Single-Phase Flow in Horizontal Pipelines

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INTRODUCTION

Horizontal single-phase flow in a pipe network occurs frequently in oilfield operations. The field engineer may need to predict with reasonable accuracy the pressure loss in these lines. Nomographs from a handbook or appropriate equations from a fluid mechanics text can be employed for these calculations.

The equations for Reynolds number and pressure loss in most textbooks will not have been derived in commonly employed field units, requiring numerous conversions which are excellent sources of error. To further complicate these calculations, Moody diagrams appear in the literature with two different sets of values for the friction factor. A calculated pressure loss would be in error by 400 per cent if the wrong friction factor were used.

In this paper the equations for Reynolds number and pressure loss are defined in field units. Examples are included to illustrate the calculations. In addition, a listing of a computer program for calculation of the Darcy friction factor is given for the engineer who has access to a computer or terminal.

REYNOLDS NUMBER

The Reynolds number in consistent units for pipe flow is a dimensionless parameter defined as

$$R_e = \frac{d\rho v}{\mu} \qquad (1)^*$$

and is one of the most important parameters in fluid mechanics. It is the ratio of the inertial to the viscous forces. The Reynolds number indicates the flow pattern from laminar (viscous) to fully developed turbulent flow.

Since the friction factor equation varies with flow regime, the proper regime must be established before further calculations are possible. Therefore, the first step in the calculation of a pressure loss is to compute the Reynolds number.

A value is not readily calculated from field data using Eq. (1). A more useful equation is

$$R_e = \frac{C_{Re} \gamma q}{\mu d} \qquad (2)$$

which applies for liquid or gas flow. The constant C_{Re} varies with the units of the volumetric flow rate and its value is given in Tables 1 and 2 for liquid and gas flow respectively. The volumetric flow rate for gas is at standard conditions which is readily available. Exact in situ computations, i.e. at the actual flowing pressure and temperature, are not justified when pipe roughness and other factors must be approximated.

The viscosity in Eq. (2) is absolute (dynamic) viscosity in centipoise. The centipoise unit was selected because of its wide use in the oil industry. If the kinematic viscosity in centistokes is known, conversion to centipoise for a liquid is accomplished by multiplying the given viscosity in centistokes times the specific gravity of the liquid. For example, the absolute viscosity would be 3.4 centipoise if the kinematic viscosity and specific gravity were 4 centistokes and 0.85 respectively. The pipe diameter in Eq. (2) is in inches.

ABSOLUTE AND RELATIVE ROUGHNESS

Absolute roughness is the average depth of the wall irregularities. A uniform value is assumed for the entire pipe wall of a given conduit; this parameter is not measured.

The relative roughness is the absolute pipe roughness divided by the pipe diameter in the same units.

$$\text{Relative Roughness} = \epsilon/d \qquad (3)$$

Since the relative roughness is a dimensionless quantity, the absolute roughness and pipe ID

*All nomenclature listed at end of paper.

must both be in the same units of length as inches, feet, etc.

The friction factor chart (Moody diagram) is based on three parameters, one of which is the relative roughness. The importance of relative roughness increases for higher Reynolds numbers and does not affect the friction factor at low Reynolds numbers where laminar flow exists.

In many handbooks and texts a relative roughness chart appears in conjunction with a Moody diagram. Curves on this chart will be labeled commercial steel or wrought iron pipe, drawn tubing, etc. Rather than using these charts it is recommended to use values for absolute roughness applicable to oilfield pipe which have been determined by several investigators. Cullender and Smith[1] found an absolute roughness of 0.0006 inches to be representative of oil well tubing. A value of 0.0007 inches was reported by the Bureau of Mines[2] and the American Gas Association[3]. These values for absolute roughness are based on fully developed turbulent flow.

FRICTION FACTOR AND FRICTION FACTOR CHART (FIG. 1)[4]

After the Reynolds number and relative roughness are calculated for the given flow conditions, the friction factor can be determined from a Moody diagram. The basis for a friction factor chart (Moody diagram) should be established before the friction factor is plugged into a pressure loss equation.

Moody diagrams appear in the literature based on two different friction factors, i.e. the Darcy and the Fanning factors.

$$\text{Darcy friction factor} = 4\,(\text{Fanning friction factor}) \tag{4}$$

If the Darcy friction factor were used in a Fanning pressure loss equation, the calculated loss would be in error 400 per cent on the high side. For this reason a friction factor cannot be used indiscriminately with all pressure loss equations.

The procedure for determining which friction factor is plotted as the ordinate of a Moody diagram is to note the f-value for a Reynolds number of 1000.

$f = 0.064$	Darcy
$f = 0.016$	Fanning

A Moody diagram based on the Fanning friction factor can be used with a Darcy (Darcy-Weisbach) equation by multiplying the chart f-value by 4 before plugging it into the equation.

The Moody diagram is a convenient display of friction factors for laminar, transition, and fully developed turbulent flow through pipes of various roughnesses. The chart eliminates the need for calculations which are very laborious for transition flow. Figure 1 is based on the Darcy friction factor.

The friction factor for laminar flow is indirectly proportional to the Reynolds number. The f-value is represented on the Moody diagram by a straight line which is independent of pipe roughness. Laminar flow is considered to exist for Reynolds numbers less than 4000 with many authors arbitrarily selecting 2000. The conditions for which laminar flow ceases are difficult to predict. A value of 2100 is used in the computer program in the Appendix. This computer program is a revised and extended version of one by Sommerfeld.[5] (See Fig. 2)

The Colebrook equation defines the friction factor in the transition range between laminar and fully developed turbulent flow. This empirical equation does not lend itself to an easy hand-calculated solution, hence the Moody diagram.

The friction factor in the fully turbulent region is defined by the Nikuradse equation. When complete turbulence is attained, the friction factor remains constant with an increasing Reynolds number.

The equations which define the Darcy friction factor for the three regions of flow are given in the Appendix.

HORIZONTAL STEADY INCOMPRESSIBLE CONFINED FLOW

The proper equation for the previously determined friction factor must be employed to calculate the pressure loss. The two force balance equations which are commonly used for describing horizontal, single-phase, isothermal, steady flow (no acceleration) of an incompressible (constant density) liquid through a pipe are the Darcy (sometimes referred to as Darcy-Weisbach) and the Fanning equations. The only difference in the two equations is a factor of 4 which may appear in the friction factor.

The equations in consistent units are generally expressed in the literature as

$$\text{Darcy} \quad \Delta p_f = \frac{fL(v_l)^2 \rho_l}{2g_c d} \tag{5}$$

and

$$\text{Fanning} \quad \Delta p_f = \frac{2fL(v_l)^2 \rho_l}{g_c d} \tag{6}$$

Unfortunately, the above formulae would require several conversions from field units to consistent units.

The Darcy Eq. (5) for pipe flow in practical field units is

$$\Delta p_f = \frac{C_l f \gamma_l L(q_l)^2}{(d)^5} \tag{7}$$

where C_l varies with the units for the liquid volumetric flow rate, and its values are given in Table 1. The remaining terms of Eq. (7) are psi for pressure loss, feet for pipe length, and inches for internal pipe diameter.

HORIZONTAL STEADY COMPRESSIBLE CONFINED GAS FLOW

An equation similar to Eq. (7) can be developed for single-phase gas flow through pipe. Since gas density varies with pressure and temperature, a relationship in terms of pressure and temperature can be substituted for gas density. Similarly, the volumetric flow rate at standard conditions can be converted to in situ conditions based on the flowing pressure and temperature in the pipe. With these substitutions, Eq. (7) for gas becomes

$$\Delta p_f = \text{Constant} \left(\frac{p_{sc}}{T_{sc}}\right)^2 \frac{f \gamma_g L \bar{T} z}{\bar{p}\,(d)^5} \left(q_{gsc}\right)^2 \tag{8}$$

where the constant would depend on the units used for pressure, length and the volumetric flow rate.

A pressure loss equation for gas flow which includes the Z-factor requires a trial-and-error solution. Many published gas flow equations are arranged to solve for the volumetric flow rate in terms of a known pressure loss which circumvents the trial-and-error solution.

Equation (8) is not in the most useful form for calculating the downstream (discharge) pressure. By letting the mean pressure ($\bar{p}$) equal $(p_1 + p_2)/2$ and the compressibility be a function of this mean pressure and an average flowing temperature for the system, Eq. (8) may be rearranged to solve for the downstream pressure.

$$p_2 = \sqrt{p_1^{\,2} - \left(\frac{p_{sc}\, q_{gsc}}{T_{sc}}\right)^2 \left(\frac{f \gamma_g L \bar{T} z}{31.8\,(d)^5}\right)} \tag{9}$$

The units are psia for pressure, degrees R for temperature, feet for length, inches for pipe diameter, and Mscf/day for volumetric gas flow rate. The specific gravity of the gas is based on an air density of 0.0764 lb_m/ft^3. Note that the gas flow rate must be converted to Mscf/day before Eq. (9) or the following Eq. (11) can be applied.

Weymouth[6] simplified the gas flow equation by assuming the gas compressibility to be unity (Z = 1.0) and the Darcy friction factor to be a constant equal to

$$f = \frac{0.032}{d^{1/3}} \tag{10}$$

where the pipe diameter is in inches. Since Weymouth used actual measured pressure losses to establish Eq. (10), the left side of the equation may be considered Zf rather than f alone. Using the above substitution in Eq. (9), the downstream pressure can be calculated directly.

$$p_2 = \sqrt{p_1^{\,2} - \left(\frac{p_{sc} q_{gsc}}{T_{sc}}\right)^2 \left(\frac{\gamma_g L \bar{T}}{992.5 d^{16/3}}\right)} \tag{11}$$

where the units are the same as for Eq. (9).

If Eq. (9) is used, Ducker[7] has shown that the Z-factor may be based on the same mean pressure assumption required to develop this form of the equation. In addition Ducker presented a method for the calculation of the Z-factor. The initial assumed mean pressure can be based on a p_2 calculated directly by Eq. (11).

For extremely long lines, a finite difference type of calculation may be employed with Eq. (9); i.e. a long pipeline can be divided into several parts. This approach would be particularly applicable to a computer solution.

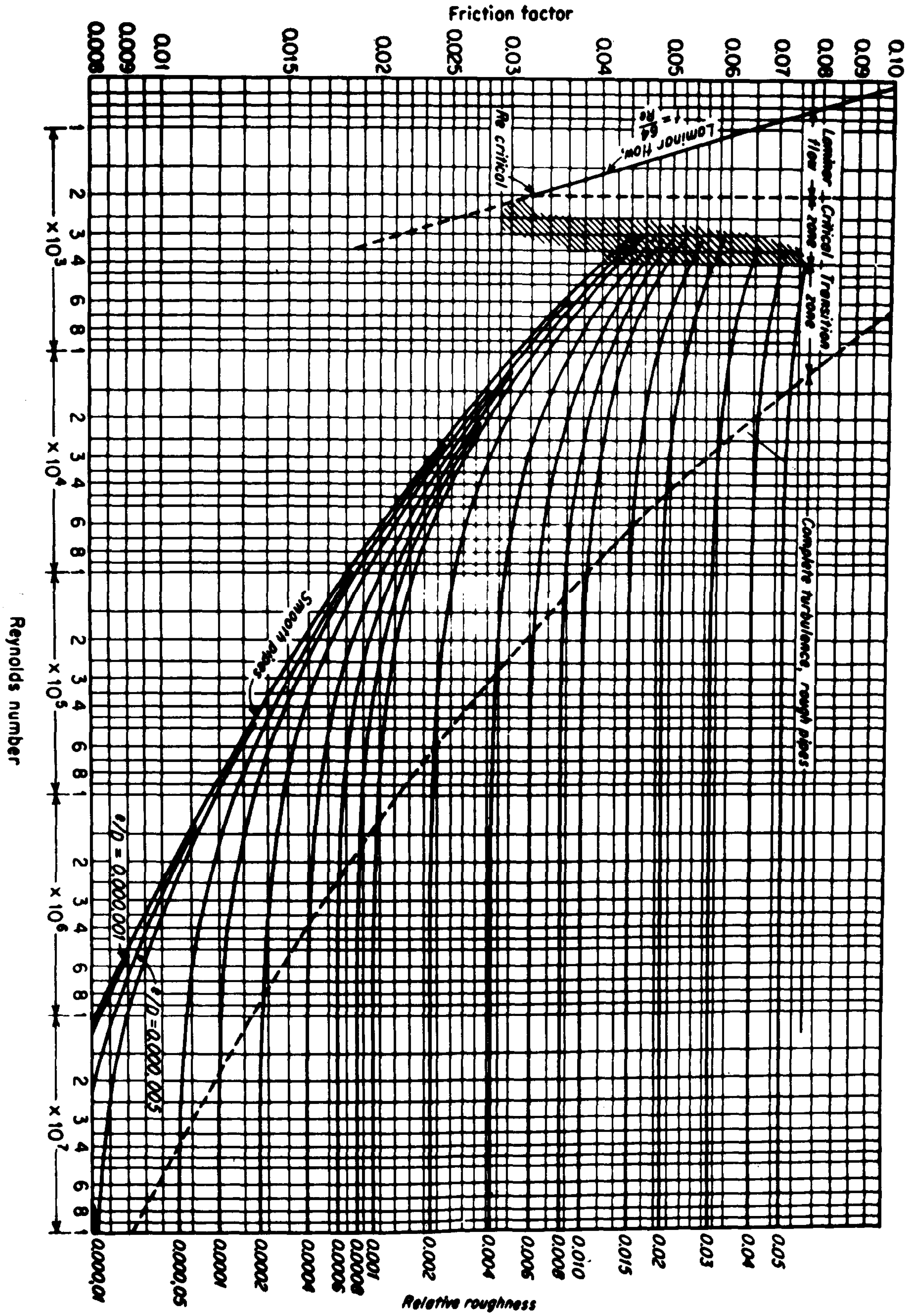

FIGURE 1
MOODY DIAGRAM

Since there is no Z-factor in Eq. (11), dividing a pipeline into short length increments will not affect the final answer.

In the Appendix the same data are used to calculate the pressure loss by Eqs. (9) and (11). These calculated losses are quite different. Advanced knowledge as to the applicability of a particular gas flow equation to a specific line size, pressure level, gas flow rate, etc is most important. Assumptions used to simplify an equation may not be valid when extrapolated well beyond the conditions for which the assumptions are based.

NOMENCLATURE

C_1 coefficient for liquid flow equation
C_{Re} coefficient for Reynolds number equation
d internal diameter of pipe
g_c gravitational constant
f friction factor
L length
p pressure
$\bar{p}$ mean pressure
Δp_f frictional pressure loss
q volumetric flow rate
Re Reynolds number
T temperature
$\bar{T}$ mean temperature
v velocity
z gas compressibility factor
γ specific gravity
ϵ absolute roughness
μ absolute viscosity
ρ density

Subscripts:

g gas
l liquid
sc standard conditions
1 upstream
2 downstream

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      SUBROUTINE CARCYF ( RE, RR, F )
      IF ( RE - 2100. ) 70, 70, 71
C     LAMINAR ( VISCOUS ) FLOW REGION
   70 F = 64. / RE
      GO TO 77
C     COLEBROOK EQUATION FOR TRANSITION
   71 A = RR / 3.7                     FLOW
      B = 2.51 /RE
      C = 0.868589
      X = - C * ALOG( A + 1.0E-12 )
   72 Y = X + C * ALOG( A + B * X )
      IF ( ABS(Y) - 1.0E-06 ) 74, 74, 73
   73 YP = 1.0 + C * B / ( A + B * X )
      X = X - Y / YP
      GO TO 72
   74 FF = 1.0 / X**2
C     NIKURADSE EQUATION FOR FULLY
C        DEVELOPED TURBULENT FLOW
      E = 1.14 - 0.86 * ALOG( RR )
      FFT = 1.0 / E**2
C     COMPARES AND USES HIGHER F-VALUE
      IF ( FF - FFT ) 75, 76, 76
   75 F = FFT
      GO TO 77
   76 F = FF
   77 CONTINUE
      RETURN
      END
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FIGURE 2
SUBROUTINE FOR CALCULATION OF DARCY FRICTION FACTOR

APPENDIX

TABLE 1
C_{Re} and C_1 for Liquid Flow*

q_L	C_{Re} in Eq. (2)	C_1 in Eq. (7)
gal/min	3160	0.0135
bbl/min	1.33×10^5	23.8
bbl/hr	2210	6.60×10^{-3}
bbl/day	92.2	1.15×10^{-5}

***Based on density of water at 60° F. of 62.37 lb_m/ft^3**

TABLE 2
C_{Re} for Gas Flow*

q_{gsc}	C_{Re} in Eq. (2)
scf/min	29.0
scf/hr	0.483
scf/day	0.0201
Mscf/day	20.1

***Volumetric gas flow rate is at standard conditions and C_{Re} is based on the density of air at 14.7 psia and 60° F. of 0.0764 lb_m/ft^3**

Equations Used to Generate The Moody Diagram

Darcy friction factor equations for steady, incompressible flow through pipe:

Laminar flow:
$$f = \frac{64}{R_e} \qquad \text{(A-1)}$$

Colebrook Equation:
$$\frac{1}{\sqrt{f}} = -0.86 \ln \left(\frac{\epsilon/d}{3.7} + \frac{2.51}{R_e\sqrt{f}} \right) \qquad \text{(A-2)}$$

Nikuradse Equation:
$$\frac{1}{\sqrt{f}} = 1.14 - 0.86 \ln\left(\frac{\epsilon}{d}\right) \qquad \text{(A-3)}$$

Example of Pressure Loss Calculations for Liquid Flow

Given Data:

d = 2.067" (2" nominal line pipe)
L = 5280 ft
ϵ = 0.00065"
q_l = 100 bbl/hr.
γ_l = 0.85
μ_l = 10 cp

Calculations:

Step 1. Calculate the Reynolds number using Eq. (2) and the appropriate C_{Re} from Table 1.

$$R_e = \frac{2210 \gamma_l q_l}{\mu_l d} \qquad (2)$$

$$R_e = \frac{2210\,(0.85)\,100}{10\,(2.067)}$$

Step 2. Calculate the relative roughness using Eq. (3)

$$\frac{\epsilon}{d} = \frac{0.00065}{2.067} = 0.00031 \qquad (3)$$

Step 3. Determine the friction factor from the Moody diagram in Fig. 1. (Since f = 0.064 for R_e = 1000, Fig. 1 is based on the Darcy friction factor).

$$f = 0.0315$$

Step 4. Calculate the pressure loss using Eq. (7) and the value of C_1 from Table 1 for the volumetric flow rate in bbl/hr.

$$\Delta p_f = \frac{0.0066\, f \gamma_l L (q_l)^2}{d^5} \qquad (7)$$

$$\Delta p_f = \frac{0.0066\,(0.0315)\,(0.85)\,(5280)\,(100)^2}{(2.067)^5}$$

Δp_f = 247 psi Answer

The pressure loss due to friction in the one mile of 2-in. nominal pipeline is approximately 250 psi for a volumetric flow rate of 100 bbl/hr. These calculations do not include the pressure change due to a difference in elevation.

Examples of Pressure Loss Calculations for Gas Flow

Given Data:

d = 2.067" (2" nominal line pipe)
L = 5280 ft
ϵ = 0.00065"
q_{gsc} = 3000 Mscf/day at p_{sc} = 14.7 psia and T_{sc} = 60°F (520°R)
γ_g = 0.65
μ_g = 0.02cp
p_1 = 800 psia (upstream pressure)
$\bar{T}$ = 100°F (560°R)

I. Calculate the pressure loss using the Weymouth equation. The downstream pressure can be obtained directly from Eq. (11).

$$p_2 = \sqrt{p_1^2 - \left(\frac{p_{sc} q_{gsc}}{T_{sc}}\right)^2 \left(\frac{\gamma_g L \bar{T}}{992.5 d^{16/3}}\right)} \qquad (11)$$

$$p_2 = \sqrt{(800)^2 - \left(\frac{14.7\,(3000)}{520}\right)^2 \left(\frac{0.65\,(5280)\,560}{992.5\,(2.067)^{16/3}}\right)}$$

p_2 = 592 psia
$\Delta p_f = p_1 - p_2$ = 800-592
Δp_f = 208 psi Answer

II. Calculate the pressure loss using the Darcy type Eq. (9).
Steps 1 - 3. Follow same steps as used to calculate the pressure loss for liquid flow.

$$R_e = \frac{20.1\ \gamma_g q_{gsc}}{\mu_g d} \qquad (2)$$

where C_{Re} = 20.1 from Table 2

$$R_e = \frac{(20.1)(0.65)(3000)}{(0.02)(2.067)} = 9.48 \times 10^5$$

$$\frac{\epsilon}{d} = \frac{0.00065}{2.067} = 0.00031 \qquad (3)$$

f = 0.0157 from Fig. 1

Step 4. Determine a Z-factor for the first assumed mean pressure and average flowing temperature for the system. The initial mean pressure can be based on p_2 calculated in Part I.

$$\bar{p} = \frac{800 + 592}{2} = 696 \text{ psia}$$

z = 0.90 (696 psia and 100°F.)

Step 5. Calculate the pressure loss using Eq. (9).

$$p_2 = \sqrt{p_1^2 - \left(\frac{p_{sc} q_{gsc}}{T_{sc}}\right)^2 \left(\frac{f \gamma_g L \bar{T} z}{31.77\, d^5}\right)} \qquad (9)$$

$$p_2 = \sqrt{(800)^2 - \left(\frac{(14.7)(3000)}{(520)}\right)^2 \times \left(\frac{(0.0157)(0.65)(5280)(560)(0.90)}{(31.77)(2.067)^5}\right)}$$

p_2 = 691 psia

Assume $\bar{p} = \frac{800 + 691}{2}$ = 745.5 psia

z = 0.89 (746 psia and 100°F.)

$$p_2 = \sqrt{(800)^2 - \left(\frac{(14.7)(3000)}{(520)}\right)^2 \times \left(\frac{(0.0157)(0.65)(5280)(560)(0.89)}{(31.77)(2.067)^5}\right)}$$

p_2 = 692 psia close enough
$\Delta p_f = p_1 - p_2$ = 800-692
Δp_f = 108 psi Answer

The pressure loss by the two equations varied primarily because of the friction factors. The Cullender and Smith equation for horizontal flow will predict nearly the same pressure loss as Eq. (9). The Weymouth equation is considered conservative; i.e. it will predict a higher than actual pressure loss for many conditions.