SIMPLIFIED GRAPHICAL SOLUTIONS OF EQUATIONS RELATING WELL PRODUCTIVITY TO WELLBORE DAMAGE AND STIMULATION

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INTRODUCTION

This paper presents simplified methods of relating the following terms commonly used by the petroleum engineer in evaluating wellbore problem conditions of damage and of stimulation:

1. Well productivity
2. van Everdingen's skin factor, s
3. Permeability change—radial extent of change

Mathematical relationships between these concepts were originally defined and applied by Muskat,[1] van Everdingen,[2] Matthews and Russell,[3] Hawkins,[4] and Grubb and Martin.[5] Since a well is generally considered to drain a reservoir by radial-type flow, each of the equations involve logarithms, and the usual graphical plots yield exponential-type curves which are difficult to extrapolate or to interpolate for intermediate values of interest.

Straight-line forms of the three equations relating (1) well productivity, (2) van Everdingen's skin factor, s, and (3) permeability change—radial extent of change—were obtained by rearranging and regrouping of terms in the original equations. The straight-line graph of each equation is also presented in Figs. 1, 2 and 3.

The practicing engineer and others interested in interpreting well production problems will find that the straight-line graphical representation is more accurately extrapolated and interpolated for intermediate values and also is considerably more easily drawn than the typical exponential curves usually given[1,5] for the relationships presented here. These should greatly assist the operations engineer in planning and designing completion, workover and stimulation procedures.

GRAPHICAL PRESENTATION

Muskat's Equation Relating Well Productivity to Permeability Change Around the Wellbore

Muskat's equation[1] is:

$$PR = \frac{q_{actual}}{q_{theoretical}} = \frac{\frac{k_s}{k}\ln\frac{r_e}{r_w}}{\frac{k_s}{k}\ln\frac{r_e}{r_s} + \ln\frac{r_s}{r_w}} \quad (1)$$

The straight-line form (y = mx + B) is obtained by taking the reciprocal of the above equation and rearranging as follows:

$$DR = \frac{q_{theoretical}}{q_{actual}} = \left[\frac{\ln\frac{r_s}{r_w}}{\ln\frac{r_e}{r_w}}\right]\frac{k}{k_s} + \frac{\ln\frac{r_e}{r_s}}{\ln\frac{r_e}{r_w}} \quad (1a)$$

where:

DR = damage ratio = $1/PR$
k_s = permeability of zone adjacent to wellbore
k = permeability of formation
r_e = radius of reservoir
r_w = radius of reservoir wellbore
r_s = radius of damaged (or stimulated) zone of permeability k_s
q_{actual} = productivity of zone = $q_{damaged}$ or $q_{stimulated}$
$q_{theoretical}$ = productivity of zone of uniform permeability k

Figure 1 is a graph of productivity ratio (PR) versus permeability change around the wellbore (expressed as the ratio of permeability near the wellbore to the reservoir permeability). Values of constant radial extent of damage (or stimulation), r_s, are shown as a series of straight lines for reservoir drainage radius, r_e, of 1320 ft and wellbore radius, r_w, of 0.25 ft. Values of r_s are shown for 1, 5, 10 and 20 ft.

Productivity ratio (PR) is the ratio of q_{actual} to $q_{theoretical}$, both measured at the same drawdown pressure under steady state flow of incompressible fluid. PR values greater than 1 indicate formation improvement or stimulation; PR values less than 1 indicate damage.

Figure 1 shows that for an undamaged well there is a maximum improvement in well productivity that can be obtained by matrix-type stimulation treatments penetrating to any *fixed* radial extent. This is demonstrated in the stimulated section of Fig. 1 where $k_{stimulated}/k_{formation}$ equals infinity. For example, a stimulation treatment of 20 radial ft (note the 20-ft radial extent line) indicates a productivity ratio of 2 where $k_{stimulated}/k_{formation}$ equals infinity. Matrix stimulation treatments to less than that depth cannot, at a maximum, double productivity of an *undamaged* well.

Unlike the effect of stimulation, damage to even a shallow radial extent can markedly reduce production, depending on the loss in permeability through the damaged zone. This is

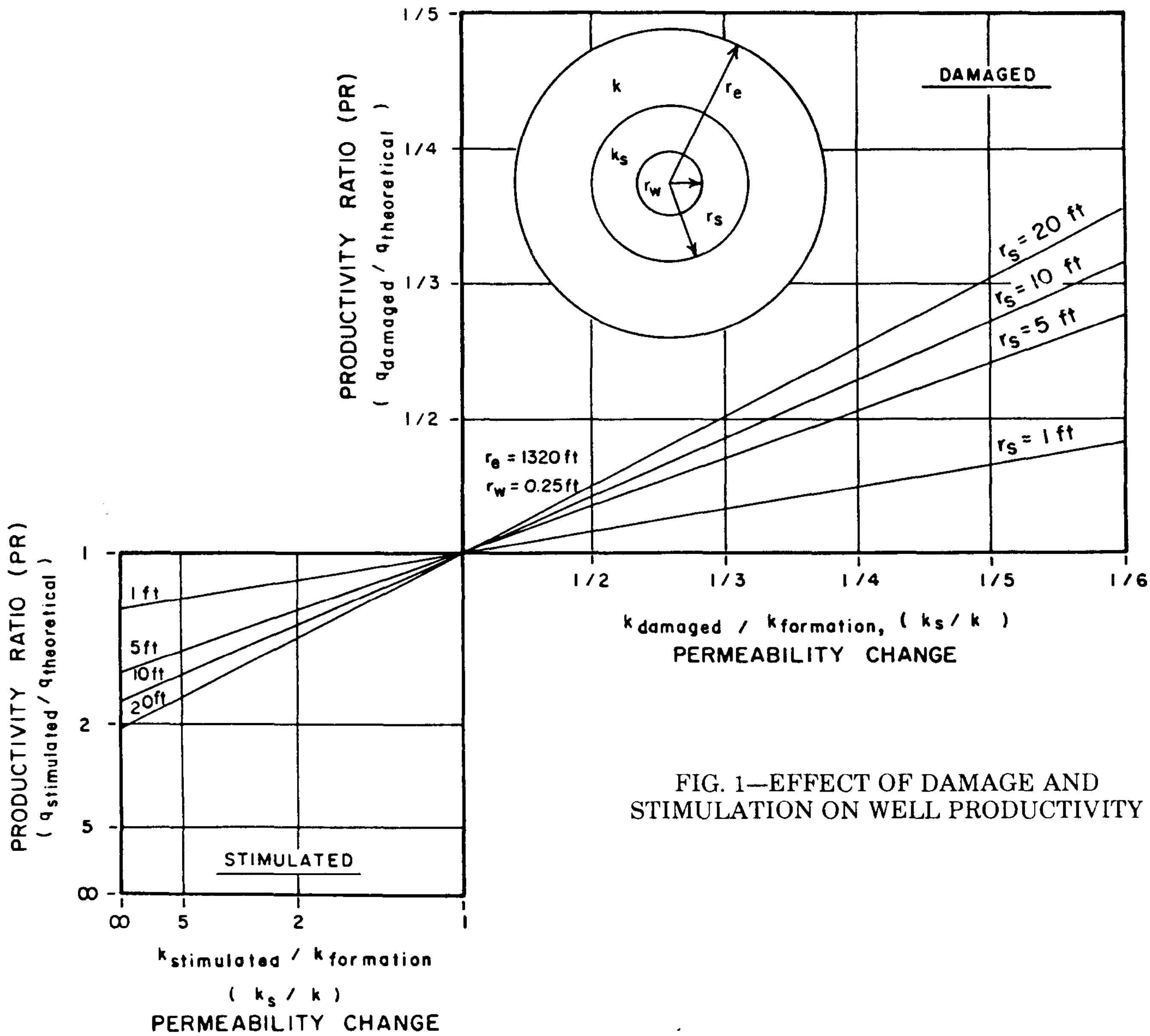

FIG. 1—EFFECT OF DAMAGE AND STIMULATION ON WELL PRODUCTIVITY

demonstrated in the damage portion of Fig. 1; for example, a 75% reduction in permeability through a 5-ft zone ($k_{damaged}$ / $k_{undamaged}$ = ¼) would reduce well productivity by about 50%.

The total maximum improvement obtained in stimulating a damaged well can be determined by using both the damaged and the stimulated portions of Fig. 1. For example, as shown in the above paragraph, removal of damage caused by a 75% reduction in permeability ($k_{damaged}$ / $k_{formation}$ = ¼) in a 5-ft zone will double productivity. If the permeability of the 5-ft zone is further increased to infinity, the well productivity will increase to 1.5, an additional increase of 50%. Thus the total maximum increase in well productivity will be from 0.5 to 1.5 or a threefold increase.*

Thus, Fig. 1 demonstrates the importance of (1) minimizing damage during completion and workover operations, (2) the maximum potential increase in productivity obtainable in stimulating damaged zones, and (3) the relatively low potential for increasing production by matrix stimulation of undamaged wells.

Figure 1 is generally sufficiently accurate for most values of reservoir drainage radius and wellbore radius found in the field. However, a similar graph may be quickly drawn for specific values of a particular reservoir drainage radius and wellbore radius.

van Everdingen's Skin Factor, s, Related to Productivity Ratio for Steady State Flow Conditions of Incompressible Fluids

The equation relating van Everdingen's skin factor, s, to well productivity [2,4] is:

$$PR = \frac{q_{actual}}{q_{theoretical}} = \frac{\ln \frac{r_e}{r_w}}{\ln \frac{r_e}{r_w} + s} \qquad (2)$$

* In very low permeability (gas) zones, less than 1 md, a long period of high production may occur during the nonsteady state (transient flow) period. In this case, the total production may be of real economic value, and the high apparent productivity ratio has significant meaning. In higher permeability zones, the transient period is relatively short, and the steady or pseudo-steady state flow conditions should be used to evaluate well performance.

The straight-line form is:

$$\frac{1}{PR} = DR = \frac{q_{theoretical}}{q_{actual}} = \left(\frac{1}{\ln \frac{r_e}{r_w}}\right) s + 1$$

Figure 2 is a straight-line graph of the equation relating skin factor to well productivity for two values of well reservoir drainage radius (660 ft and 10,000 ft) and a wellbore radius of 0.25 ft. Well productivity is shown both as productivity ratio (PR) and as damage ratio (DR).

Using as an example the 660-ft reservoir drainage radius, a negative skin factor of -3 indicates that the well productivity is about 1.6 times the theoretical (undamaged) productivity. A positive skin factor of +8 indicates that damage (or limited perforated interval) has reduced the well's productivity to about one-half of its undamaged (theoretical) value.

van Everdingen's Skin Factor, s, Related to Permeability Changes Around the Wellbore

The equation relating van Everdingen's skin factor, s, to permeability change around the wellbore and depth of change[2,4] is:

$$s = \left(\frac{k}{k_s} - 1\right) \ln \frac{r_s}{r_w} \qquad (3)$$

The straight-line form is:

$$s = \left(\ln \frac{r_s}{r_w}\right) \frac{k}{k_s} - \ln \frac{r_s}{r_w} \qquad (3a)$$

Figure 3 is a straight-line graph of Eq. (3a) relating skin factor to permeability change around the wellbore and the depth of damage or stimulation. By expressing radial extent of damage values in terms of wellbore radius, only one graph is required for all values of wellbore radius.

In many cases, after skin factor values are obtained from well test data, questions arise about the depth or radial extent of the zone of damage or improved permeability. Figure 3

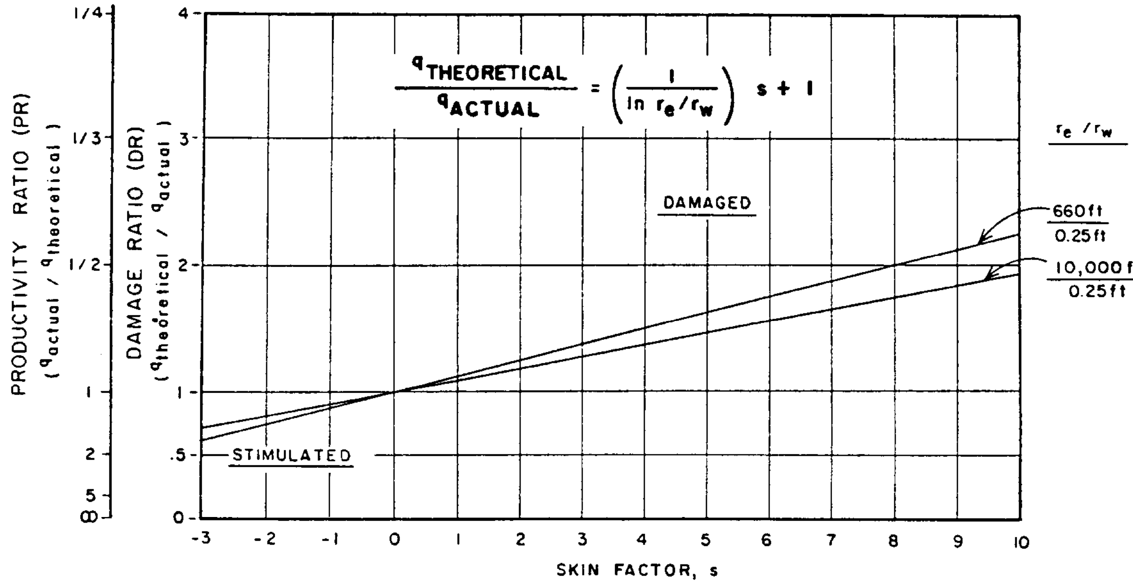

FIG. 2—EFFECT OF SKIN FACTOR ON PRODUCTIVITY RATIO

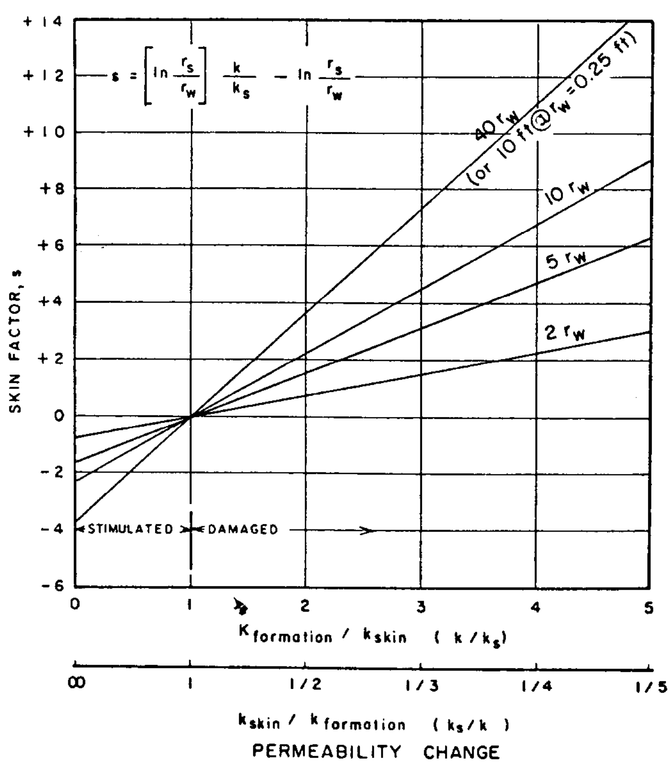

FIG. 3—EFFECT OF RADIAL EXTENT AND PERMEABILITY CHANGE ON SKIN FACTOR

shows how the skin factor depends both on radial extent and on the change in permeability of the zone around the wellbore. For example, if a well test indicates a positive skin factor of +6 and if laboratory tests indicate that the completion practice (drilling mud, cement, completion fluids, etc.) has reduced the permeability in the zone around the wellbore to about one-fourth of its original value, it can be seen from Fig. 3 that the damaged zone extends about $7r_w$ or about 3 ft in a 6-in. diameter well. Thus, a matrix-type stimulation treatment should be sized to treat at least to this depth in order to correct the damage condition.

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