HYDRAULIC FRACTURING MODELING; PAST, PRESENT, FUTURE

Ali Rezaei, Mehdi Rafiee and M. Y. Soliman
Texas Tech University,
Steven Mores, Statoil Gulf Services LLC

ABSTRACT

Hydraulic fracturing simulations initiated after early years of the first application of this technology in the oil industry. Since then, several methodologies have been examined to model this complicated process. These approaches resulted in significant progresses in the modeling of hydraulically created fractures. Features such as out of plane propagation, proppant transport, fluid flow, thermal stress, etc., were added to the models and made them more reliable. As the number of stimulated wells increased due to combination of hydraulic fracturing and horizontal drilling in unconventional reservoirs, prediction of final fracture geometry before the actual operation became necessary. Thus, an effective model is needed to obtain a successful hydraulic fracture treatment.

Two dimensional hydraulic fracture models were between the early analytical models for predicting the final fracture geometry. Inefficiency of these models in predicting fracture geometry in reservoirs with complex layers caused the development of so called P3D models. Later on, P3D short comes resulted in the development of fully 3D models. Several 3D models have been developed since then by coupling fluid flow equations with fracture mechanics. Although 3D models give more accurate results than others, out of plane propagation is not considered in majority of them. They simply ignore the near-wellbore effects of deviated wells and assume a planar starting crack that has extended beyond this region. This problem was solved later using true 3D models that still suffer expensive computation time.

In this study, a historical background of hydraulic fracture modeling is presented. An overview of analytical models is presented first. Then, improvements in numerical modeling within last decades are investigated in detail. Common assumptions in modeling of hydraulic fractures are presented and mathematical equations for each assumption is explained. Coupling of rock behavior equations with fluid flow equations is described. In summary, suggestions for potentially future improvements in modeling are highlighted.

INTRODUCTION

Hydraulic fracturing is the process of creating a fracture in the rock using a pressurized fluid. The technique has been used for more than 60 years in petroleum engineering to increase production from low permeability reservoirs and has wide applications in other geomechanics fields such as disposal of waste drill cuttings, heat production from geothermal reservoirs, fault reactivation and measurement of in situ stresses. Another application of hydraulic fracturing is in unconventional reservoirs such as tight gas, CBM, and shale. The potential of these reservoirs in production of oil and gas makes it imperative that oil and gas industry find the best plausible technique to exploit these reservoirs efficiently and economically. Hydraulic fracturing of tight gas started in late 1970's and proved to be the best way to produce from unconventional reservoirs. Fracturing of shale formations started in the mid 90's and it is shown that the performance of these reservoirs is highly dependent on design of the hydraulic fracturing [1].

Typically, hydraulic fracturing process starts with pumping a high-pressure fluid to pressurize a sealed section of the wellbore and break the rock. By injecting more fluids into the newly created crack, it tends to propagate or turn into the direction of maximum horizontal far field stress. As the hydraulic fracture reaches its final length, a fine grain material, called 'proppant' is placed in the fracture in order to keep the newly created fracture open after the treatment is finished. Hydraulic fracturing can be used to bypass the induced damage from the wellbore or to increase production of tight reservoirs such as shale.

For many reasons (e.g. high cost of the fracturing fluid and proppant), estimation of the final fracture geometry is needed prior to the real fracturing job. Therefore, prediction of the growth behavior of hydraulic fracture in a specific time domain became the objective of large number of studies since the early years of its application. In majority of these studies, a fluid pressure or an injection rate at the wellbore is specified and fracture dimension, proppant placement and fracture closure pressure is calculated. The solution for these problems is one of the key components that is needed to properly design the hydraulic fracturing process. Solving these problems require a comprehensive understanding of the processes that are involved in a typical hydraulic fracturing process.

Generally, hydraulic fracturing is a three dimensional problem which, even in its most basic form, requires the coupling of at least five practices including: (i) mechanical response of the rock due to hydraulic load on its surface; (ii) pressure distribution caused by a fluid motion inside the fracture; (iii) Fracture propagation regime; (iv) fracturing fluid diffusion into the rock (Leak off); (v) thermal exchange between fluid and reservoir rock and fluid. The problem can be much more complicated by taking into account the effect of other components of a real hydraulic fracturing process. Composite layering effect, multi layers formation, far-field stress magnitude/direction change, and presence of a free surface, natural fractures etc. are some examples of such components that may or may not exist during hydraulic fracturing. In addition to the components discussed earlier, uncertainty of the problem tends to the classification of the problem as "data limited" based on Holling's classification [2]. All of these processes make hydraulic fracturing a complex problem in which obtaining a closed form analytical solution may not be possible. Therefore, some of these components are neglected and a relatively simplified hydraulic fracturing problem is usually studied.

HISTORY AND BACKGROUND

The first non-acid hydraulic fracturing treatment for reservoir stimulation was tested in Hugoton field in Kansas in 1947 on a gas well [3]. Currently, tens of thousands of wells are stimulated hydraulically to ensure the economical production. Most of these reservoirs need an expensive mass injection of fluid and sand (or proppant), therefore an optimized hydraulic fracture design is crucial.

Hydraulic fracture models can broadly classify into two man categories; Analytical modeling and numerical simulations. Early models were mainly analytical and starting from 60's the numerical simulations became impotent. Generally speaking, besides of the method used for solving such a problem, there are four main processes to be addressed that are fracture initiation, fracture propagation, fluid flow and proppant transport, and fluid leak-off into the rock matrix.

In fracturing, a portion of fluid leaks off into formation due the nature of porous rock that is under pressure. Depending on permeability and effective porosity, the leaked off fluid can increase the pore pressure near the fracture surface. Therefore, compressive stress on the surface of the fracture will increase. This may be accounted for by incorporating the poroelasticity law. However, this phenomenon is almost transient in low permeable rocks with milidarcy permeability range. Thus, neglecting the poroelasticity in fracture modeling is not far from reality in these reservoirs and thus, is not considered in this work.

ANALYTICAL MODELS

The first simple analytical models were developed in 1950s in an effort to obtain a close form solution for the geometry of hydraulic fracture based on reservoir and fracturing fluid properties and as a function of pumping parameters and the state of stress [4]. There are three main types of 2D analytical models that have been used for quick fracture analysis: i) PKN model, ii) KGD model, and iii) Radial model. One of the pioneer works in this area was the paper published by Pekins and Kern [5], which introduced so called PK model. This model is composed of a vertical ellipse and a horizontal ellipse. Nordgren [6], improved PK model to account for the effect of fluid loss. The modified model is called PKN. Since PKN model defines the fracture into vertical sections that act independently, the fracture tip becomes very important in the calculation of propagation path. Figure 1 shows a schematic of PKN model geometry.

Another well-known model was developed by Khristianovic and Zheltov [7] and Geertsma and de Klerk [8]. This model, called KGD [7], describes the fracture into horizontal layers, emphasizing on the importance of fracture height (Figure 2). PKN model is suitable for larger fracture length/height ratio while KGD models are applicable to short fractures where plain strain conditions are pertinent in horizontal sections [9]. Using 2D fracture models assuming a constant height in each step, fracture length and width can be calculated (Figure 4 and Figure 5). Both, Perkins and Kern and Geertsma and de Klerk derived radial solutions for their models. These modes are known as radial PKN and radial KGD respectively.

The equations of radial fracture geometry, with assumption of constant pressure was solved by Sneddon [10]. Radial models, assume a penny shape fracture with constant pressure (Figure3). Several variations of these analytical models have been developed that account for a power-law fluid [11], leak off and fracture toughness [12], an impermeable rock [13]; a zero toughness and a leak-off dominated case [14]. These models are different also in other aspects. PKN model is appropriate when the length of the fracture is much higher than its height. On the other hand, KGD model is useful for the cases where the fracture length is short and horizontal plain strain is considered.

The radial model is applicable, when pressure distribution is constant and injection is a point source. Although, the usage of 2D analytical models reduced in 1990s due to emerging of pseudo 3D models, they are still used for the purpose that discussed earlier. A shortcoming of 2D models is that they assume that the height of the fracture is constant. Therefore, height growth cannot be estimated using these models.

Pseudo 3D models were developed in 1980s by removing the assumption of the constant height [15, 16]. In pseudo 3D solution height is a function of fracture half-length and elapsed time. The assumption in these models is that fracture length is greater that its height. Another major difference between P3D models and 2D models is the two dimensional fluid flow in the fracture. Fluid flow in the 2D models in one dimensional along the fracture length, but in 3 D fracture length also the vertical fluid movement is considered. There are two basic Pseudo 3D models: cell based and lumped models. In the cell based approach, the length of the fracture is divided into PKN type rectangles with their own heights (Figure 6), while in the lumped models, fracture consists of two half ellipses on top of each other (Figure 7). A detailed comparison between 2D and P3D fracture models can be found in Warpinski et a [17]. P3D models like 2D models cannot estimate the geometry of fractures with arbitrary shapes and direction. They assume that the fracture a simple 2 wing fracture initiated and propagating in direction of maximum horizontal stress. In order to predict the geometry of fractures other than this situation a three-dimensional fracture is needed.

First attempts for developing three dimensional models started in late 1980s [4]. The first developed models for this purpose were planar 3D models [18–22] because they couldn't predict the out of plane propagation. These models presented the fracture surface by a moving triangular grid (Figure 8) or a fixed rectangular mesh (Figure 9). Later on out of plane propagation added to the planar 3D models [21, 23, 24]. Also, the effect of near wellbore region modeled using a 3D simulator by Carter et al. [25].

In recent years, combination of the horizontal drilling technology and horizontal completion has opened up a large potential of hydrocarbon production in uncongenial reservoirs. Hydraulic fracturing in these reservoirs is much more complex compared to vertical wells. Usually horizontal wells are completed by placement of multiple fractures. Each of these fractures affects others and results in the change of stress distribution in the system. The complex state of stress created by presence of multiple fractures results in complex propagation of the fractures in the system and may cause impeding the injection of proppant and leading to premature Screenout [26]. Therefore, a fully 3D hydraulic fracture simulator is needed.

NUMERICAL SIMULATIONS

Analytical models cannot be used for more complicated fracture geometries. Thus, a numerical method is usually used for solving the problem of fracture with complicated geometries. The most used numerical methods for solving hydraulic fracture problems are finite element method, extended finite element method, boundary element method, and finite difference method. Examples of their application in hydraulic fracturing is abundant and can be found in the literature (see for example [21, 27] for finite difference, and [23, 28–30] for boundary element method).

Each of these methods has their advantages and disadvantages. Finite element method has the advantage of efficiency in modelling of mediums with anisotropy, heterogeneity and non-linearity. The matrices are symmetric in this method but because of the need for discretizing the rock domain, the amount of computation to solve the resulting system of equations is prohibitively excessive. Another disadvantage of this method is in problems with moving boundaries. Since the boundaries of the fracture move when propagation occurs at the end of each time step, re-meshing of the whole domain is needed which is inherently inefficient. Extended finite element method is a modified version of finite element method that is more efficient than the original FEM. In this method, no modification to discretization is required; mesh is created independent of the fracture geometry and its location. Also, the convergence rate is improved compared to regular finite element method.

Another widely used numerical method for hydraulic fracturing is boundary element method in which the equations are derived from boundary integral equation. In this method, only boundary of the crack needs to be discretized which results in reducing the dimension of the problem by one. A special form of boundary element method, called "displacement discontinuity method" [31] thought to be more suited to this kind of problem. In this method, only the surface of the fracture needs to be discretized and the main unknowns are fracture opening and stress change.

ASSUMPTIONS IN MODELLING HYDRAULIC FRACTURES

Before clarifying assumptions in hydraulic fracturing, mathematical equations that are used to model hydraulic fracture behavior should be understood. There are five equations that govern hydraulic fracturing behavior; i) the elasticity equation, ii) fluid flow equation, iii) leak-off equation, iv) proppant transport equation, v) fracture propagation equation. Elasticity equation is the mechanical response of the rock due to a hydraulic load on the fracture surface. Fluid flow equation is mass conservation, which relates mass entering fracture to pressure on the surface of the fracture. Leak off equations describe the effect of fracturing fluid entering to the reservoir due to difference between injection pressure and pore fluid pressure. Propanat transport describe the mass concentration of the proppant with time and in each position inside the fracture. Probably, the most important equations in hydraulic fracturing are the equations for fracture growth. Fracture propagation criteria, direction of propagation, and rate of the propagation are governed by these equations. Solution of hydraulic fracturing problem requires coupling of these five equations (if more complications be ignored) in the same model. Location of the fracture front, fracture width, fluid pressure, and proppant concentration during treatment are parameters that should be known during treatment. Having all of these equations together makes the model inefficient. Therefore, some simplifications are needed in order to get the required solution.

First assumption in problem of hydraulic fracturing is that the rock layers are parallel to each other having different mechanical properties. Usually rock is assumed to be linearly elastic and natural discontinuities (i.e. natural fractures, faults, etc.) are ignored. Reservoir medium is presumed homogeneous and stress field is in the reservoir is the same everywhere. Also, the effect of stress change due to poroelastic effect and preexisting fractures are neglected. Majority of modeling approaches assume that hydraulic fracture is two symmetric wing fracture. Few approaches consider fracture propagation in a system of natural fractures. Fracturing fluid is usually considered incompressible and fluids are assumed to be immiscible. Typically a simple fluid rheology such as Newtonian or power law is used for the fluid flow inside the fracture. However, the actual fracturing fluid in some practices has more complicated rheologies. Commonly, a simple one-dimensional is used for leak off effect and interaction between proppant particles is neglected.

LINEAR ELASTICITY EQUATIONS

In order to understand the mechanism of hydraulic fracturing, two critical key factors, i.e. rock mechanics and fluid mechanics, should be understood well. Rock mechanical properties that is described by elasticity equation dictate the stress and strain distribution in the rock body, and fluid pressure is the controlling factor of change in stresses and displacements [32]. Combination of rock and fluid mechanics control the fracture geometry. Therefore, these two factors are the key elements that should be considered as a priority in hydraulic fracturing. Elasticity equation relates the displacement of the rock to the pressure on the surface of the fracture due to injection of the fracturing fluid. The integral form of the elasticity equation can be written as [4]:

$$C_w = \int C(x,y;\xi,\eta)\, w(\xi,\eta,t)\, d\xi\, d\eta = p(x,y,t) - \sigma_c(x,y) \tag{1}$$

where p is the fluid pressure, σ_c is confining stress, and w is the fracture width. C is the kernel function that contains the information about elastic medium. For simplified fracture geometries and considering some simplifications, equation (1) can be solved analytically. For more complicated forms a numerical method is used to solve the equation.

FRACTURE PROPAGATION

Study of fracture propagation is the most important part of every hydraulic fracture model because of the dynamic essence of problem. Final estimation of the dimension of hydraulic fracture depends on the chosen fracture propagation mode. Generally, three different fracture propagation modes exist in fracture mechanics, namely, Mode I or opening mode, Mode II or sliding mode, and Mode III or tearing mode. Figure 10 shows a schematic of these three propagation modes.

There are three different fracture propagation criteria to predict the direction of propagation in fracture mechanics; Maximum tensile stress criterion, or σ-criterion [33]; Minimum strain energy density criterion or S-criterion [34]; Maximum energy release rate criterion or G-criterion [35]. Modified G criteria for predicting mixed fracture propagation direction was introduced by Shen et al. [36]. Behnia [37] compared the result of these three propagation criteria. The result of their study is shown in the Figure 11. Because of its simplicity and reasonable results, σ-criterion seems to be most popular criterion in mixed mode fracture propagation studies. This criterion is capable of predicting the mixed mode (shear and tensile) fracture propagation.

Stress intensity factor and fracture toughness are two parameters that control initiation and propagation of fracture. Similar to the relationship between stress and strength, fracture toughness is the critical form of stress intensity factor. Certain failure criterion needs to be used in order to predict the fracture propagation direction. For each step of loading on fracture, stress intensity factor should be examined to see if the propagation occurs. Since the geometry of the problem continually changes during the propagation process, the stress intensity factor needs to be calculated numerically.

In a system of uniaxial loading, a fracture will propagate along a straight line perpendicular to the direction of applied load when:

$$K_I > K_{IC} \tag{2}$$

In a problems with a complex load, the mixed mode fracture propagation criterion introduced by Erdogen [33] may be used to predict both shear and tensile fracture propagation. The condition for crack propagation based on this criterion is:

$$\cos\frac{\theta}{2}\left[K_I cos^2\frac{\theta}{2} - \frac{3}{2} K_{II}\, sin\frac{\theta}{2}\right] = K_{Ic} \tag{3}$$

where θ is crack propagation angle and is defined as:

$$\theta = 2 \arctan[\frac{1}{4}\left(\frac{K_I}{K_{II}}\right) - \frac{1}{4}\sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8} \tag{4}$$

At the crack tip the following condition should be satisfied:

$$K_I \sin\theta_0 + K_{II}\,(3\,cos\theta_0 - 1) = 0 \tag{5}$$

FLUID FLOW WITHIN THE FRACTURE AND LEAK OFF

Hydraulic fracturing may be considered as two parallel irregular surfaces in which the width varies along the length as a result of a fracturing fluid pressure gradient along the fracture. The fluid flow inside the fracture by itself is a complex problem because of a non-Newtonian nature of the fracturing fluid and the possibility of reaching turbulent flow. Considering the fracturing fluid diffusion into the rock matrix and the heat transfer between fracturing and pore fluid would add to the complexity of the problem. A non-linear differential equation form of lubrication theory usually models the fluid flow inside fracture. These equations relate the fluid velocity, crack opening and pressure profile inside the fracture.

Reynolds' equation is used for solving fluid flow equation in the fracture [4]. Two-dimensional Reynold's equation is written as:

$$\frac{\partial w}{\partial t} = \nabla.[D(w)(\nabla p - \rho g)] + \delta(x,y)Q \tag{6}$$

where ρ is the fluid density, g is the gravitational acceleration, δ is the Dirac delta function, Q is the injection rate, and . D equals to:

$$D(w) = \frac{w^3}{12\,\mu} \tag{7}$$

where μ is the Newtonian fluid viscosity. If the effect of leak off be considered, equation (6) we be:

$$\frac{\partial w}{\partial t} = \nabla.[D(w)(\nabla p - \rho g)] + \delta(x,y)Q - \frac{2\,C_L}{\sqrt{t - t_0(x,y)}} - 2s_0\delta(t - t_0(x,y)) \tag{8}$$

where C_L is the Carter leak off coefficient and $t_0(x,y)$ is the time for starting leak off and:

$$D(w,p) = N'\, w^{\frac{(2n'+1)}{n'}}|\nabla p - \rho g|^{\frac{(1-n')}{n'}} \tag{9}$$

$$N' = \left(\frac{n'}{2n'+1}\right)\left(\frac{1}{2^{n'+1}K'}\right) \tag{10}$$

$$|\nabla p - \rho g| = \sqrt{\left(\frac{\partial p}{\partial x} - \rho g_x\right)^2 + \left(\frac{\partial p}{\partial y} - \rho g_y\right)^2} \tag{11}$$

where for a Newtonian fluid n'=1 and $K' = \mu$. In most of the hydraulic fracturing simulators it is assumed that the leak off is one dimensional and it is orthogonal to the fracture surface [4]. Fluid flow equations are usually solved using finite difference method. The simplest case is for one-dimensional fluid flow inside the fracture and assuming a Newtonian fluid. Since steady state condition is assumed, fluid injection is assumed to be constant and the net pressure is assumed to be zero at the tip of fracture as the result of the fluid lag Figure 12. In the lag area, which is located at the tip element, fluid pressure is zero. Boundary conditions for the problem are:

$$q(0,t) = q_i = constant \tag{12}$$

$$p_f(x_l,t) - \sigma_n = 0 \tag{13}$$

where σ_n is far-field stress normal to the fracture surface.

PROPPANT TRANSPORT

Proppant transport in the fracture is usually modeled by assuming a mixture of solid particles (proppant) and fracturing fluid. Therefore, fluid flow equations discussed in the previous section needs to be modified for the presence of proppant. The solution for proppant transport is usually given by the volumetric concentration of solid as the function of time and location [38]. For the modelling of proppant in the fracture three assumptions are considered; i) proppant and fracturing fluid are incompressible, ii) proppant size is small compared to the width of the fracture, iii) if the gravity force be neglected, proppant and fluid move at the same velocity [4].

COUPLING OF THE FLUID FLOW AND ROCK MECHANICAL DEFORMATION

Fracturing fluid affects the fracture behavior in two ways; it may increase the fracture width or even cause the fracture propagation. On the other hand change in fracture dimension results in change of fluid pressure. Therefore, two-way interaction between fluid and rock should be understood in order to solve problems of hydro-mechanical coupling correctly. This interaction is the key factor of coupling part of any hydraulic fracture model.

Interaction between rock and fluid may be solved mathematically in two different ways, implicitly and explicitly. In the first method equations of fluid flow and rock mechanics are solved together. Solutions that are based on Darcy's law are solved in this way. Finite element is the common numerical method that is used for this kind of solution. Zhang [39] used boundary element method for solving this equations. The second method, which is simpler than implicit method, is the method in which equations of fluid flow and rock mechanics are solved separately, then solutions are coupled explicitly. Answer of each part effect the solution for the other part. Therefore, an iteration process is needed between fluid flow solutions and rock mechanical solutions. The computational time for the second method is longer than the first method that is a disadvantage for this method.

An example of the explicit method for coupling solutions obtained from a boundary element method and finite difference method is given below. As discussed earlier displacement discontinuity method is widely accepted for fracture problem. In this example this method is used to calculate the displacement and stresses on the boundary of a fracture due to an applied load. A standard forward finite difference is also used to relate the injection of the fracturing fluid to the distribution of the pressure in the fracture. The procedure for iteration is given below.

NUMERICAL CONSIDERATIONS

Two adjacent displacement discontinuity elements are considered as two hydraulic domains that are connected hydraulically. Figure 13 shows how elements are considered for hydro-mechanical coupling. Fluid flow is proportional to the pressure difference between two domains. Pressure and width are calculated at each iteration as shown in Figure 14. Because of symmetry, only one wing of fracture is shown here. As it may be seen at end of each iteration process, pressure and width distribution inside fracture are calculated. Iteration scheme shown in Figure 15 and the procedure below needs to be followed in order to get a correct answer.

A constant flow rate is injected from the middle of fracture (first element at the beginning of each half-length). On the other hand net pressure is zero in last element. q unlike pressure and width, will be measured at beginning and end of each element, i.e. beginning and end of each hydraulic domain.

Two main parts of coupling process are change of fracture width as the result of fluid pressure on fracture surface and change in pressure inside the fracture that is related to fracture width. The iteration process for solving these two unknowns is as follows.

Step 1 - The injection takes place at that wellbore is at the middle of fracture. Fluid flow inside fracture is calculated using cubic law. Since width of fracture is assumed to be zero at the first time step, an arbitrary fluid pressure profile is assumed inside the fracture. An arbitrary time step also will be chosen and it controls the amount of fluid that inters the fracture.

Step 2 - Injection of fluid into fracture changes the fluid pressure. Pressure profile will change from maximum at the wellbore to zero at the fracture tip. Pressure distribution is used in this step to calculate the new fracture width using the geomechanical solver. Displacement discontinuity method is used again to calculate the width in this stage.

Step 3 - As the result of pressure change inside fracture, width of fracture will change. Change in the fracture width, will be given to fluid flow solver to calculate the new pressure profile. A comparison between two sets of pressures/displacements indicates the end of iteration process, i.e. whether a convergence is reached. A relaxation factor can be used in this section in order to shorten the iteration process.

Step 4 - Changes in fracture width results in fracture volume change. New fracture volume, hence changes the fluid pressure and therefore a new pressure profile needs to be calculated again. In this step injected volume and fracture volume is compared in order to make sure the correct time step were chosen. Time step is adjusted and all the iteration process is repeated again.

After reaching convergence in each iteration, pressures and displacements will be checked for propagation. If propagation condition is met, an element is added to the fracture tip as discussed in chapter three. Iteration should be repeated again to get the new fracture pressures/width. This step is the end of one time step and all the procedure is needed to be followed again for next time step.

Fluid Time Step -A proper time step is needed for the iteration described above, otherwise in next time step fluid flow will be reversed and numerical solution would not converge. To achieve the convergence in each time step, pressure difference of a domain must be less than original pressure difference between two domains [40]. This can be expressed as below:

$$\Delta P_1 = \frac{Q\,\Delta t}{l\,w} < \Delta P_1 = Q\frac{12\,\mu l}{w^3}$$

where l is domain (element) length

SUMMARY

A review on the problem of hydraulic fracture modeling presented in this paper. General assumptions that are considered in modeling of hydraulic fracture discussed. The equations used for modeling of hydraulic fracture problems briefly reviewed. It showed that there are at least five equations that need to be solved simultaneously in order to get a reliable solution to the problem of hydraulic fractures. Coupling part is believed to be the main constraint for solving hydraulic fracturing problems. One solution to this problem may be the implicit iteration of equations that needs derivation of new equations. It is believed that the process of hydraulic fracture in not understood thoroughly yet. Therefore, another newly area in hydraulic fracture modeling that is propagation of hydraulic fractures in the systems of natural fractures skipped in this study. Finally, some features in hydraulic fracturing modeling that has not been answered yet is introduced:

- Current linear elastic hydraulic fractures are not efficient in modeling of coal bed methane fracturing and a new approach is needed for modeling of these formations.
- Propagation of hydraulic fractures in unconsolidated sand stone cannot be done using current hydraulic fracture simulators. Research on this area has been started with a limited success in modeling of these reservoirs
- More researches on the cohesive zone in front of the fracture is necessary. Usually, this region is implemented in hydraulic fracture models by assuming a fluid lag region.

- A proper model for study the behavior of hydraulic fractures approaching a natural discontinuity is needed.
- As mentioned in the text, coupling of different equations is the main obstacle to have a sophisticated hydraulic fracture. current fully three dimensional are computationally expensive therefore, a faster numerical method is required to have more efficient simulator

REFERENCES

1. Soliman MYY, Daal J, East L. Fracturing unconventional formations to enhance productivity. J Nat Gas Sci Eng [Internet]. 2012 Sep [cited 2015 Jan 16];8:52–67. Available from: http://www.sciencedirect.com/science/article/pii/S187551001200008X

2. Starfield AM, Cundall PA. Towards a methodology for rock mechanics modelling. Int J Rock Mech Min Sci Geomech Abstr [Internet]. 1988 Jun;25(3):99–106. Available from: http://www.sciencedirect.com/science/article/pii/0148906288922929

3. Howard GC, Fast CR. Hydraulic Fracturing: Monograph 2 of the Henry L. Doherty Series. New York: Society of Petroleum Engineers of AIME; 1970.

4. Adachi J, Siebrits E, Peirce a., Desroches J. Computer simulation of hydraulic fractures. Int J Rock Mech Min Sci. 2007 Jul;44(5):739–57.

5. PERKINS TK, KERN LR. Widths of Hydraulic Fractures. Journal of Petroleum Technology 1961.

6. Nordgren R. Propagation of a vertical hydraulic fracture. Soc Pet Eng J. 1972;

7. Khristianovic S, Zheltov Y. Formation of vertical fractures by means of highly viscous fluids. Proc 4th World Pet Congr Rome [Internet]. Rome; 1955;5:579–86. Available from: http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Formation+of+vertical+fractures+by+means+of+highly+viscous+liquid#0

8. Geertsma J, Klerk F De. A rapid method of predicting width and extent of hydraulically induced fractures. J Pet Technol. 1969;

9. Adachi JI, Detournay E. Self-similar solution of a plane-strain fracture driven by a power-law fluid. Int J Numer Anal Methods Geomech. 2002;26(December 2001):579–604.

10. Sneddon I, Elliot H. The opening of a Griffith crack under internal pressure. Appl Math. 1946;4:262–7.

11. Daneshy A. On the Design of Vertical Hydraulic Fractures. Journal of Petroleum Technology. 1973.

12. Spence DA, Sharp P. Self-Similar Solutions for Elastohydrodynamic Cavity Flow. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences. 1985. p. 289–313.

13. Detournay E, Garagash DI. The near-tip region of a fluid-driven fracture propagating in a permeable elastic solid. J Fluid Mech. 2003;494:1–32.

14. Lenoach B. The crack tip solution for hydraulic fracturing in rock of arbitrary permeability. Proc Rock Mech Pet Eng [Internet]. 1994;391–7. Available from: http://www.spe.org/elibrary/servlet/spepreview?id=00028075

15. Settari A, Cleary MP. Development and Testing of a Pseudo-Three-Dimensional Model of Hydraulic Fracture Geometry. SPE Prod Eng. Society of Petroleum Engineers; 1986;1.

16. Morales RH, Abou-Sayed AS. Microcomputer Analysis of Hydraulic Fracture Behavior With a Pseudo-Three-Dimensional Simulator. SPE Production Engineering. 1989.

17. Warpinski NR, Moschovidis Z a., Parker CD, Abou-Sayed IS. Comparison Study of Hydraulic Fracturing Models—Test Case: GRI Staged Field Experiment No. 3 (includes associated paper 28158). SPE Prod Facil. 1994;9(3):7–18.

18. Clifton RJ, Abou-Sayed AS. On the computation of the three-dimensional geometry of hydraulic fractures. Proc SPE Symp on Low Permeability Gas Reservoirs, Denver. 1979. p. 307–13.

19. Barree RD. A Practical Numerical Simulator for Three-Dimensional Fracture Propagation in Heterogeneous Media. SPE Reservoir Simulation Symposium [Internet]. 1983. Available from: http://www.spe.org/elibrary/servlet/spepreview?id=00012273

20. Touboul E, BenNaceur K, Thiercelin M. Variational Methods in the Simulation of Three-Dimensional Fracture Propagation. The 27th US Symposium on Rock Mechanics (USRMS). 1986. p. 659–68.

21. Advani SH, Lee TS, Lee JK. Three dimensional modeling of hydraulic fractures in layered media: {F}inite element formulations. ASME J Energy Res Tech. 1990;112:1–18.

22. Gu H, Leung KH. 3D Numerical Simulation of Hydraulic Fracture Closure With Application to Minifracture Analysis. J Pet Technol. 1993;45(March):206–12.

23. Vandamme L, Schlumberger D, Curran JH, Toronto U. Pressure Distribution in Three- Dimensional Hydraulic Fractures. 1988;181–6.

24. Sousa JLS, Carter BJ, Ingraffea AR. Numerical Simulation of {3D} Hydraulic Fracture Using {N}ewtonian and Power-Law Fluids. Int J Rock Mech Min Sci. 1993;30:1265–71.

25. Carter BJ, J D, Ar I, PA W. Simulating fully 3D hydraulic fracturing [Internet]. Modeling in …. 2000. Available from: http://www.cfg.cornell.edu/~bruce/papers/modeling_geomechanics_2000_carter_etal.pdf

26. Soliman M, Hunt J, Azari M. Fracturing Horizontal Wells in Gas Reservoirs. SPE Prod Facil. 1999;14(November):277–83.

27. Boone TJ. An investigation of poroelastic effects related to hydraulic fracture propagation in rock and stress measurement techniques. 1989;(2):73–80.

28. Detournay E, Fairhurst C. Two-dimensional elastoplastic analysis of a long, cylindrical cavity under non-hydrostatic loading. Int J Rock Mech Min Sci Geomech Abstr [Internet]. 1987;24:197–211. Available from: http://www.sciencedirect.com/science/article/pii/0148906287901756

29. Scavia C. A numerical technique for the analysis of cracks subjected to normal compressive stresses. Int J Numer methods … [Internet]. 1992;33. Available from: http://onlinelibrary.wiley.com/doi/10.1002/nme.1620330504/abstract

30. Ghassemi a. Three Dimensional Poroelastic Hydraulic Fracture Simulation Using Displacement Discontinuity Method. 1996.

31. Crouch SL, Starfield SL. Boundary Element Methods in Solid Mechanics~: With applications in rock mechanics and geological engineering. 1983.

32. Valko P, Economides MJ. Hydraulic Fracture Mechanics.

33. Erdogan F, Sih GCC. On the Crack Extension in Plates Under Plane Loading and Transverse Shear. J Basic Eng. 1963;85:519.

34. Sih GCC. Strain-energy-density factor applied to mixed mode crack problems. Int J Fract. 1974;10(3):305–21.

35. Palaniswamy K, Knauss WG. Propagation of a crack under general, in-plane tension. Int J Fract Mech. 1972;8:114–7.

36. Shen B, Stephansson O. Numerical Analysis of Mixed Mode I and Mode II Fracture Propagation. Int J Rock Mech Min Sci Geomech Abstr. 1993;30:861–7.

37. Behnia M, Goshtasbi K, Marji MF, Golshani a. On the crack propagation modeling of hydraulic fracturing by a hybridized displacement discontinuity/boundary collocation method. J Min Env [Internet]. 2011;2(1):1–16. Available from: http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:On+the+crack+propagation+modeling+of+hydraulic+fracturing+by+a+hybridized+displacement+discontinuity/boundary+collocation+method#0

38. Drew JE, Passman SL. Theory of Multicomponent Fluids. 1998.

39. Zhang X, Jeffrey RG. The role of friction and secondary flaws on deflection and re-initiation of hydraulic fractures at orthogonal pre-existing fractures. Geophys J Int. 2006;166:1454–65.

40. Shen B, Guo H, Ko T, Lee S. Coupling rock-fracture propagation with thermal stress and fluid flow. Int J …. 2012;(December):794–808.

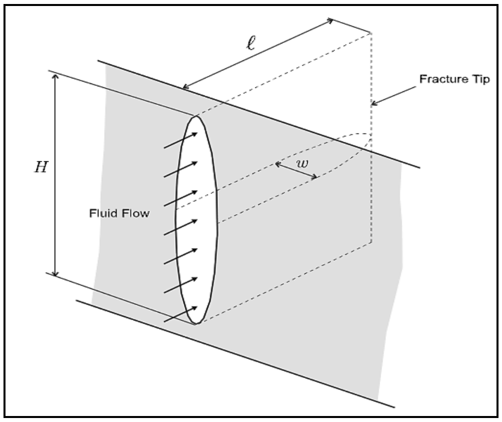

Figure 1 - PKN geometry

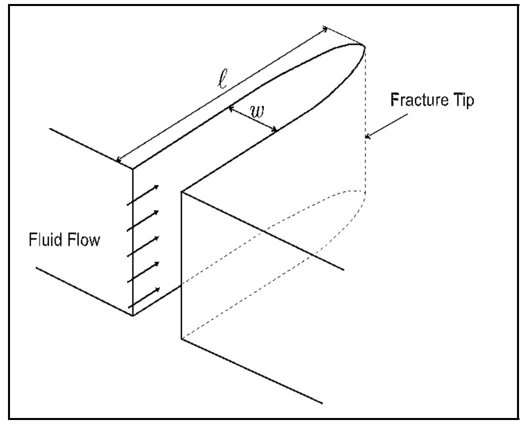

Figure 2 - KGD geometry

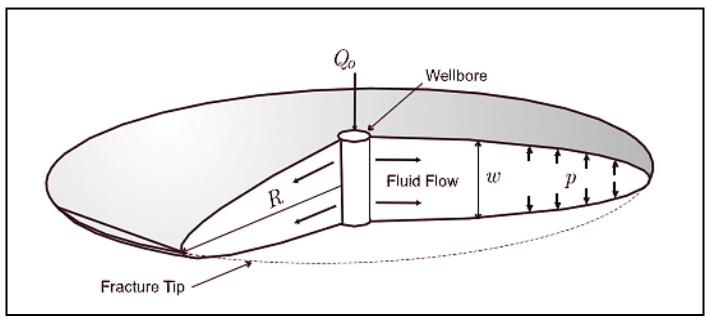

Figure 3 - Radial geometry

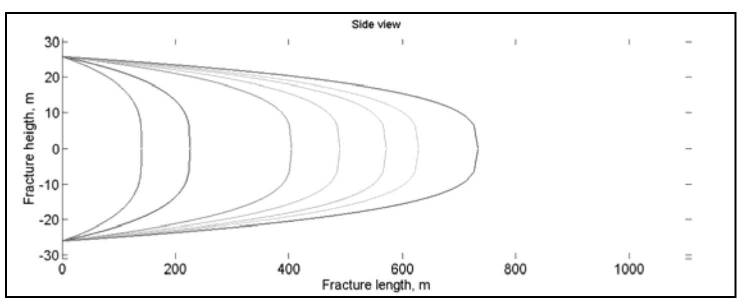

Figure 4 - PKN fracture propagation (side view)

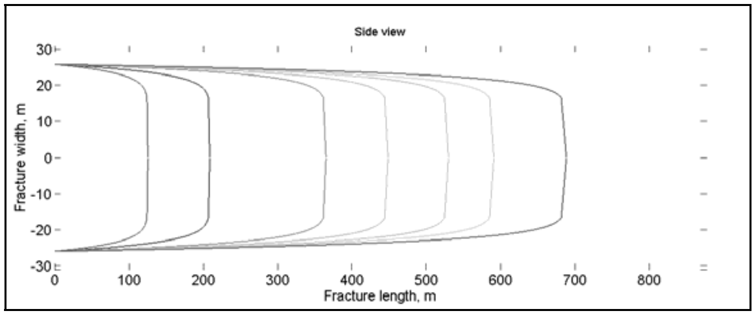

Figure 5 - KGD fracture propagation (side view)

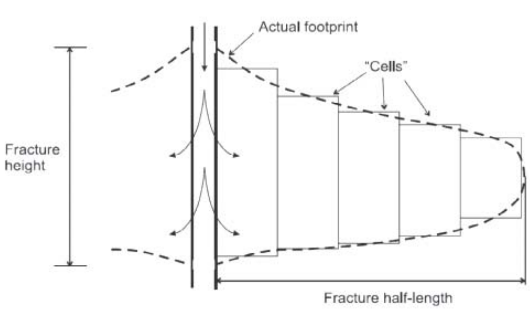

Figure 6 - Cell-based pseudo 3D model

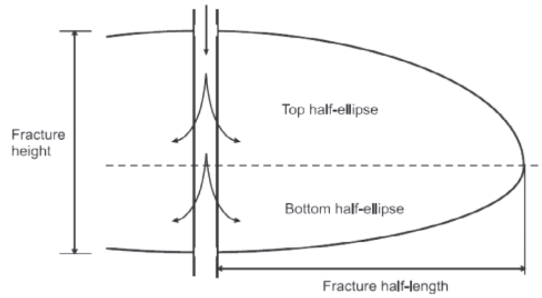

Figure 7 - Lumped elliptical pseudo 3D model

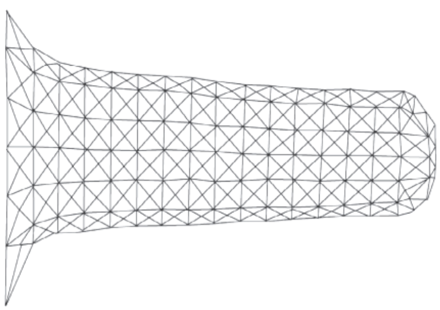

Figure 8 - Planar 3D model using moving grid

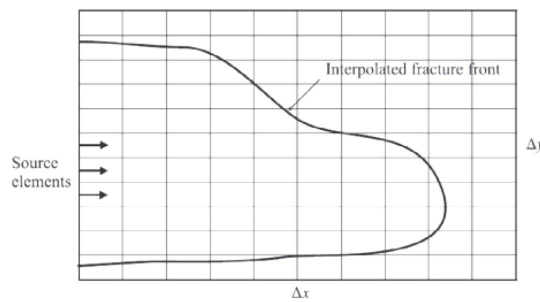

Figure 9 - Planar 3D model using fixed grid

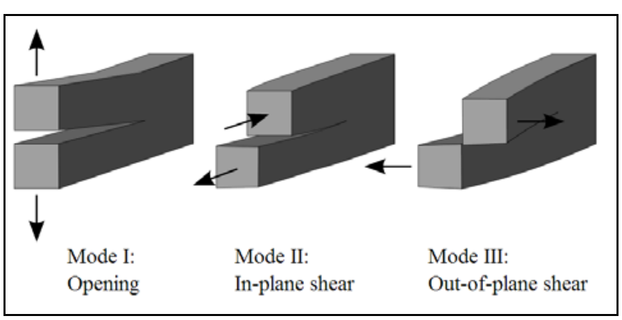

Figure 10 - Different fracture propagation modes

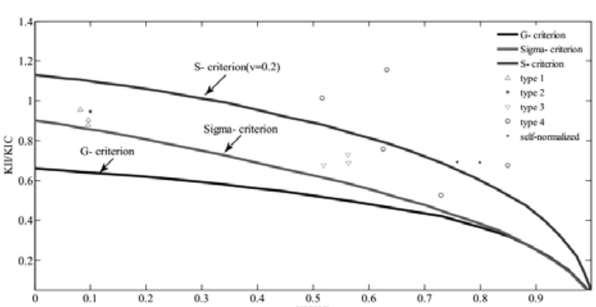

Figure 11 - Comparison of different fracture propagation criteria (after [37])

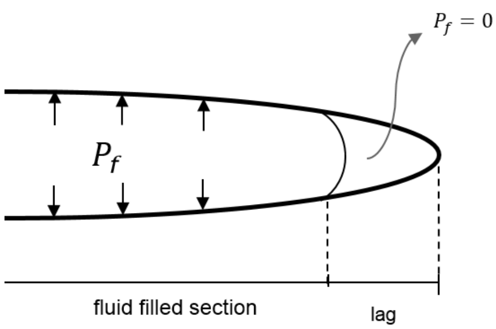

Figure 12 - The fluid filled section of the crack and a lag

Figure 13 - Fluid flow between two adjacent elements

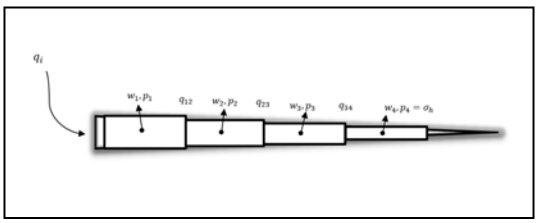

Figure 14 - The discretization of a fracture to elements used for coupling

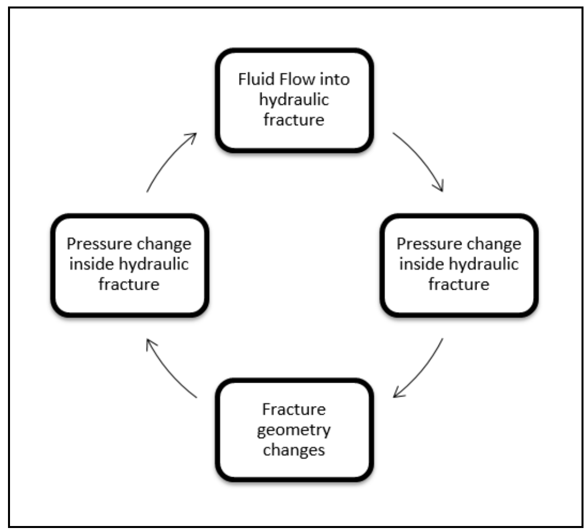

Figure 15 - Hydro-mechanical iteration process for coupled model