hydraulic Oil Well Pumps", Kobe, Inc., Huntington Park, Calif., 1961.

GLOSSARY

AN	Flow area of the nozzle (in ²).
AS	Throat annulus area (AT-AN) (in ²).
ASM	Minimum throat annulus flow area to avoid cavitation (in ²).
AT	Flow area of the throat (in ²).
D	Vertical depth of well (ft).
D1	Inside diameter of tubing or casing (in).
D2	OD of inner tubing in annular flow (in).
Eff	Jet pump efficiency.
GD	Gradient of mixed power fluid and produced fluid returning to surface (psi/ft).
GLR	Gas-liquid ratio in return flow to surface (scf/BBL).
GN	Gradient of power fluid passing through nozzle (psi/ft).
GO	Gradient of produced oil (psi/ft).
GOR	Gas-oil ratio (scf/BBL).
GS	Gradient of well produced fluid (psi/ft).
GW	Gradient of water (psi/ft).
HP	Horsepower.
KN	Nozzle loss coefficient.
KTD	Throat-diffuser loss coefficient.
L	Tubing length (ft).
M	Dimensionless mass flow ratio.
N	Dimensionless pressure recovery ratio.
PD	Pump discharge pressure (psi).
PF	Friction loss in tubing (psi/ft).
PFN	Friction loss in power fluid tubing (psi/ft).
PFD	Friction loss in return conduit (psi/ft).
PN	Pressure at the nozzle entrance (psi).
PS	Pump suction pressure (producing bottom hole pressure) (psi).
PT	Surface operating pressure (triplex pressure) (psi).
PWH	Flow line pressure at the wellhead (psi).
QD	Flow rate from pump discharge (BPD).
QG	Flow rate of gas through pump (BPD).
QN	Flow rate through the nozzle (BPD).
QS	Flow rate to pump suction (production flow rate) (BPD).
QSC	Maximum non-cavitating pump suction flow rate (BPD).
R	Dimensionless ratio of nozzle area to throat area.
WC	Production water cut (50% water cut is entered as .50).
WCD	Water cut in return flow to surface.
μ_o	Viscosity of return fluids (cp).
μ_o	Viscosity of oil (cp).
μ_w	Viscosity of water (cp).

TYPICAL SINGLE SEAL

JET PUMP

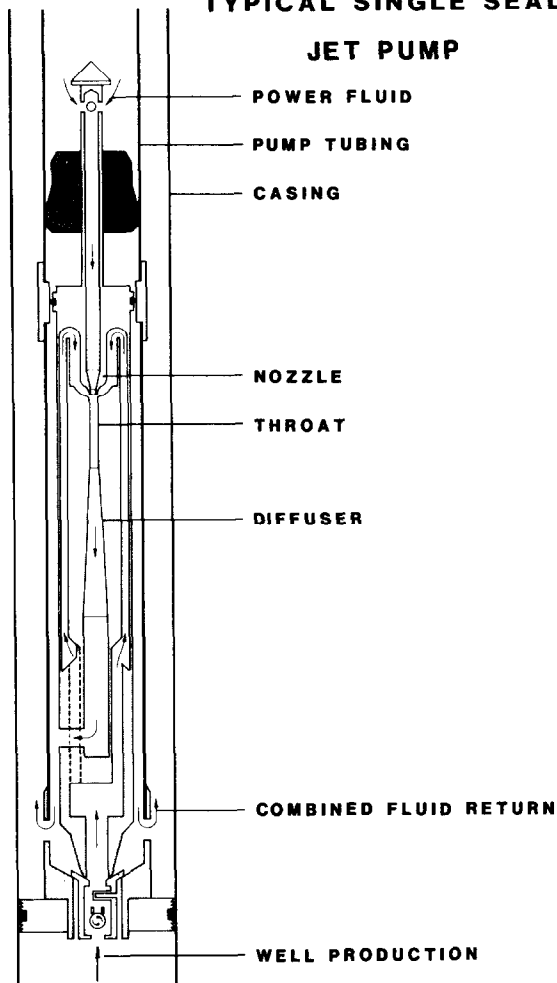


FIGURE 1

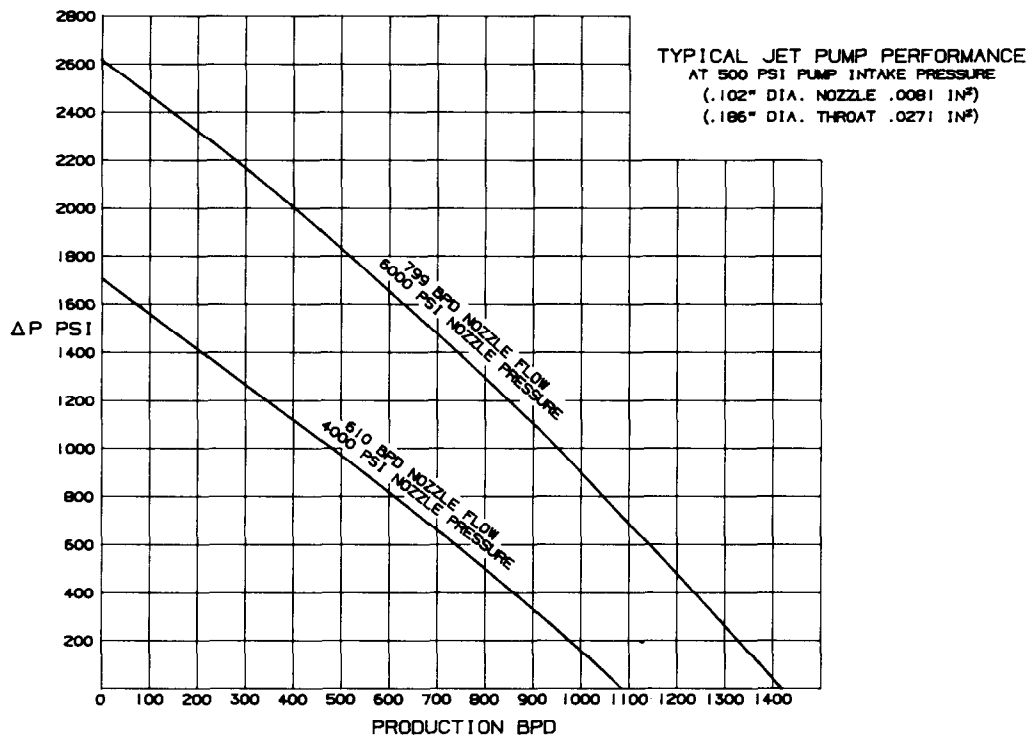


FIGURE 2

TYPICAL DIMENSIONLESS PERFORMANCE CURVES

$$KN = .03, KTD = .2$$

$$N = \frac{PD - PS}{PN - PD}$$

$$M = \frac{QS}{QN}$$

$$EFFICIENCY = N \times M$$

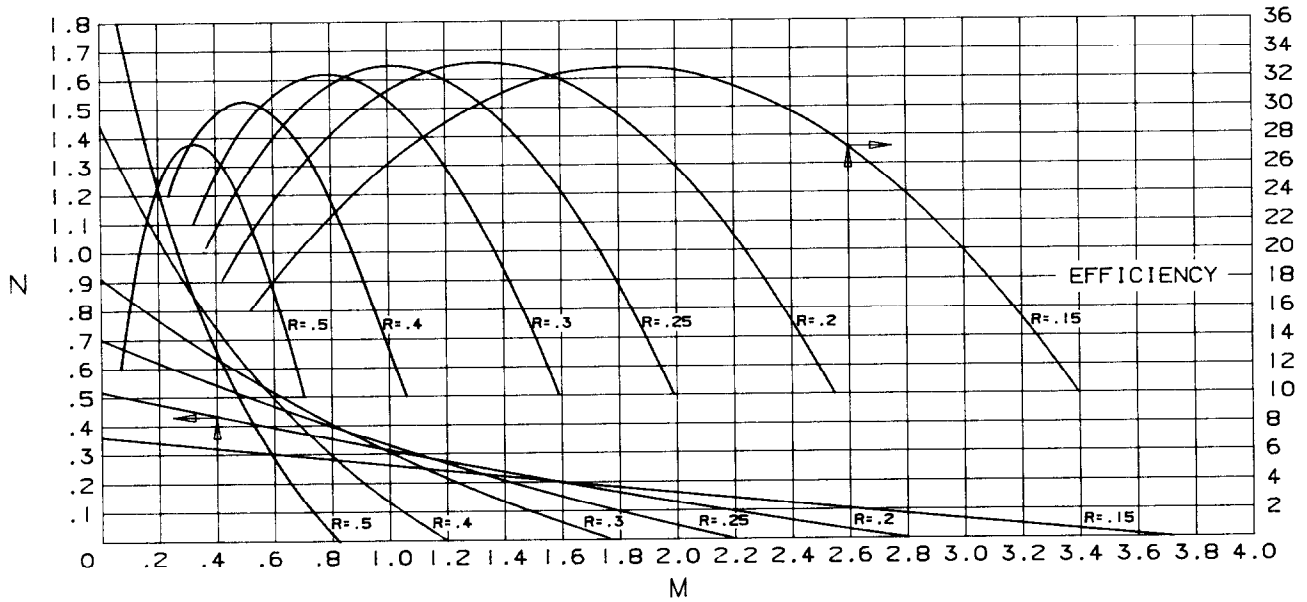


FIGURE 3

JET PUMP NOMENCLATURE

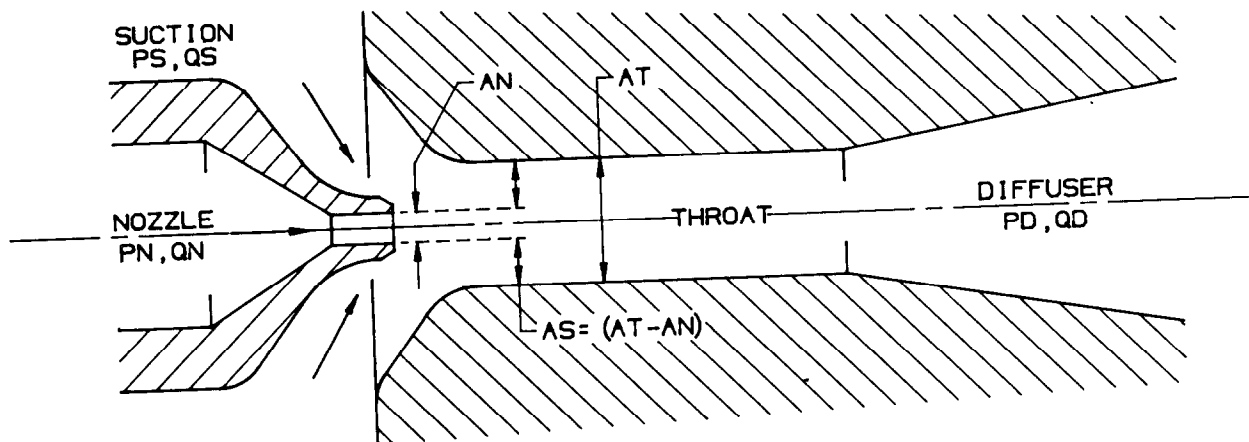
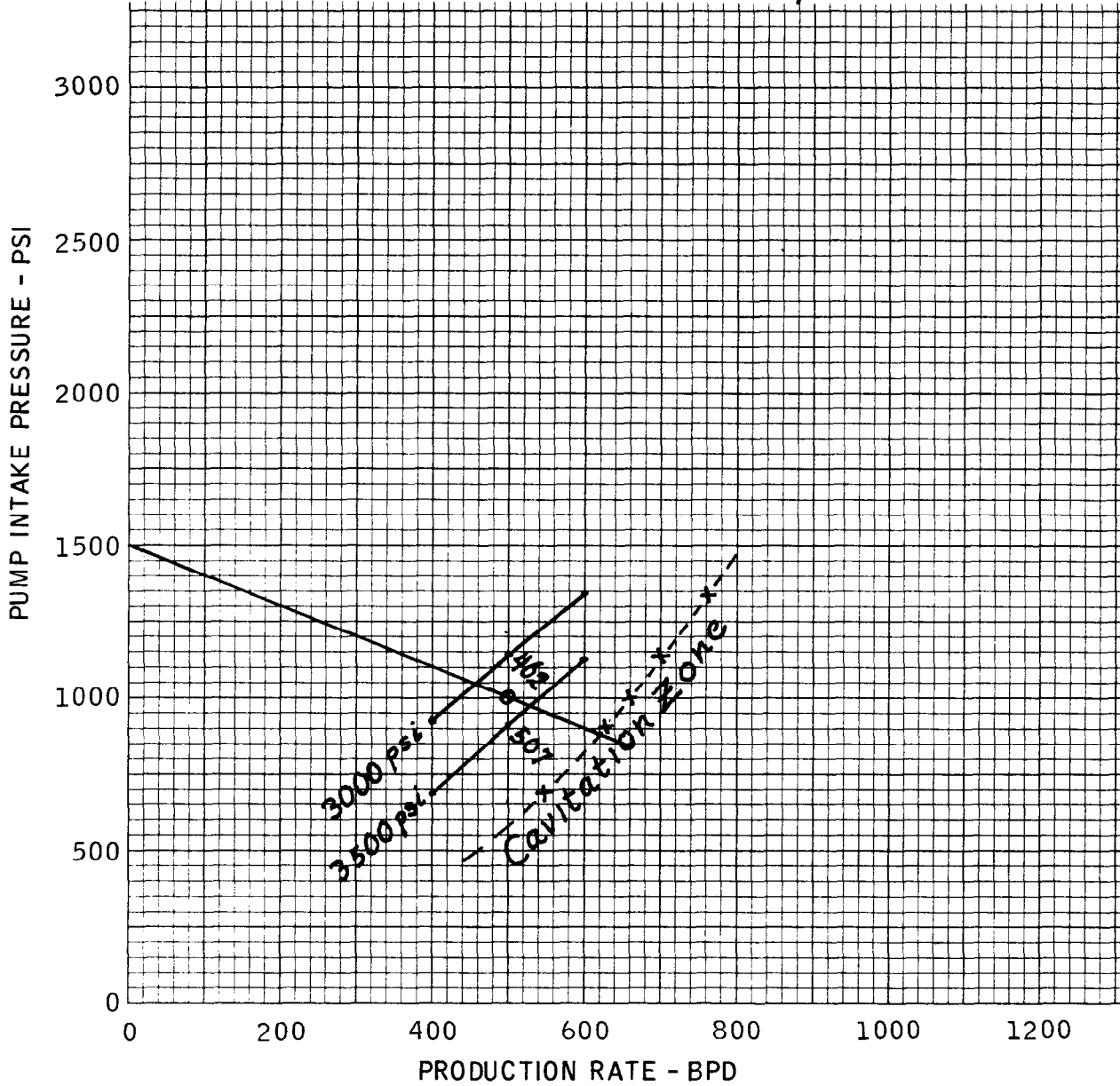


FIGURE 4

A-3 2" JET PUMP PRODUCTION UNIT PERFORMANCE

DEPTH OF PUMP 5000 FT. TUBULARS 2³/₈ x 5¹/₂
 OIL 42 °API .8155 S.G. WATER 1.03 S.G. 30 %
 GAS 0 GOR SCF/BBL POWER FLUID Oil
 WELLHEAD FLOW LINE 100 PSI DATE _____ BY _____

Calculator Prediction for Example



NOTE - NUMBERS ADJACENT TO CURVES ARE POWER FLUID - BPD
 - JET PUMP OPERATION MUST BE ABOVE & LEFT OF CAVITATION LINE

FIGURE 5

SHORT COURSE
 LUBBOCK
 DEMO.
 DEMO.
 DATE 2-2-83
 PROD TARG., B/D 2000.
 PUMP SIZE 3 X11-B
 G.O.R. 1250.
 GAS S.G. .908
 PLOT NO. 1 9125
 SEPARATOR PRESS., PSI 40.
 FLOW LINE PRESS., PSI 75.
 WATER CUT 98.
 PRODUCTION H2O S.G. 1.030

PRODUCTION OIL API 42.100
 POWER FL. OIL 42.100
 PIP TARGET, PSI 1500.
 WELL HEAD TEMP, F 90.
 B. H. TEMP, F 184.
 VAPOR PRESSURE, PSI 0.
 VERT DEPTH, FT. 7000.
 TBG LENGTH, FT. 7000.
 CASE I.D., IN. 6.094
 TUBING O.D., IN. 2.375
 COUPLING O.D., IN. 3.063
 TUBING I.D., IN. 1.995
 UPSTRING 7 X 2-3/8
 DOWNSTRING 2-3/8 FIXED

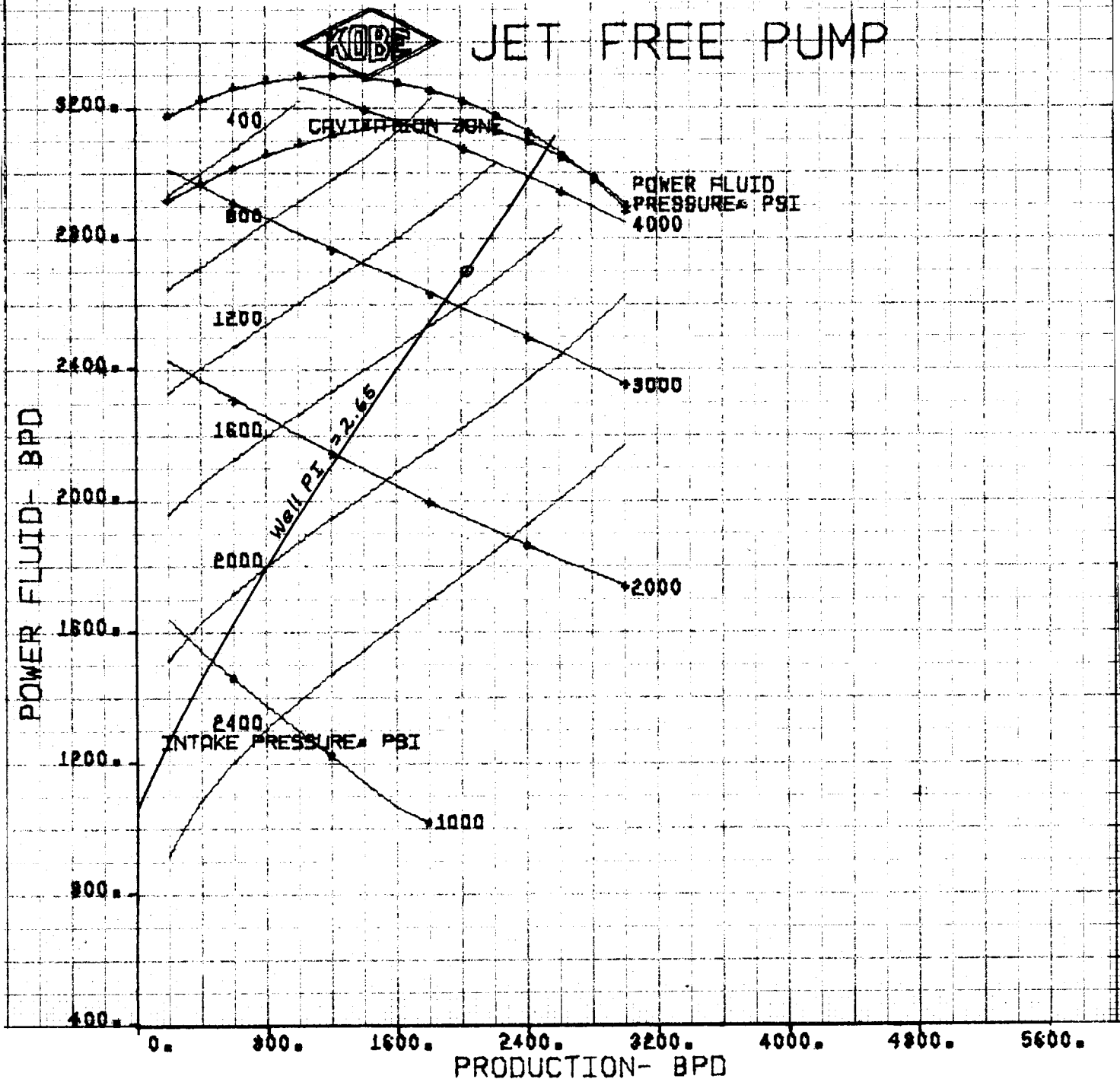
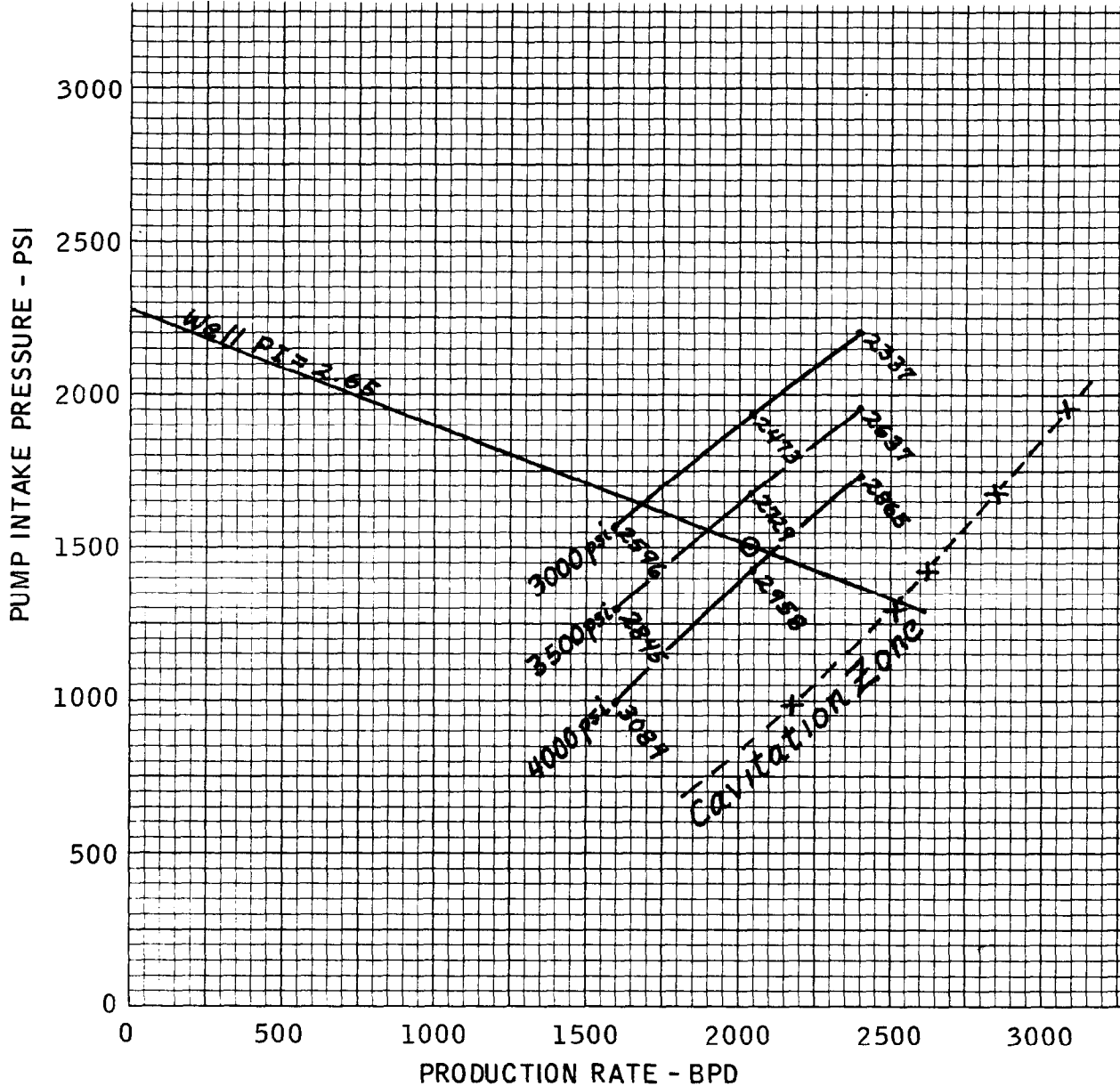


FIGURE 6

11-B 3" JET PUMP PRODUCTION UNIT PERFORMANCE

DEPTH OF PUMP 7000 FT. TUBULARS 2 3/8 x 7" Fixed
 OIL 42.1 °API .8151 S.G. WATER 1.03 S.G. 98 %
 GAS 1250 GOR SCF/BBL POWER FLUID Oil
 WELLHEAD FLOW LINE 75 PSI DATE _____ BY _____

Calculator Prediction for Kobe Well



NOTE - NUMBERS ADJACENT TO CURVES ARE POWER FLUID - BPD
 - JET PUMP OPERATION MUST BE ABOVE & LEFT OF CAVITATION LINE

FIGURE 7

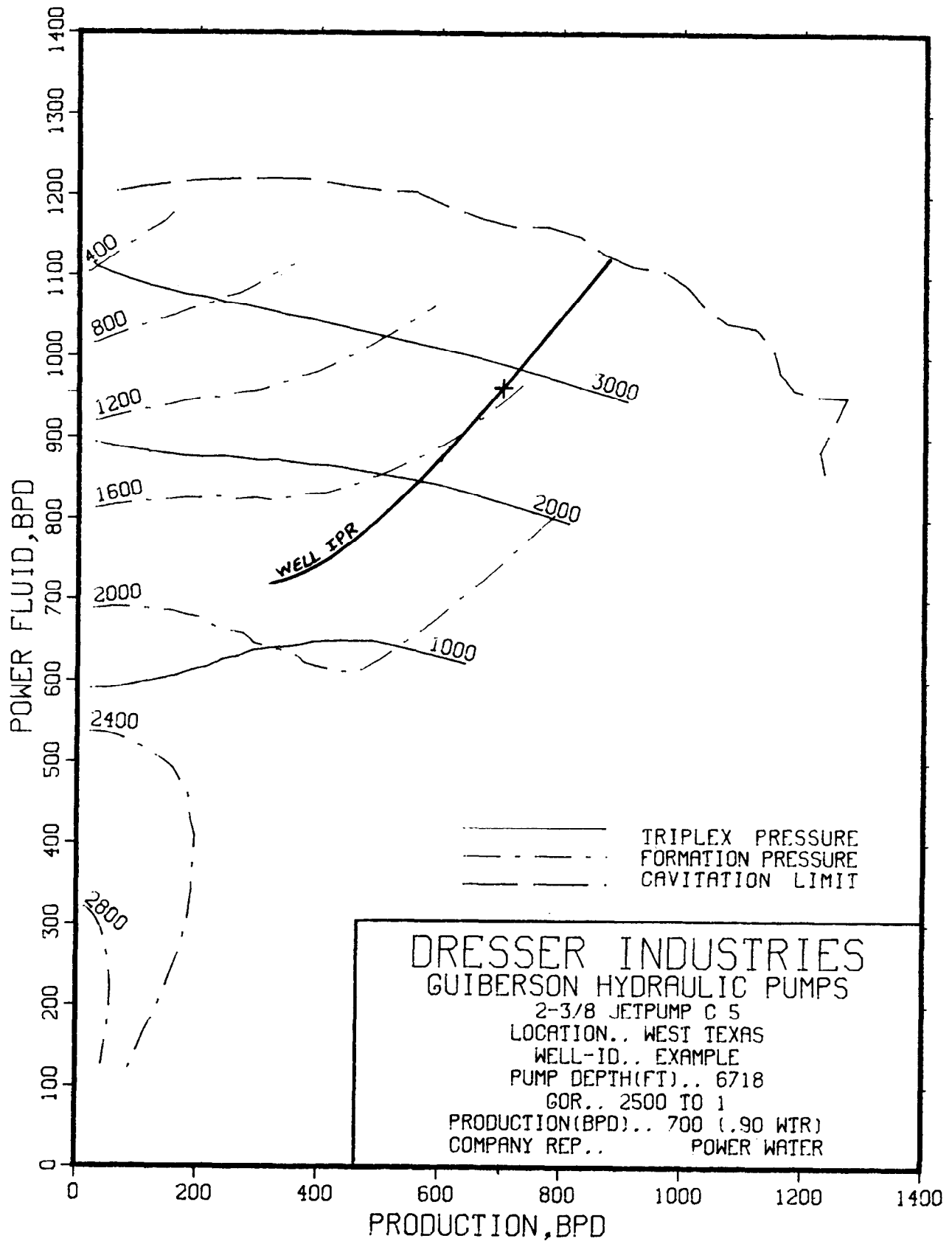
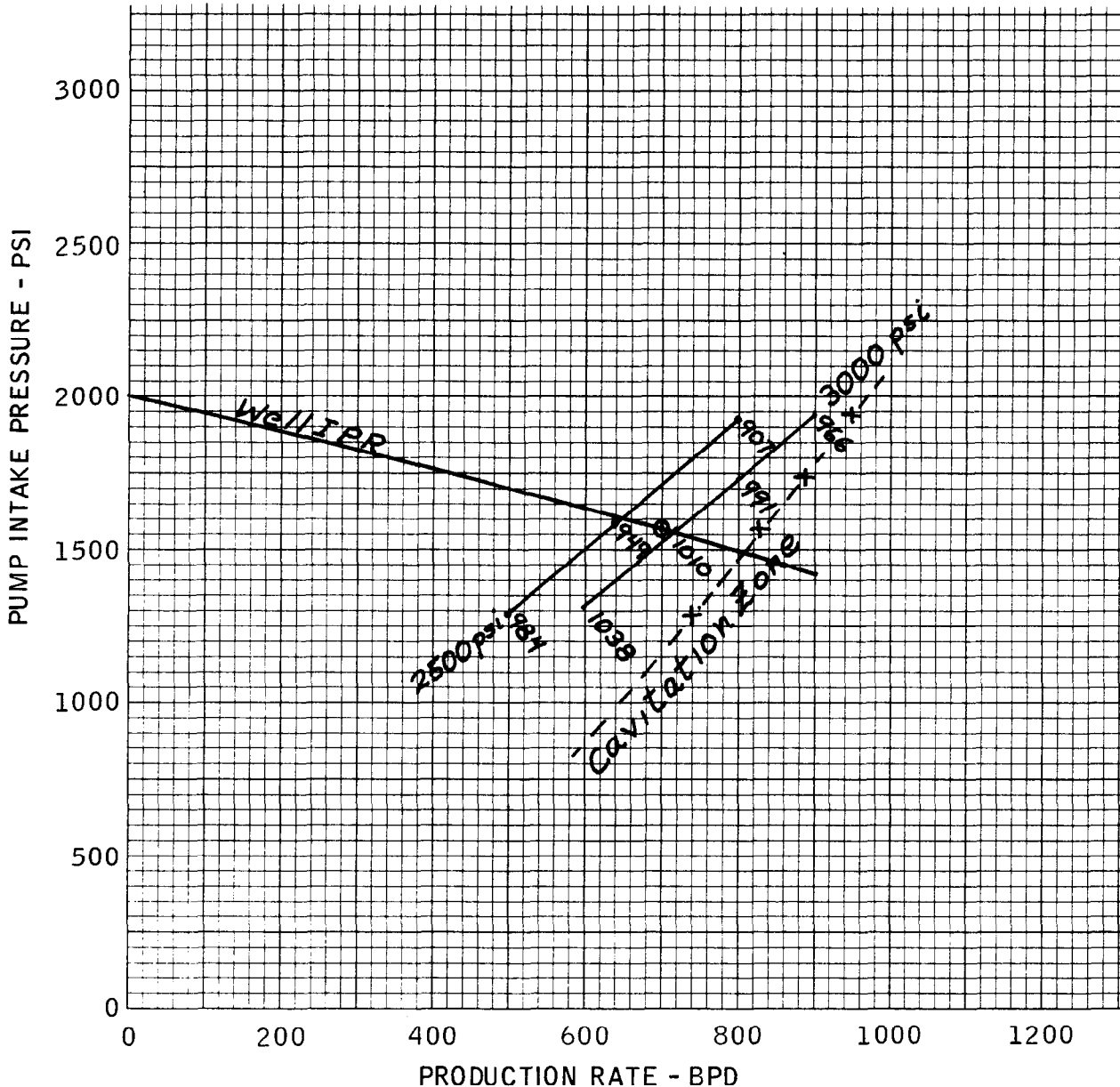


FIGURE 8

C-5 2" JET PUMP PRODUCTION UNIT PERFORMANCE

DEPTH OF PUMP 6718 FT. TUBULARS 2 3/8 x 4 1/2"
 OIL 52 °API .771 S.G. WATER 1.05 S.G. 90 %
 GAS 2500 GOR SCF/BBL POWER FLUID Water
 WELLHEAD FLOW LINE 100 PSI DATE _____ BY _____

Calculator Prediction for Guiberson Well



NOTE - NUMBERS ADJACENT TO CURVES ARE POWER FLUID - BPD
 - JET PUMP OPERATION MUST BE ABOVE & LEFT OF CAVITATION LINE

FIGURE 9

6A JET PUMP PERFORMANCE

NATIONAL WELL

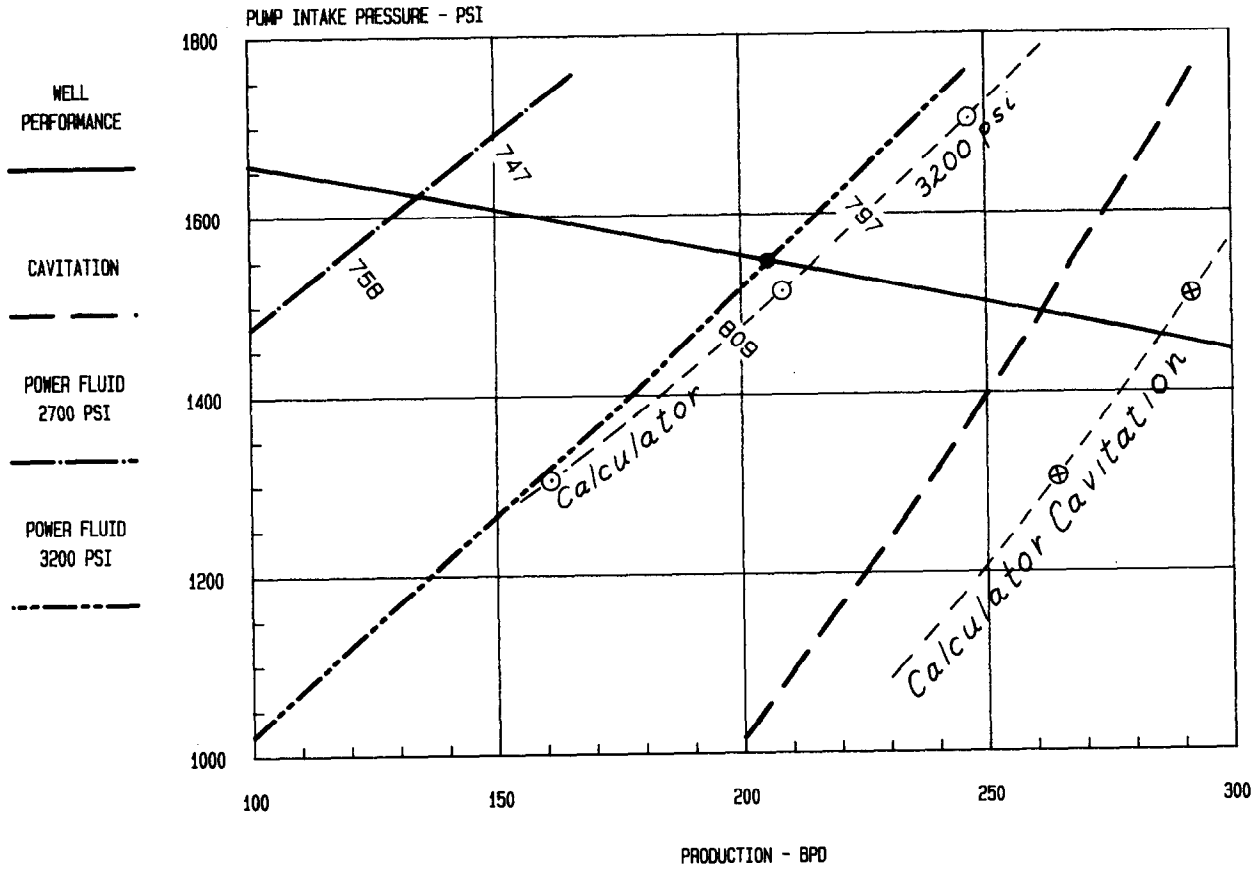


FIGURE 10

PROGRAM DESCRIPTION

1. Input and storage of well data.
2. Calculation and storage of GS, friction constant for return, friction constant for power fluid, (1-Water Cut)
3. Initialize Storage Register 25 as 0.40

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Partition to 719.29	3 OP 17	R/S	719.29
2	Cards 1,2,3 (CLR before each)			1,2,3
3	Start		A	DATA
4	Enter or Accept	Depth	R/S	Depth
5	"	Tbg Length	"	Tbg Length
6	"	Tbg ID	"	Tbg ID
7	" (0 for parallel rtn)	Tbg OD	"	Tbg OD
8	"	Rtn ID	"	Rtn ID
9	"	Flowline Press	"	Flowline Press
10	"	Pwr Grad	"	Pwr Grad
11	"	Oil Grad	"	Oil Grad
12	"	Wtr Grad	"	Wtr Grad
13	"	Oil Visc	"	Oil Visc
14	"	GOR	"	GOR
15	"	Water Cut	"	Water Cut
16	Calculate after entering Water Cut			0
17	Load "Jet Pump Calculation" if data correct. Return to 3 for corrections.			

USER DEFINED KEYS	DATA REGISTERS (INV INV)	LABELS (Op 08)
A Start	⁰ GS	¹⁰ Oil Visc (μ_o)
B	¹ Depth (D)	¹¹ GOR
C	² Tbg Length (L)	¹² Water Cut (WC)
D	³ Tbg ID	¹³ D1P
E	⁴ Tbg OD	¹⁴ D2
A'	⁵ Rtn ID	¹⁵ (D1-D2)
B'	⁶ Flowline Press	¹⁶ (D1 ² - D2 ²)
C'	⁷ Pwr Grad (GN)	¹⁷ Fric Const Rtn
D'	⁸ Oil Grad (GO)	¹⁸ Fric Const Pwr
E'	⁹ Water Grad(GW)	¹⁹ (1-WC)
FLAGS 0	1	2
	3	4
	5	6
	7	8
	9	

FIGURE 11

APPENDIX 1 TI-59 PROGRAM LISTING (PART. 719.29)

This listing is an improved version of the program used for the examples in the text.

282	01	01	355	53	(428	28	28	501	55	+	574	54)	647	55	+
283	69	DP	356	43	RCL	429	33	X²	502	02	2	575	45	YX	648	43	RCL
284	05	05	357	16	16	430	65	X	503	95	=	576	01	1	649	16	16
285	43	RCL	358	65	X	431	43	RCL	504	42	STD	577	93	.	650	65	X
286	15	15	359	43	RCL	432	24	24	505	25	25	578	07	7	651	43	RCL
287	99	PRT	360	01	01	433	33	X²	506	87	IFF	579	09	9	652	20	20
288	03	3	361	54)	434	54)	507	01	01	580	54)	653	95	=
289	03	3	362	95	=	435	55	+	508	44	SUM	581	95	=	654	99	PRT
290	01	1	363	42	STD	436	53	(509	25	CLR	582	42	STD	655	03	3
291	06	6	364	27	27	437	01	1	510	43	RCL	583	29	29	656	04	4
292	07	7	365	76	LBL	438	75	-	511	27	27	584	76	LBL	657	03	3
293	01	1	366	15	E	439	43	RCL	512	75	-	585	33	X²	658	06	6
294	69	DP	367	43	RCL	440	24	24	513	53	(586	03	3	659	69	DP
295	01	01	368	20	20	441	54)	514	43	RCL	587	03	3	660	01	01
296	69	DP	369	65	X	442	33	X²	515	25	25	588	03	3	661	43	RCL
297	05	05	370	43	RCL	443	54)	516	65	X	589	07	7	662	20	20
298	69	DP	371	00	00	444	95	=	517	53	(590	69	DP	663	69	DP
299	00	00	372	55	+	445	75	-	518	43	RCL	591	01	01	664	05	05
300	43	RCL	373	43	RCL	446	53	(519	14	14	592	69	DP	665	99	PRT
301	27	27	374	13	13	447	01	1	520	75	-	593	05	05	666	03	3
302	91	R/S	375	55	+	448	93	.	521	43	RCL	594	43	RCL	667	03	3
303	99	PRT	376	43	RCL	449	02	2	522	27	27	595	29	29	668	03	3
304	42	STD	377	07	07	450	65	X	523	54)	596	99	PRT	669	06	6
305	27	27	378	65	X	451	43	RCL	524	54)	597	03	3	670	69	DP
306	61	GTD	379	53	(452	24	24	525	95	=	598	04	4	671	01	01
307	15	E	380	53	(453	33	X²	526	42	STD	599	03	3	672	43	RCL
308	76	LBL	381	53	(454	65	X	527	21	21	600	01	1	673	21	21
309	14	D	382	43	RCL	455	53	(528	43	RCL	601	69	DP	674	69	DP
310	43	RCL	383	11	11	456	01	1	529	29	29	602	01	01	675	05	05
311	26	26	384	55	+	457	85	+	530	61	GTD	603	43	RCL	676	99	PRT
312	65	X	385	43	RCL	458	43	RCL	531	23	LNx	604	13	13	677	91	R/S
313	93	.	386	21	21	459	28	28	532	76	LBL	605	69	DP	678	76	LBL
314	05	5	387	54)	460	54)	533	44	SUM	606	05	05	679	89	↑
315	05	5	388	45	YX	461	33	X²	534	53	(607	99	PRT	680	53	(
316	85	+	389	01	1	462	54)	535	43	RCL	608	02	2	681	53	(
317	53	(390	93	.	463	95	=	536	27	27	609	03	3	682	43	RCL
318	01	1	391	02	2	464	42	STD	537	75	-	610	03	3	683	21	21
319	75	-	392	65	X	465	28	28	538	43	RCL	611	03	3	684	55	+
320	43	RCL	393	02	2	466	55	+	539	21	21	612	69	DP	685	43	RCL
321	26	26	394	93	.	467	53	(540	54)	613	01	01	686	00	00
322	54)	395	08	8	468	01	1	541	55	+	614	69	DP	687	54)
323	65	X	396	85	+	469	93	.	542	43	RCL	615	05	05	688	34	FX
324	43	RCL	397	01	1	470	00	0	543	25	25	616	43	RCL	689	35	1/X
325	10	10	398	54)	471	03	3	544	85	+	617	13	13	690	55	+
326	55	+	399	65	X	472	75	-	545	43	RCL	618	65	X	691	06	6
327	43	RCL	400	43	RCL	473	43	RCL	546	27	27	619	43	RCL	692	09	9
328	16	16	401	19	19	474	28	28	547	95	=	620	29	29	693	01	1
329	95	=	402	85	+	475	54)	548	61	GTD	621	55	+	694	85	+
330	45	YX	403	43	RCL	476	95	=	549	34	FX	622	05	5	695	53	(
331	93	.	404	12	12	477	42	STD	550	76	LBL	623	02	2	696	43	RCL
332	02	2	405	54)	478	26	26	551	24	CE	624	09	9	697	19	19
333	01	1	406	95	=	479	75	-	552	22	INV	625	01	1	698	65	X
334	65	X	407	42	STD	480	43	RCL	553	87	IFF	626	00	0	699	43	RCL
335	43	RCL	408	28	28	481	25	25	554	01	01	627	95	=	700	11	11
336	16	16	409	25	CLR	482	95	=	555	33	X²	628	99	PRT	701	55	+
337	65	X	410	43	RCL	483	55	+	556	43	RCL	629	03	3	702	02	2
338	53	(411	24	24	484	43	RCL	557	14	14	630	04	4	703	04	4
339	43	RCL	412	65	X	485	25	25	558	75	-	631	03	3	704	06	6
340	15	15	413	02	2	486	95	=	559	53	(632	06	6	705	05	5
341	45	YX	414	95	=	487	50	I×I	560	43	RCL	633	01	1	706	00	0
342	01	1	415	85	+	488	32	X!T	561	07	07	634	05	5	707	55	+
343	93	.	416	53	(489	93	.	562	65	X	635	69	DP	708	43	RCL
344	07	7	417	53	(490	00	0	563	43	RCL	636	01	01	709	21	21
345	09	9	418	01	1	491	00	0	564	01	01	637	69	DP	710	54)
346	54)	419	75	-	492	05	5	565	54)	638	05	05	711	54)
347	65	X	420	43	RCL	493	77	GE	566	85	+	639	71	SBR	712	65	X
348	43	RCL	421	24	24	494	24	CE	567	53	(640	89	↑	713	43	RCL
349	17	17	422	65	X	495	43	RCL	568	43	RCL	641	43	RCL	714	20	20
350	95	=	423	02	2	496	26	26	569	18	18	642	22	22	715	95	=
351	85	+	424	54)	497	85	+	570	65	X	643	75	-	716	42	STD
352	43	RCL	425	65	X	498	43	RCL	571	53	(644	43	RCL	717	16	16
353	06	06	426	53	(499	25	25	572	43	RCL	645	23	23	718	92	RTN
354	85	+	427	43	RCL	500	95	=	573	13	13	646	95	=	719	00	0

FIGURE 15

Appendix 2 HP-41C		INSTRUCTIONS FOR "JET"		SIZE: 030	
Step	Instructions	Input	Function	Display	
1	Set status and key in in the program				
2	Start program		XEQ "JET"	Data	
3	Start Data Entry		R/S		
4	Enter or Accept	Depth	R/S	Depth=___	
5	Enter or Accept	Tbg-Length	R/S	TBG Length=___	
6	Enter or Accept	Tbg-I.D.	R/S	TBG ID=___	
7	Enter or Accept	Tbg-O.D.	R/S	TBG OD=___	
8	Enter or Accept	Return I.D.	R/S	Return ID=___	
9	Enter or Accept	Well Head Press	R/S	Well Head Press=___	
10	Enter or Accept	Pwr Grad.	R/S	PWR Grad=___	
11	Enter or Accept	Prod. Oil Grad.	R/S	Oil Grad=___	
12	Enter or Accept	Water Grad.	R/S	Water Grad=___	
13	Enter or Accept	Oil Visc.	R/S	Oil Visc=___	
14	Enter or Accept	Gas Oil Ratio	R/S	GOR=___	
15	Enter or Accept	Water Cut	R/S	Water Cut=___	
16	Calculates GS, Friction Constants and (1-WC)			Jet	
17	Start Jet Calculation		R/S		
18	Enter or Accept	Prod. Rate	R/S	QS=___	
19	Enter or Accept	Suct. Press	R/S	PS=___	
20	Calculates minimum suction area		R/S	ASM=___	
21	Enter or Accept	Nozzle Area	R/S	AN=___	
22	Enter or Accept	Throat Area	R/S	AT=___	
23	Set flag 01 for operating pressure required at given PS and QS. Do not set Flag 01 for constant operating pressure.	SF01			
24	Enter or Accept	Operating Pressure	R/S	PT=___	
25	It GLR in Return ≥ 10 , Display GLR, WC, GD, QD Enter PD		R/S	GLR=___, WC=___, GD=___, QD=___, PD=?	
26	Displays solution, Operating Press., Nozzle Flow, Horsepower, Cavitation Limit, Prod. Rate, Pump Suction Pressure			PT=___, QN=___, HP=___, QSC=___, QS=___, PS=___	
27	New Point		R/S	Jet(Step 17)	
28	New Data		EXQ"JET"	Data(Step 2)	
STORAGE					
R00	GS	R10	Visc. Oil(μ o)	R20	QS
R01	Depth(D)	R11	Gas Oil Ratio(GOR)	R21	PS
R02	Tbg. Length(L)	R12	Water Cut(WC)	R22	AT
R03	Tbg. I.D.	R13	D1, QN	R23	AN
R04	Tbg. O.D.	R14	D2, PN	R24	R
R05	Rtn. I.D.	R15	(D1-D2), QD	R25	N
R06	Flowline Press(PWH)	R16	(D1 ² -D2 ²), GD, ASM	R26	WCD,N(temp)
R07	Grad.Pwr(GN)	R17	Fric. Const. Rtn.	R27	PD
R08	Grad. Oil(GD)	R18	Fric. Const. Pwr.	R28	M, Numer. N
R09	Grad Water(GW)	R19	(1-WC)	R29	PT

FIGURE 16

APPENDIX 2
HP-41C PROGRAM LISTING (SIZE 030)

This listing is an improved version of the program used for
the examples in the text.

01*LBL "JET"	52 "PROD OIL GRAD="	103 STO 10	154 /	205 "FLAG?"
02 "DATA"	53 ARCL X	104 GTO C	155 .1	206 AVIEW
03 AVIEW	54 AVIEW	105*LBL D	156 Y+X	207 STOP
04 STOP	55 STOP	106 RCL 10	157 /	208 /
05 FIX 0	56 STO 00	107 RCL 00	158 RCL 16	209 STO 24
06 RCL 01	57 RCL 09	108 /	159 ENTER↑	210 FIX 0
07 "DEPTH="	58 "WATER GRAD="	109 .21	160 *	211 RCL 29
08 ARCL X	59 ARCL X	110 Y+X	161 /	212 "PT="
09 AVIEW	60 AVIEW	111 RCL 00	162 RCL 15	213 ARCL X
10 STOP	61 STOP	112 *	163 /	214 AVIEW
11 STO 01	62 STO 09	113 RCL 17	164 RCL 02	215 STOP
12 RCL 02	63 FIX 2	114 *	165 *	216 STO 29
13 "TBG LENGTH="	64 RCL 10	115 STO 10	166 202 E-8	217*LBL 01
14 ARCL X	65 "OIL VISC="	116*LBL C	167 *	218 RCL 07
15 AVIEW	66 ARCL X	117 RCL 05	168 STO 17	219 RCL 01
16 STOP	67 AVIEW	118 STO 13	169 RTN	220 *
17 STO 02	68 STOP	119 RCL 04	170*LBL B	221 +
18 FIX 3	69 STO 10	120 STO 14	171 "JET"	222 RCL 13
19 RCL 03	70 RCL 11	121 -	172 AVIEW	223 1.79
20 "TBG ID="	71 "GOR="	122 STO 15	173 STOP	224 Y+X
21 ARCL X	72 ARCL X	123 RCL 13	174 FIX 0	225 RCL 18
22 AVIEW	73 AVIEW	124 ENTER↑	175 RCL 20	226 *
23 STOP	74 STOP	125 *	176 "QS="	227 -
24 STO 03	75 STO 11	126 RCL 14	177 ARCL X	228*LBL 02
25 RCL 04	76 FIX 2	127 ENTER↑	178 AVIEW	229 STO 14
26 "TBG OD="	77 RCL 12	128 *	179 STOP	230 RCL 21
27 ARCL X	78 "WATER CUT="	129 -	180 STO 20	231 -
28 AVIEW	79 ARCL X	130 STO 16	181 RCL 21	232 RCL 07
29 STOP	80 AVIEW	131 XEQ E	182 "PS="	233 /
30 STO 04	81 STOP	132 1	183 ARCL X	234 SORT
31 RCL 05	82 STO 12	133 RCL 12	184 AVIEW	235 RCL 23
32 "RETURN ID="	83 RCL 03	134 -	185 STOP	236 *
33 ARCL X	84 STO 13	135 STO 19	186 STO 21	237 832
34 AVIEW	85 STO 15	136 RCL 00	187 FIX 4	238 *
35 STOP	86 ENTER↑	137 *	188 XEQ a	239 STO 13
36 STO 05	87 *	138 RCL 09	189 "ASM="	240 RCL 20
37 FIX 0	88 STO 16	139 RCL 12	190 ARCL X	241 +
38 RCL 06	89 XEQ E	140 *	191 AVIEW	242 STO 15
39 "WELL HEAD PRES="	90 .433	141 +	192 STOP	243 1/X
40 ARCL X	91 RCL 07	142 STO 00	193 RCL 23	244 RCL 00
41 AVIEW	92 X<=Y?	143 .2	194 "AN="	245 RCL 20
42 STOP	93 GTO D	144 STO 25	195 ARCL X	246 *
43 STO 06	94 ENTER↑	145 GTO B	196 AVIEW	247 RCL 07
44 FIX 3	95 .55	146*LBL E	197 STOP	248 RCL 13
45 RCL 07	96 X<>Y	147 RCL 16	198 STO 23	249 *
46 "PWR GRAD="	97 /	148 RCL 15	199 RCL 22	250 +
47 ARCL X	98 .21	149 /	200 "AT="	251 *
48 AVIEW	99 Y+X	150 .21	201 ARCL X	252 STO 16
49 STOP	100 *	151 Y+X	202 AVIEW	253 RCL 20
50 STO 07	101 RCL 17	152 RCL 13	203 STOP	254 RCL 12
51 RCL 08	102 *	153 RCL 15	204 STO 22	255 *

FIGURE 17

APPENDIX 2
HP-41C PROGRAM LISTING (SIZE 030)

256 RCL 15	307 STOP	358 RCL 12	409 2	460 AVIEW
257 /	308 STO 27	359 +	410 /	461 STOP
258 STO 26	309 GTO E	360 *	411 STO 25	462 *
259 .43	310*LBL D	361 STO 28	412 FS? 01	463 52910
260 RCL 07	311 RCL 26	362 RCL 24	413 GTO 03	464 /
261 X<=Y?	312 .55	363 2	414 RCL 14	465 "HP="
262 GTO C	313 *	364 *	415 RCL 27	466 ARCL X
263 RCL 13	314 1	365 ENTER↑	416 -	467 AVIEW
264 RCL 15	315 RCL 26	366 ENTER↑	417 RCL 25	468 STOP
265 /	316 -	367 1	418 *	469 RCL 22
266 ST+ 26	317 RCL 10	368 X<>Y	419 RCL 27	470 RCL 23
267*LBL C	318 *	369 -	420 X<>Y	471 -
268 RCL 19	319 +	370 RCL 28	421 -	472 XEQ a
269 RCL 20	320 RCL 16	371 RCL 24	422 STO 21	473 /
270 *	321 /	372 *	423 RCL 29	474 RCL 20
271 RCL 11	322 .21	373 X+2	424 GTO 01	475 *
272 *	323 Y↑X	374 *	425*LBL 03	476 "QSC="
273 RCL 15	324 RCL 16	375 1	426 RCL 27	477 ARCL X
274 /	325 *	376 RCL 24	427 RCL 21	478 AVIEW
275 10	326 RCL 15	377 -	428 -	479 STOP
276 X>Y?	327 1.79	378 X+2	429 RCL 25	480 RCL 20
277 GTO D	328 Y↑X	379 /	430 /	481 "QS="
278 "GLR="	329 *	380 +	431 RCL 27	482 ARCL X
279 ARCL Y	330 RCL 17	381 1.2	432 +	483 AVIEW
280 AVIEW	331 *	382 RCL 24	433 GTO 02	484 STOP
281 PSE	332 RCL 06	383 X+2	434*LBL 04	485 RCL 21
282 PSE	333 +	384 *	435 FS? 01	486 "PS="
283 FIX 2	334 RCL 16	385 1	436 GTO 05	487 ARCL X
284 RCL 26	335 RCL 01	386 RCL 28	437 GTO 06	488 AVIEW
285 "MC="	336 *	387 +	438*LBL 05	489 STOP
286 ARCL X	337 +	388 X+2	439 RCL 14	490 GTO B
287 AVIEW	338 STO 27	389 *	440 RCL 07	491*LBL a
288 PSE	339*LBL E	390 -	441 RCL 01	492 RCL 00
289 PSE	340 RCL 20	391 ENTER↑	442 *	493 RCL 21
290 RCL 16	341 RCL 00	392 ENTER↑	443 -	494 /
291 "GD="	342 *	393 1.03	444 RCL 13	495 SORT
292 ARCL X	343 RCL 13	394 X<>Y	445 1.79	496 691
293 AVIEW	344 /	395 -	446 Y↑X	497 /
294 PSE	345 RCL 07	396 /	447 RCL 18	498 RCL 19
295 PSE	346 /	397 STO 26	448 *	499 RCL 11
296 FIX 0	347 RCL 11	398 RCL 25	449 +	500 *
297 RCL 15	348 RCL 21	399 -	450 STO 29	501 24650
298 "QD="	349 /	400 RCL 25	451*LBL 06	502 /
299 ARCL X	350 1.2	401 /	452 RCL 29	503 RCL 21
300 AVIEW	351 Y↑X	402 ABS	453 "PT="	504 /
301 PSE	352 2.8	403 .005	454 ARCL X	505 +
302 PSE	353 *	404 X>Y?	455 AVIEW	506 RCL 20
303 RCL 27	354 1	405 GTO 04	456 STOP	507 *
304 "PD=?"	355 +	406 RCL 26	457 RCL 13	508 STO 16
305 ARCL X	356 RCL 19	407 RCL 25	458 "QH="	509 RTN
306 AVIEW	357 *	408 +	459 ARCL X	510 .END.

FIGURE 18

TABLE 1
NOZZLE AND THROAT SIZES

KOBE			NATIONAL			GUIBERSON		
Nozzle #	Area	Throat # Area	Nozzle #	Area	Throat # Area	Nozzle #	Area	Throat # Area
1	.0024	1 .0060	1	.0024	1 .0064	DD	.0016	000 .0044
2	.0031	2 .0077	2	.0031	2 .0081	CC	.0028	00 .0071
3	.0040	3 .0100	3	.0039	3 .0104	BB	.0038	0 .0104
4	.0052	4 .0129	4	.0050	4 .0131	A	.0055	1 .0143
5	.0067	5 .0167	5	.0064	5 .0167	B	.0095	2 .0189
6	.0086	6 .0215	6	.0081	6 .0212	C	.0123	3 .0241
7	.0111	7 .0278	7	.0103	7 .0271	D	.0177	4 .0314
8	.0144	8 .0359	8	.0131	8 .0346	E	.0241	5 .0380
9	.0186	9 .0464	9	.0167	9 .0441	F	.0314	6 .0452
10	.0240	10 .0599	10	.0212	10 .0562	G	.0452	7 .0531
11	.0310	11 .0774	11	.0271	11 .0715	H	.0661	8 .0661
12	.0400	12 .1000	12	.0346	12 .0910	I	.0855	9 .0804
13	.0517	13 .1292	13	.0441	13 .1159	J	.1257	10 .0962
14	.0668	14 .1668	14	.0562	14 .1476	K	.1590	11 .1195
15	.0863	15 .2154	15	.0715	15 .1879	L	.1963	12 .1452
16	.1114	16 .2783	16	.0910	16 .2392	M	.2463	13 .1772
17	.1439	17 .3594	17	.1159	17 .3046	N	.3117	14 .2165
18	.1858	18 .4642	18	.1476	18 .3878	P	.3848	15 .2606
19	.2400	19 .5995	19	.1879	19 .4938			16 .3127
20	.3100	20 .7743	20	.2392	20 .6287			17 .3750
		21 1.0000						18 .4513
		22 1.2916						19 .5424
		23 1.6681						20 .6518
		24 2.1544						
Nozzle	Throat	R	Nozzle	Throat	R	Guiberson Ratios Listed in Table 2		
N	N-1	.517 A-	N	N-1	.483 X			
N	N	.400 A	N	N	.380 A			
N	N+1	.310 B	N	N+1	.299 B			
N	N+2	.240 C	N	N+2	.235 C			
N	N+3	.186 D	N	N+3	.184 D			
N	N+4	.144 E	N	N+4	.145 E			

TABLE 2 - GUIBERSON RATIOS AND THROAT ANNULUS AREAS (in²)

Nozzle DD	Throats	000	00						
	R	.36	.22						
	AS	.0028	.0056						
CC	Throats	000	00	0	1				
	R	.64	.40	.27	.20				
	AS	.0016	.0043	.0076	.0115				
BB	Throats	00	0	1	2				
	R	.54	.37	.27	.20				
	AS	.0032	.0065	.0105	.0150				
A	Throats	0	1	2	3				
	R	.53	.39	.29	.23				
	AS	.0048	.0088	.0133	.0185				
B	Throats	0	1	2	3	4	5	6	
	R	.92	.66	.50	.40	.30	.25	.21	
	AS	.0009	.0048	.0094	.0145	.0219	.0285	.0357	
C	Throats	1	2	3	4	5	6	7	
	R	.86	.65	.51	.39	.32	.27	.23	
	AS	.0020	.0066	.0118	.0191	.0257	.0330	.0408	
D	Throats	3	4	5	6	7	8	9	
	R	.74	.56	.46	.39	.33	.27	.22	
	AS	.0064	.0137	.0203	.0276	.0354	.0484	.0628	
E	Throats	4	5	6	7	8	9	10	11
	R	.77	.63	.53	.45	.36	.30	.25	.20
	AS	.0074	.0140	.0212	.0290	.0420	.0564	.0722	.0954
F	Throats	6	7	8	9	10	11	12	
	R	.69	.59	.48	.39	.33	.26	.22	
	AS	.0138	.0217	.0346	.0490	.0648	.0880	.1138	
G	Throats	8	9	10	11	12	13	14	
	R	.68	.56	.47	.38	.31	.26	.21	
	AS	.0208	.0352	.0510	.0742	.1000	.1320	.1712	
H	Throats	10	11	12	13	14	15	16	
	R	.69	.55	.45	.37	.30	.25	.21	
	AS	.0302	.0534	.0792	.1112	.1504	.1945	.2467	
I	Throats	11	12	13	14	15	16	17	
	R	.72	.59	.48	.40	.33	.27	.23	
	AS	.0339	.0597	.0917	.1309	.1750	.2272	.2895	
J	Throats	13	14	15	16	17	18	19	
	R	.71	.58	.48	.40	.34	.28	.23	
	AS	.0515	.0908	.1349	.1871	.2493	.3256	.4167	
K	Throats	15	16	17	18	19	20		
	R	.61	.51	.42	.35	.29	.24		
	AS	.1015	.1537	.2160	.2922	.3833	.4928		
L	Throats	16	17	18	19	20			
	R	.63	.52	.44	.36	.30			
	AS	.1164	.1787	.2549	.3460	.4555			
M	Throats	17	18	19	20				
	R	.66	.55	.45	.38				
	AS	.1287	.2050	.2961	.4055				
N	Throats	18	19	20					
	R	.69	.57	.48					
	AS	.1395	.2306	.3401					
P	Throats	19	20						
	R	.71	.59						
	AS	.1575	.2670						

TABLE 3 KOBE						
NOZZLE	THROAT ANNULUS AREA (IN ²)					
	A-	A	B	C	D	E
1		.0036	.0053	.0076	.0105	.0143
2	.0029	.0046	.0069	.0098	.0136	.0184
3	.0037	.0060	.0089	.0127	.0175	.0231
4	.0048	.0077	.0115	.0164	.0227	.0308
5	.0062	.0100	.0149	.0211	.0293	.0397
6	.0080	.0129	.0192	.0273	.0378	.0513
7	.0104	.0167	.0248	.0353	.0488	.0663
8	.0134	.0216	.0320	.0456	.0631	.0856
9	.0174	.0278	.0414	.0589	.0814	.1106
10	.0224	.0360	.0534	.0760	.1051	.1428
11	.0289	.0464	.0690	.0981	.1358	.1840
12	.0374	.0599	.0891	.1268	.1749	.2382
13	.0483	.0774	.1151	.1633	.2265	.3076
14	.0624	.1001	.1482	.2115	.2926	.3974
15	.0806	.1287	.1920	.2731	.3780	.5133
16	.1036	.1668	.2479	.3528	.4881	.6629
17	.1344	.2155	.3203	.4557	.6304	.8562
18	.1735	.2784	.4137	.5885	.8142	1.1058
19	.2242	.3595	.5343	.7600	1.0516	1.4282
20	.2896	.4643	.6901	.9817	1.3583	1.8444

TABLE 4 - NATIONAL						
NOZZLE	THROAT ANNULUS AREA (IN ²)					
	X	A	B	C	D	E
1		.0040	.0057	.0080	.0108	.0144
2	.0033	.0050	.0073	.0101	.0137	.0183
3	.0042	.0065	.0093	.0129	.0175	.0233
4	.0054	.0082	.0118	.0164	.0222	.0296
5	.0068	.0104	.0150	.0208	.0282	.0377
6	.0087	.0133	.0191	.0265	.0360	.0481
7	.0111	.0169	.0243	.0338	.0459	.0612
8	.0141	.0215	.0310	.0431	.0584	.0779
9	.0179	.0274	.0395	.0548	.0743	.0992
10	.0229	.0350	.0503	.0698	.0947	.1264
11	.0291	.0444	.0639	.0888	.1205	.1608
12	.0369	.0564	.0813	.1130	.1533	.2046
13	.0469	.0718	.1035	.1438	.1951	.2605
14	.0597	.0914	.1317	.1830	.2484	.3316
15	.0761	.1164	.1677	.2331	.3163	.4223
16	.0969	.1482	.2136	.2968	.4028	.5377
17	.1234	.1888	.2720	.3779	.5128	
18	.1571	.2403	.3463	.4812		
19	.2000	.3060	.4409			
20	.2546	.3896				

TABLE 5 — SAMPLE TI-59 TAPES

DATA ENTRY	JET WITH FLAG SET	JET WITHOUT FLAG 3000 psi	JET WITHOUT FLAG 3500 psi
DATA	JET	JET	JET
DEPTH	QS 500.	QS 500.	QS 500.
TBG LENGTH	PS 1000.	PS 1000.	PS 800.
TBG ID	ASM 0141220402	ASM .0141220402	ASM .0157889209
TBG OD	AN 0.0055	AN 0.0055	AN 0.0055
RETURN ID	AT 0.0241	AT 0.0241	AT 0.0241
WELL HEAD PRESS	FLAG?	FLAG?	FLAG?
100.	PT 3000.	PT 3000.	PT 3500.
PWR GRAD	PT 3283.531495	PT 3000.	PT 3500.
0.353	QN 489.	QN 462.	QN 507.
PROD OIL GRAD	HP 30.40739505	HP 26.24379796	HP 33.56968078
0.353	QSC 658.5450736	QSC 702.9793516	QSC 627.1974491
WATER GRAD	QS 500.	QS 500.	QS 500.
0.446	PS 1000.	PS 1139.	PS 907.
OIL VISC			
2.5			
GOR			
0.			
WATER CUT			
0.3			

TABLE 6 — WELL DATA FOR NATIONAL WELL

Pump Depth	9706 ft	Oil Viscosity	7 cp
Tubing Length	9706 ft	Gas-Oil Ratio	1000 scf/BBL
Tubing I.D.	1.900 in	Water Cut	.19
Tubing O.D.	2.375 in	Bottom Hole Temp.	182° F
Return I.D.	6.500 in	Well Head Temp	80° F
Well Head Press.	120 psi	Static Bottom Hole Press	1800 psi
Power Fluid	30° API	Flowing Bottom Hole Press	1550 psi
Oil Gravity	30° API	Production Rate	195-205 BPD
Water Gravity	.455 psi/ft	Gas Gravity	.65

TABLE 7 — WELL DATA FOR GUIBERSON WELL

Pump Depth	6718 ft	Oil Viscosity	1.6 cp
Tubing Length	6718 ft	Gas-Oil Ratio	2500 scf/BBL
Tubing I.D.	1.995 in	Water Cut	.90
Tubing O.D.	2.375 in	Bottom Hole Temp	120° F
Return I.D.	4.05 in	Well Head Temp	80° F
Well Head Press.	100 psi	Static Bottom Hole Press.	2000 psi
Power Fluid	Water	Flowing Bottom Hole Press	1566 psi
Oil Gravity	52° API	Production Rate	700 BPD
Water Gradient	.455 psi/ft	Gas Gravity	.85