

SOLVING ROD BUCKLING

Greg L. Mendenhall
Texland Petroleum Inc.

R.E. Ott
Mobil E&P U.S. Inc.

ABSTRACT

Producing companies and operators are universally concerned with reducing failures in rod pumping systems. Costly failures effect a company's profitability. A leading cause of rod pumping failures is rod buckling. This paper discusses the problems, causes, and solutions of sucker rod buckling. Various methods of determining the buckling force are presented. These methods include computer modelling, physical downhole measurement, and empirical models. Furthermore, a calculation scheme is detailed for predicting buckling modes, pitches, bending moments, and peak stresses in the rod string. Buckling hysteresis and friction are also discussed. The results of the buckling predictions are supported by actual stress measurements from a downhole load cell tool. Lastly, considerations for preventing severe rod buckling are offered. All material is presented for practical field application.

INTRODUCTION

When analyzing or designing a sucker rod string, most engineers consider the maximum loading of each rod taper. This loading is evaluated with respect to tensile strength and fatigue. The maximum loading normally occurs in the top rods of each taper. However, when buckling is exhibited in the rod string, bottom rod loading can approach or surpass the axial loads experienced in any taper's top rod. The buckling is revealed in the form of severe rod coupling or tubing wear and possibly rod parts from fatigue. For this reason, rod buckling must be approached methodically by rod system designers.

Many misconceptions exist in the understanding of rod buckling. Rod buckling is sometimes considered solely a function of rod string dynamics. Also, rod buckling is often attributed to hydrostatic pressure within the tubing string. In reality, buckling is straight forward and defined by a very specific set of characteristics. Buckling is dependent on the rod string's stability and is defined multi-axially. When buckling is properly understood, the information is valuable in optimizing pumping systems and decreasing rod failures.

UNDERSTANDING BUCKLING

In an attempt to quantify buckling, the effecting factors must be ascertained. Rod dynamics alone do not define buckling. The rod's characteristics relate to both the pump's dynamics

and rod inertia. The velocity and acceleration of the pump, as well as the fluid properties and pump clearance define the major contributing compressive load which can cause buckling. Therefore, the work accomplished by Lubinski and others, showing that hydrostatic effects within the tubing string (acting on the rods) negate any true compressive force, dispels buckling's reliance on hydrostatic pressure. In other words, the compressive stress acting to compress the rod exactly equals the compressive stress which tends to straighten the rod. For this reason, hydrostatic pressure has no effect in the consideration of buckling. The compressive force which directly effects buckling is related mainly to the resistance offered at the pump. This force most greatly impacts the rod above the pump. The compressive load created through rod string inertia during force wave propagation has a secondary effect. This loading appears higher in the rod string. Once the compressive load is found, a meaningful buckling analysis can be accomplished.

FINDING THE LOAD

Now comes the million dollar question. What is the magnitude of the compressive load which effects buckling? This value of course depends upon the specific rod installation and is determined in one of three ways. The compressive load can be measured, modelled, or empirically determined. The most accurate means is measurement of the loading as a function of time and/or position at the pump (Figure 1). Measurement is accomplished through the use of a downhole load cell device (DHLC). The authors have extensively used this technique in the evaluation of buckling as well as comparisons of commercial software programs. However, the use of the DHLC is both time consuming and expensive for routine analysis. For this reason, modelling is a popular alternative (Figure 2). Modelling is effective when software programs have been calibrated through comparison with the DHLC or experience. Modelling is considered a viable alternative to actual measurement. Lastly, the least expensive and time consuming method of determining buckling force is through empirical data. This method involves analyzing polished rod velocity and acceleration. The pump dynamics are then related and a compressive load is estimated. This method is not recommended because the complexity of rod string dynamics is ignored. Through one of these three methods, the maximum compressive load is determined and used in the buckling calculations.

DEFINING BUCKLING

When modelling a rod string, stresses are generally viewed uni-axially (stress along the rod axis). Buckling analysis, on the otherhand, is a tri-axial problem. Three stress components exist. In cylindrical coordinates, the stresses are axial, radial, and tangential to the rod string (Figure 3). The axial stress occurs along the rod's axis and is caused by tensile and compressive loads. The radial stress occurs normal to the rod's body and is caused by deflection of the rod from the straight, vertical axis. The tangential stress occurs tangentially to the rod body and is caused by torsion. In this discussion of rod buckling, the tangential stress component is considered negligible, since rod strings are initially straight (or semi-

straight), no restoring moment exists within the pump, stresses usually remain in the elastic range of the material, and friction is negated by vibration. Therefore, the problem of buckling analysis is accurately described on a bi-axial basis. Figure 4 graphically displays actual load versus time measurements in a three strain-gauge DHLC tool. The figure shows the occurrence of buckling, while the tool freely rotates about its vertical axis (no torsion).

CRITICAL LOADS

Buckling is the unstable deflection of the sucker rod placed under a critical, compressive load. Critical, compressive loading keeps the rod in a slightly bent form. This statement holds true if the rod is perfectly elastic and the yield strength of the rod is not exceeded. To quantify this critical, compressive load, Euler's column formula is presented for a rod string enclosed in tubing. This formula defines buckling characteristics for a finite length rod and is dependent on rod stiffness, weight, and tubing size (similar to a column on an elastic foundation). Equations for buckling in inclined boreholes as presented by Lubinski and Woods are given.

Most work on rod and pipe buckling centers on post-buckling helical behavior. While applicable, the rod designer and analyzer is concerned with all forms that buckling may take. If the compressive load seen by a rod is considerably large, the rod string will undergo several mode changes. For instance, as the pump's plunger just begins to move downward, the compressive load may be relatively small. The rod will be straight about its vertical axis. As acceleration begins to increase and pump resistance increases, the compressive load may exceed the load necessary to drive the rod into a sinusoidal shape. The critical, sinusoidal buckling load is exceeded and the rod is buckled sinusoidally. Also, as the maximum downward acceleration is experienced by the pump and its resistance has peaked, the rod may take the shape of a helix (spring). The critical, helical buckling load is exceeded and the rod buckles helically (Figure 5). In the cycle, as the pump and rods begin their upstroke, the rod reverses its shape again to a sinusoid and then straight. Depending on the compressive loads experienced throughout the pumping cycle, the rod may only buckle sinusoidally or not at all. This scenerio occurs in straight or inclined boreholes.

SINUSOIDAL BUCKLING

From the discussion, there are two critical loads of concern. These loads create sinusoidal buckling and helical buckling. Euler determined the critical sinusoidal load by solving the differential equation for the rod deflection curve with "Free-Fixed" boundary conditions (Figure 6). "Free-Fixed" conditions relate the rod's deflection to a column fixed on an elastic foundation at one end and free to move radially (within the tubing) at the other. This condition best describes the physical condition of the bottom rod. According to Euler's derivation, the following applies for a slightly bent rod;

$$F_{crs} = (\pi^2 EI) / (4 L^2) \quad (1)$$

For any other sinusoidal shape assumed by the rod;

$$F_{crs} = [(2n-1) \pi / (2L)]^2 EI \quad (2)$$

Euler's equations apply to a finite length rod. For sinusoidal buckling of an infinite rod in an inclined borehole (with the same boundary conditions), Dawson and Paslay's equation is used;

$$F_{crs} = EI (n' \pi / L)^2 + \{w \sin(a) / r\} \{L / (n' \pi)\}^2 \quad (3)$$

The term n' is the number of half sine waves in the buckled rod section and is used to quantify the half wave length;

$$n' = L / l_w \quad (4)$$

After Den Hartog's work, the half-wave length for a long beam with the elastic foundation constant $(k) = w \sin(a) / r$, then;

$$l_w = \pi (EI / k)^{1/4} \quad (5)$$

Equations 3, 4, and 5 are used to calculate the sinusoidally buckled rod shape.

HELICAL BUCKLING

The transition between sinusoidal and helical buckling forces is very small. For this reason, most rods will assume a full helix in buckling. In this discussion, the critical force required to buckle a rod into a helix is quantified.

The critical, helical buckling force calculation is based on strain energy theory. Cheatham and Pattillo introduced the method for inclined boreholes. Through this theory, the minimum force required to bend the rod into a helix is;

$$F_{crh} = (8EI w \sin(a) / r)^{1/2} \quad (6)$$

The solution to experimental data acquired by Lubinski and Woods verifies the theoretical solution;

$$F_{crh} = 2.85 (EI)^{0.504} w^{0.496} (\sin(a) / r)^{0.511} \quad (7)$$

NEUTRAL POINT

Neutral point is usually defined as the point in the rod string where the loading goes from tension to compression. The concept of a static neutral point does not exist in a dynamic rod string. The dynamic neutral point changes throughout the pumping cycle and is necessary in a buckling analysis to determine the length of the rod string effected by the compressive force. The best solution for quantifying the neutral point is through computer simulation. However, when this solution is not available, the neutral point is assumed to be the distance up the rod string (from the pump) where the submersed rod weight equals the compressive load;

$$\text{distance to NP} = F / w \quad (8)$$

FATIGUE

When buckling occurs, high bending stresses are encountered. If a rod buckles sinusoidally, the maximum bending stress can be twice the critical buckling stress. During helical buckling, stresses can be four times the critical buckling stress. Consequently, buckling mode is important in calculating the additional stress experienced by the rod. This stress translates into rod fatigue if the endurance limit of the material is exceeded.

In buckling, if the maximum stress seen by the rod is less than the elastic limit of the steel, the rod will become straight when the load is removed. Thus, the conclusion is drawn that a rod can be loaded infinitely if the elastic limit is not surpassed. This assumption is not correct for repeated loadings. Fatigue will cause ruptures in the rod at stresses much lower than the static breaking strength under repeated cyclical loads. The fatigue appears from cold working of the steel, making the rod brittle.

Fatigue is relative to buckling since the rods undergo complete cycles of load changes (compression to tension). This form of loading is more detrimental to rod life than cyclical tension loading (as seen in the taper top rods). Also, other factors contribute to fatigue failures and rupture. In the cases of rods in a corrosive environment and rod body eccentricity, the endurance limit stress should be considered far less than the ultimate strength of the rod. The endurance limit, for instance, of 1020HR structural steel is about thirty thousand psi (Figure 7). The material will withstand over a billion cycles of repeated loading at this stress load. However, if placed in an environment of hydrogen sulfide or weak acids, the material will fail after far less loading cycles.

To evaluate the potential for fatigue, the maximum stress applied to the rod must be quantified. Buckling pitch and radius of curvature for a helix are calculated to determine the maximum bending stress for the rod. Buckling pitch is defined as the axial length of one helix;

$$pitch (p) = \{ 8 \pi^2 E I / F_{buck} \}^{1/2} \quad (9)$$

For helical buckling, the radius of curvature for the helix is as follows;

$$\rho = \{ p^2 + 4 \pi^2 r^2 \} / \{ 4 \pi^2 r \} \quad (10)$$

The moment created in the helically buckled rod is;

$$M = E I / \rho \quad (11)$$

The maximum stress is then defined by the following (c is half the rod diameter);

$$S_c = M c / I \quad (12)$$

The maximum rod compressive stress is;

$$S_{max} = S_z + S_c \quad (13)$$

The minimum rod compressive stress is;

$$S_{min} = S_z - S_c \quad (14)$$

As is apparent from these equations, a rod may experience compressive loading on one side of its body, while tensile loading may take place on the opposite side (Figure 8).

FRICTION

Friction is difficult to define with respect to axial loading on the rods in a buckled state. However, because of rod string vibration, friction is negligible in creating buckling hysteresis. Theoretically, friction imparts an additional force for the rods to overcome when "unbuckling". Because of the vibrations which exist downhole, hysteresis effects can be ignored. Therefore, the main concern associated with friction is rod coupling and/or tubing wear. When rods buckle sinusoidally, contact area between rod and tubing is small. Wear is accelerated. When rods buckle helically, contact (bearing) area is greater. Wear may be more or less in this mode than in the sinusoidal mode. One thing is certain, friction is detrimental to the longevity of the rod string.

PREVENTING BUCKLING

Once a maximum compressive load is determined, then the critical buckling loads (sinusoidal and helical) are calculated for the rod string. The actual load is compared to the critical loads

and the buckling shape is defined. From this shape, the stresses experienced by the rod string are calculated and the possibility of fatigue is evaluated. If the loads are unacceptable, then the rod string must be modified to prevent buckling.

Several means are available to prevent or reduce buckling. If a given compressive load cannot be appreciably reduced, then the designer must modify the rod string's characteristics. This modification is accomplished by increasing the stability of the rod string through shortening the effective rod string length (using rod guides) or increasing the rigidity of the string (heavy weight sinker bars).

When using rod guides, an attempt is made to maintain the rod string in a straight, stable condition. The compressive force must not exceed the critical sinusoidal buckling force. Through proper spacing of the rod guides, the critical buckling force can be increased to a value higher than the compressive force. For guided rods, new boundary conditions in the solution to Euler's equation must be used. The boundary conditions assume a "fixed-roller fixed" configuration. The rod has a small amount of lateral movement, but the guide offers a restoring moment.

For the new boundary condition, Euler's solution is;

$$F_{crs} = (\pi^2 E I) / (l_g^2) \quad (15)$$

If the actual compressive force is assumed to be the critical force, then the rod guides must be spaced as follows;

$$l_g = \pi \{ E I / F_{crs} \}^{1/2} \quad (16)$$

Although rod guides are a solution to buckling elimination, guides can create increased friction and concentrated wear if applied improperly.

Heavy weight sinker bars provide rod stability to prevent buckling. When designing with heavy weight sinker bars, bars with sufficient diameter and weight must be used to avoid the critical buckling force. As rod diameter and weight increase, the compressive force required to buckle the rod greatly increases. The use of large diameter sinker bars drastically changes the buckling tendency of the string (Figure 9). Also, the dynamic neutral point is driven closer to the plunger. Thus, less of the rodstring is effected by compressive forces (in most cases). While using sinker bars, modelling must be accomplished to insure the dynamics of the rod string have not appreciably changed. Because of the difficulty associated with modelling short sections of heavy weight sinker bars, a reliable, calibrated software program is necessary.

CONCLUSIONS

Buckling of rod strings has several detrimental effects. Tubing wear, coupling wear, fatigue failures, and decreased rod fall are all associated with severe buckling.

The prevention of rod buckling is accomplished by increasing the rod string's stability. Rod diameter increases, weight increases, design modifications, and rod guide placement will effect buckling.

Buckling can be systematically analyzed to increase the longevity of rod strings.

NOMENCLATURE

E	= Young's Modulus, psi
F	= Force, lbs _f
F_{buck}	= Buckling force influencing rods, lbs _f
F_{crs}	= Critical buckling force - sinusoidal - lbs _f
F_{crh}	= Critical buckling force - helical - lbs _f
I	= Minimum second moment of area for rod section, in ⁴
L	= Length of rod section, inches
M	= Moment, in - lbs _f
S_c	= Maximum stress from bending, psi
S_{min}	= Minimum rod compressive stress, psi
S_{max}	= Maximum rod compressive stress, psi
S_z	= Axial rod stress, psi
a	= Borehole inclination from vertical, degrees
c	= Half rod diameter, inches
k	= Elastic foundation constant, psi
l_g	= Critical spacing for rod guides, inches
l_w	= Length of half-wave for long beam, inches
n	= Number of full sine waves in rod section, (-)
n'	= Number of half sine waves in rod section, (-)
p	= Helical buckling pitch, inches
π	= Radial constant (3.1416)
r	= Radial clearance between rod diameter and tubing wall, inches
ρ	= Radius of curvature for helix, inches
w	= Weight of rods in fluid, lbs _f / in

ACKNOWLEDGEMENT

The authors wish to thank the management of Texland Petroleum, Inc., and Mobil Exploration & Producing U.S., Inc., for permission to publish this work.

REFERENCES

1. Beer, F.P. & Johnston, E.R. Jr.: Mechanics of Materials, First Edition, New York: McGraw-Hill Company, 1981.
2. Cheatham, J.B. & Pattillo, P.D.: *Helical Postbuckling Configuration of a Weightless Column Under the Action of an Axial Load*, SPEJ, August 1984.
3. Dawson, R. & Paslay, P.R.: *Drillpipe Buckling in Inclined Holes*, JPT, October 1984.
4. Den Hartog, J.P.: Advanced Strength of Materials, First Edition, New York: McGraw-Hill Book Company, 1952.
5. Hibbeler, R.C.: Engineering Mechanics - Statics and Dynamics, Third Edition, New York: MacMillan Publishing Co., Inc., 1983.
6. Lubinski, A., Althouse, W.S. & Logan, J.L.: *Helical Buckling of Tubing Sealed in Packers*, JPT, June 1962.
7. Lubinski, A. & Blenkarn, K.A.: *Buckling of Tubing in Pumping Wells, Its Effects and Means for Controlling It*, SPE, Petroleum Branch Fall Meeting, Los Angeles, October 1956.
8. Ott, R.E. & Mendenhall, G.L.: *Making the Most of Rod Design - A Comparison of Measured and Modeled Rod String Stresses*, SPE, Permian Basin Oil & Gas Recovery Conference, Midland, Texas, March 1994.
9. Timoshenko, S.P. & Gere, J.M.: Theory of Elastic Stability, Second Edition, New York: McGraw-Hill Book Company, 1961.

Theoretical Pump Card

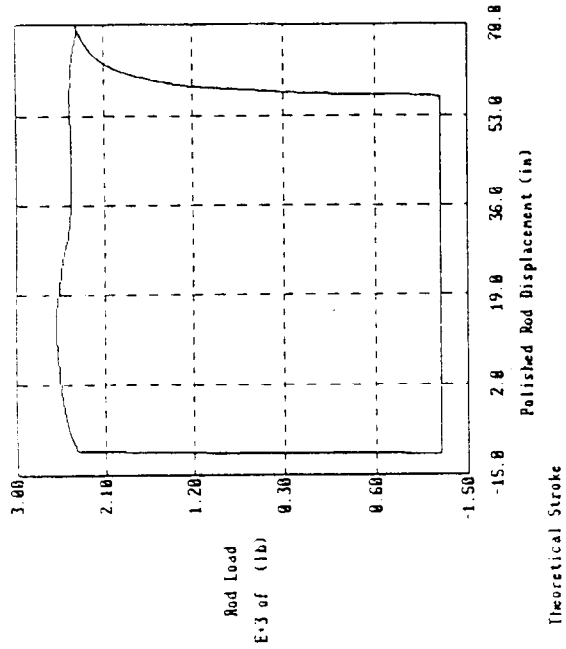


Figure 2

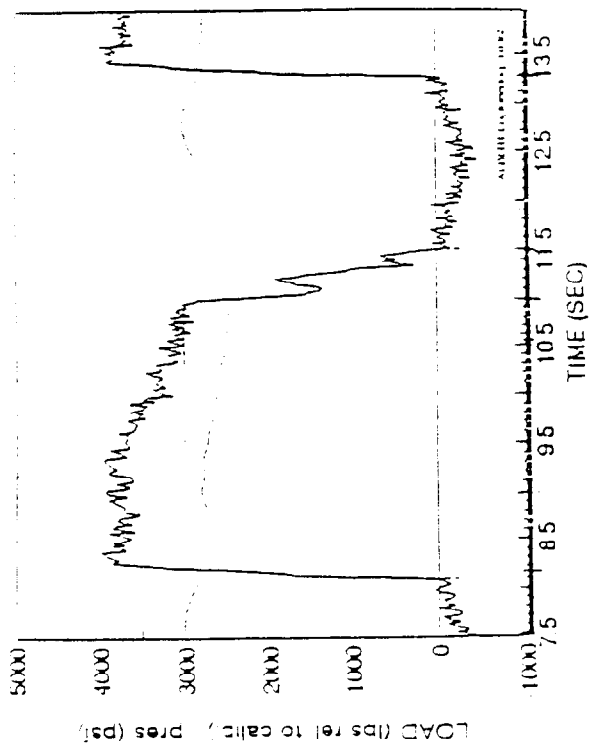


Figure 1

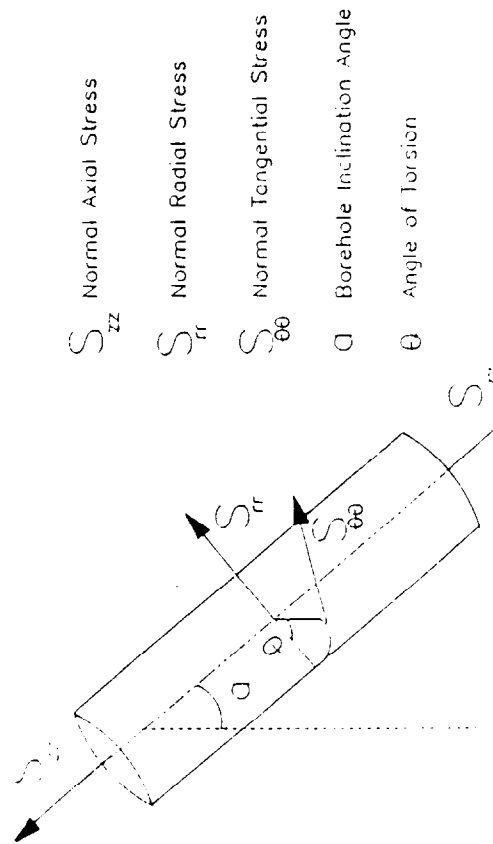


Figure 3 - Cylindrical Coordinate Stress Field

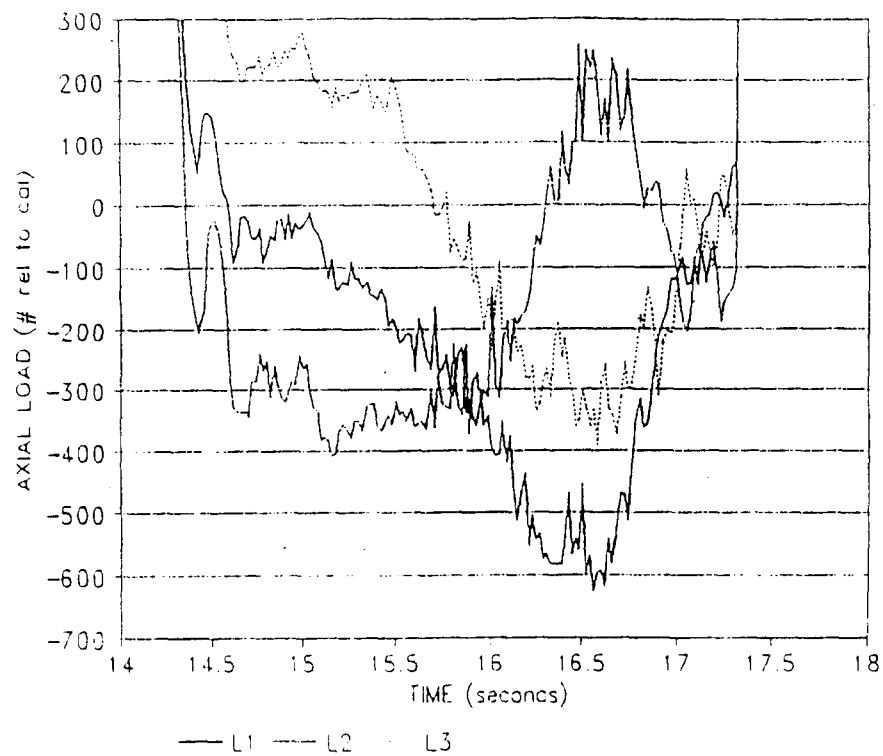


Figure 4

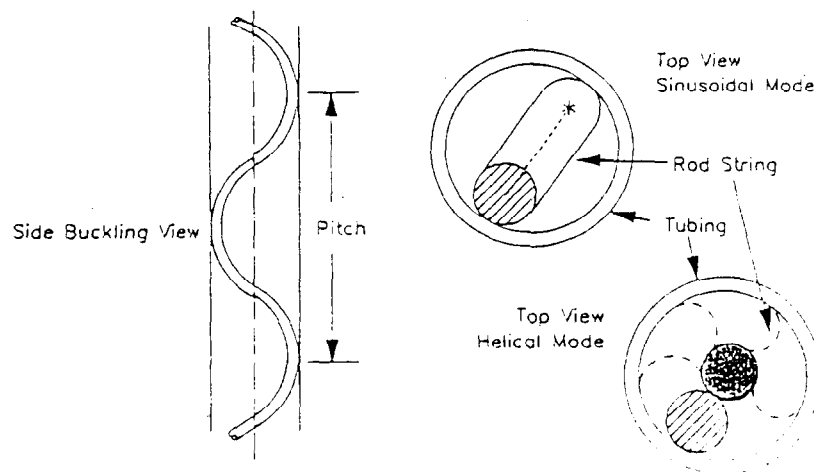


Figure 5 - Sinusoidal and Helical Buckling Views

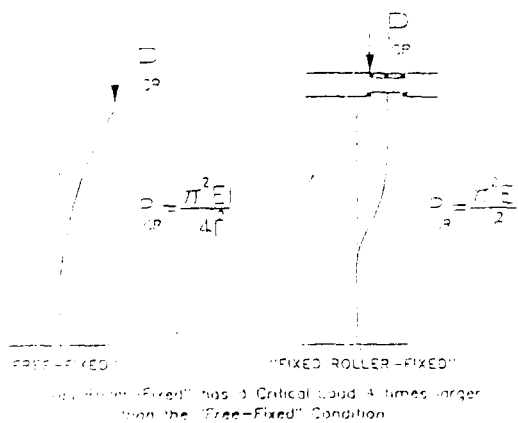


Figure 6 - Euler's Equation Boundary Conditions

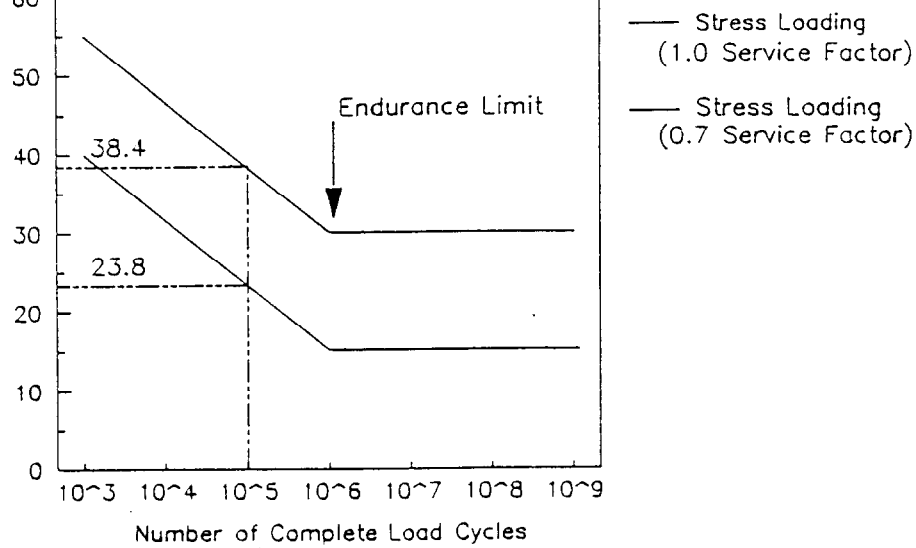


Figure 7 - Fatigue in Rod Strings
(Effects of Load Reversals)

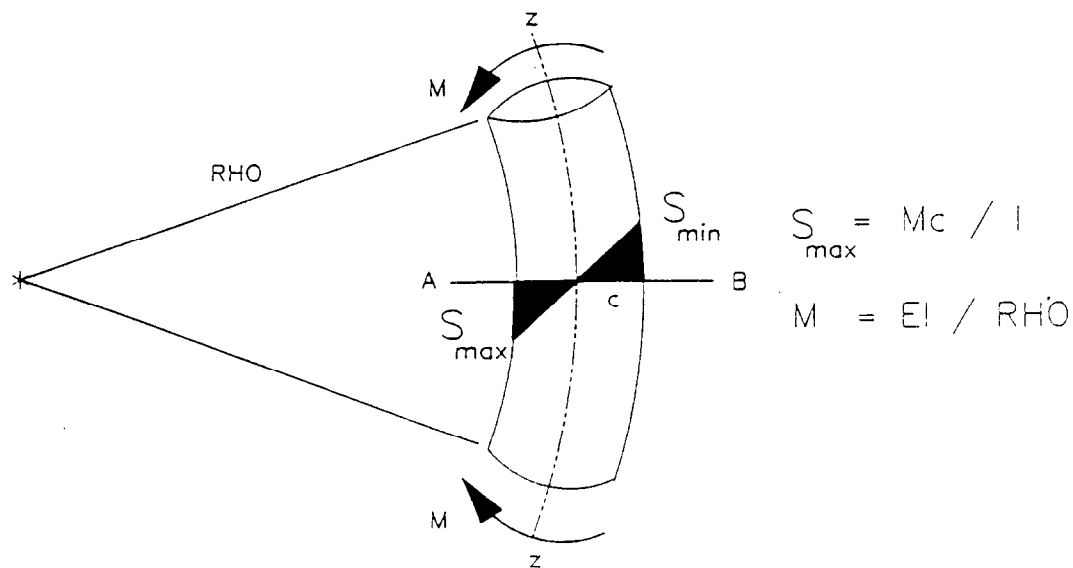


Figure 8 - Bending Stress Schematic

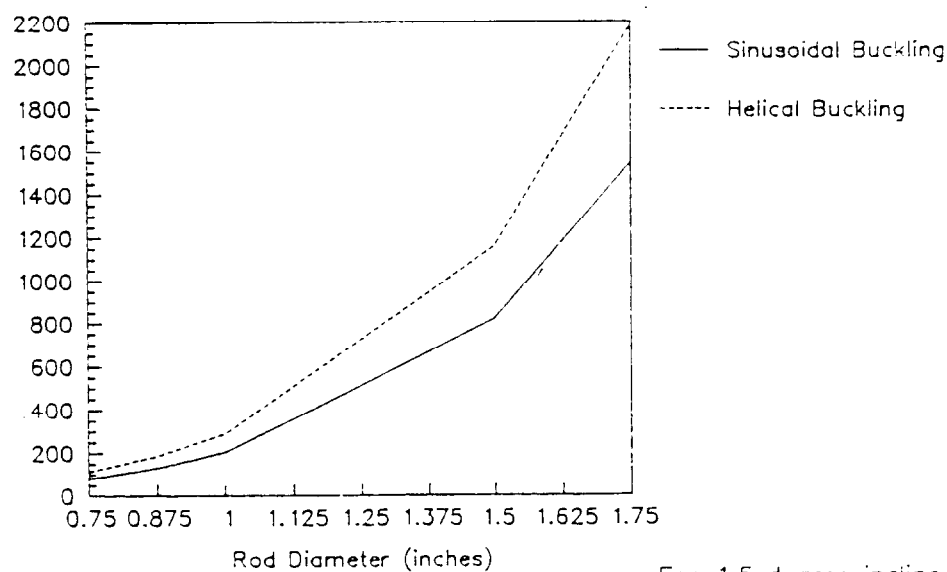


Figure 9 - Critical Buckling Loads—Helix and Sinusoid
As a Function of Rod Diameter and Weight

For 1.5 degree incline
300' Rod Section