Short Cuts to Sucker Rod Calculations

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In designing sucker rod strings and specifying operating conditions, we often neglect or estimate certain parameters because their true value must be obtained through the use of time consuming formulas. This paper presents a fast means of determining synchronous pumping speeds for tapered as well as straight strings, based on the Slonneger formulas, but using slightly different constants. Fast means of calculating A.P.I. maximum loads are also presented.

SYNCHRONOUS PUMPING SPEEDS

Whether or not pumping at synchronous speeds adversely affects sucker rod performance and life, is a controversial question. Some operators disregard synchronous speeds entirely, feeling that they have no real effect. Others claim to have eliminated otherwise unexplained failures by moving to a non-synchronous speed. The fact remains, however, that the shape of dynamometer cards is clearly affected by synchronous speeds. It is the author's feeling that on deep heavily loaded strings, pumping at synchronous speeds can have an adverse effect. On lightly loaded strings, such speeds can conceivably be helpful in increasing the effective plunger travel. In any event, a means of quickly and easily determining synchronous and non-synchronous pumping speeds in straight and, in particular, in tapered strings is highly desirable.

In 1937, Mr. J. C. Slonneger's Classic article "Vibration Problems in Oil Wells" was published in A.P.I. DRILLING AND PRODUCTION PRACTICE, 1937 edition. In this article, for the first time, Mr. Slonneger developed formulas and charts to determine pumping speeds which are in phase with the natural harmonic frequency vibrations of the rod string. In developing the acoustic velocity in sucker rods which is the first step in deriving these formulas, he used certain constants which have changed slightly, since that time, with changes in the type of steel and refinements in the joint configuration.

In 1960, Midwest Research Institute measured the physical characteristics of sucker rods with a high degree of accuracy.[1] This work was done in connection with a research program sponsored by "Sucker Rod Pumping Research, Inc.", an organization composed of representatives from both oil companies and equipment manufacturers. One of the constants measured was "E", Young's modulus of elasticity. It was determined to be 30,750,000 lbs/sq. in., which represents the over-all elasticity of the rod, including the joint. This is an average value and was determined from measurements of 45 rods including the four common sizes, from four manufacturers. This value compares with 29,000,000 lbs/sq. in. used by Slonneger in 1937. The average weight of the rods per foot, divided by the area of the rod body has been determined to be 3.657 lbs for today's rods. Since the fluid surrounding the joint would add slightly to the effective mass in movement during vibration, we have increased this value by two per cent to give the final value of W_r/a as 3.73 lbs. This compares with 3.8 lbs used by Slonneger in 1937

Using the above up-dated values, Slonneger's synchronous speed formulas would now be derived as follows:

(1) Nomenclature:

Let V = Velocity of stress transmission, ft./sec.

F = Fundamental frequency or 1st order harmonic.

λ = Length of full wave in ft.

d = Density or mass per unit volume.

E = Young's modulus of elasticity, p.s.i.

$= 30.75 \times 10^6 \times 144$, lbs./sq.ft.

W_r = Weight of rods per ft., lbs.

a = Area of rods, in^2.

g = Acceleration due to gravity, ft./sec.

= 32.2 ft./sec.

D = Length of rod string in ft./1000
N = Harmonic order

(2) Basic Equations:

$$F = \frac{V}{\lambda} \quad (1)$$

$$V = \sqrt{\frac{E}{d}} \quad (2)$$

$$\lambda = 4D \times 10^3 \text{(One end only fixed)} \quad (3)$$

$$d = \frac{W_r D}{g\left(\frac{a}{144}\right)D} = \frac{144\ W_r}{g\ a} \quad (4)$$

(3) As Applied to Sucker Rods:

$\frac{W_r}{a} = 3.73$ average for all rod sizes.

Substituting in (4):

$$d = \frac{144 \times 3.73}{32.2} = 16.68$$

Substituting in (2)

$$V = \sqrt{\frac{30{,}750{,}000 \times 144}{16.68}}$$

$$= 16{,}280 \text{ ft./sec. (acoustic velocity)} \quad (5)$$
$$= 976{,}800 \text{ ft./min.} \quad (5a)$$

Substituting in (1):

$$F = \frac{976.8}{4D} = \frac{244}{D} \quad (6)$$

Formulas (5) and (5a) also represent the velocity of stress transmission in steel sucker rods including the joints.

Formula (6) compares with 237/D as developed in 1937.

When tapered strings are used, formula (6) does not apply since the mass is no longer evenly distributed throughout the full length of the string. It has been shown elsewhere, however, that the fundamental frequency of a rod system, straight or tapered is inversely proportioned to the square root of its static elongation due to its own weight when hanging freely.[2] Using the constants given above, the static elongation is:

$$e = 0.714\ D^2$$

Solving for D and substituting in formula (6) we have:

$$F = 206/\sqrt{e}$$

But the harmonic order is:

$$F/\text{s.p.m.} = N$$

Then:

$$\left(N = 206/\text{s.p.m.}\right)\left(\sqrt{e}\right) \quad (7)$$

This formula can be used to determine the harmonic order of tapered rod strings if we first determine "e" which we defined as being the **static** elongation due to its own weight. Following are the required formulas:

Let W = weight per foot, lbs.
L = length of section, ft./1000
Subscript 1, largest section.
and 2, smallest section

Two-Way Taper

$$e = 0.714\ (L_1^2 + L_2^2) + \frac{L_1 L_2 W_2}{0.7 W_1}$$

$$= 0.714\ (L_1^2 + L_2^2) + L_1 L_2 K$$

Where K = 0.8976 for 5/8" - 1/2" taper
= 1.0026 for 3/4" - 5/8" taper
= 1.0599 for 7/8" - 3/4" taper
= 1.0741 for 1" - 7/8" taper
= 1.1120 for 1-1/8" - 1" taper

Three-Way Taper

$$e = 0.714\ (L_1^2 + L_2^2 + L_3^2) + \left(\frac{W_2 L_2 + W_3 L_2}{0.7\ W_1}\right) L_1\ \frac{W_3\ L_3\ L_2}{0.7\ W_2}$$

$$= 0.714\ (L_1^2 + L_2^2 + L_3^2) + K_a L_1 L_2 + K_b L_1 L_3 + K_c L_2 L_3$$

For 3/4" - 5/8" - 1/2" taper:

$K_a = 1.003$, $K_b = 0.630$, $K_c = 0.898$

For 7/8" - 3/4" - 5/8" taper:

$K_a = 1.060$, $K_b = 0.744$, $K_c = 1.003$

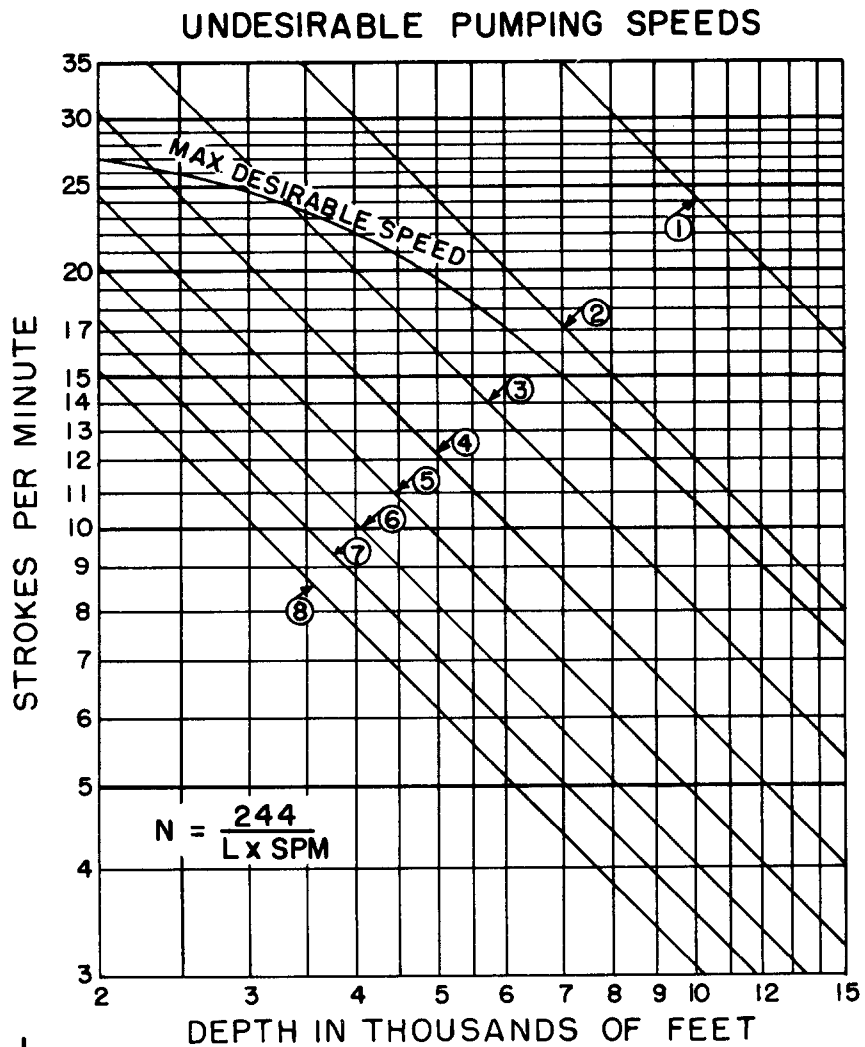
For 1" - 7/8" - 3/4" taper:

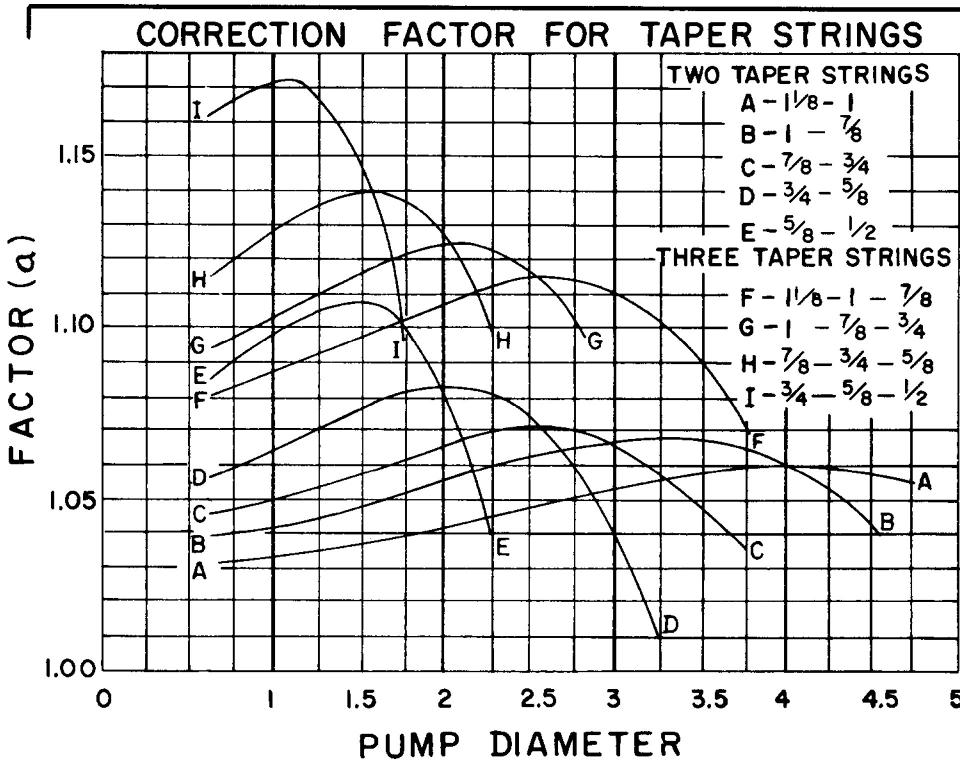
$K_a = 1.074$, $K_b = 0.799$, $K_c = 1.060$

For 1-1/8" - 1" - 7/8" taper:

$K_a = 1.112$, $K_b = 0.838$, $K_c = 1.074$

The above, while simplified, still presents a cumbersome set of figures for rapid calculations. It can be made quite simple by using the graph in the lower part of Fig. 1. This gives a factor which can be applied to the harmonic order of a straight string to obtain the harmonic order of the tapered string being considered.


UNDESIRABLE PUMPING SPEEDS
MAX. DESIRABLE SPEED
STROKES PER MINUTE
DEPTH IN THOUSANDS OF FEET
N = 244 / L x SPM


CORRECTION FACTOR FOR TAPER STRINGS
TWO TAPER STRINGS
A - 1 1/8 - 1
B - 1 - 7/8
C - 7/8 - 3/4
D - 3/4 - 5/8
E - 5/8 - 1/2
THREE TAPER STRINGS
F - 1 1/8 - 1 - 7/8
G - 1 - 7/8 - 3/4
H - 7/8 - 3/4 - 5/8
I - 3/4 - 5/8 - 1/2
FACTOR (a)
PUMP DIAMETER

The pump size used in the graph, does not influence the harmonics of the string. The harmonic order is a function of the percentage of each size rod in the string. Since the pump size governs this percentage, correction factors can be plotted against pump size, if we make certain that the percentages of each size rod in the string, corresponds with reasonable closeness to the percentages given in Table 1. A similar graph was devised by Midwest Research Institute in the sucker rod work mentioned earlier.[3]

Illustrating the use of these correction factors, consider a 6200 ft straight string of rods operating at 15 s.p.m. It would be operating at the 2.6 harmonic order and should be satisfactory. If the string was tapered 7/8 in., 3/4 in. and 5/8 in., however, and with the proportions used for a 1-1/2 in. pump, the correction factor would be (from Curve "H"), 1.139 and the correct order would be: 2.6 x 1.139 or 3, the third order harmonic. Speed should be increased or decreased.

The percentages given in Table 1, differ slightly from some other tables used for the same purpose. In the past some authors, when calculating the fluid load, have considered the full area of the plunger.[4] This gives higher fluid loads than actual, and higher proportions of the larger rods than required for equal stresses. This may be justified in actual operation. However, we have used the net area of the plunger just above the pump; i.e., the area of the plunger minus the area of the smallest rod, in calculating the fluid load. This, too, gives higher than actual fluid loads, since the larger rods displace more fluid than the small rods but we feel this is closer to the actual requirements and have used these percentages for many years. It is worth noting that if the percentages being used in the field do not agree with the percentages shown in the table for the pump size being used, fairly good accuracy can be obtained by reading the correction factor above the pump size shown opposite the actual percentage. For instance, if 30 per cent of 7/8 in. rods in a 7/8 in, 3/4 in taper was being used with a 1-1/4 in. pump, read the correction factor 1.058, shown for a 1-1/2 in. pump.

A.P.I. MAXIMUM ALLOWABLE LOADS

A.P.I. Standard 11-B - Sixteenth Edition,

TABLE 1

CORRECT LENGTHS OF COMPONENT PARTS OF TAPERED STRINGS

Pump	5/8 - 1/2	3/4 - 5/8	7/8 - 3/4	1 - 7/8	1-1/8 - 1	3/4 - 5/8 - 1/2		7/8 - 3/4 - 5/8		1 - 7/8 - 3/4		1-1/8 - 1 - 7/8	
Size	% 5/8	% 3/4	% 7/8	% 1	% 1-1/8	% 3/4	% 5/8	% 7/8	% 3/4	% 1	% 7/8	% 1-1/8	% 1
3/4	30.8	25.1	21.7	18.7	16.7	21.5	25.1	18.4	20.7	15.8	18.3	14.1	16.0
7/8	33.4	26.5	22.7	19.3	17.1	23.4	27.3	19.5	21.9	16.5	19.1	14.5	16.5
1	36.4	28.2	23.7	20.0	17.6	25.5	29.7	20.7	23.2	17.3	20.0	15.1	17.1
1-1/16	38.0	29.1	24.3	20.4	17.8	26.6	31.1	21.4	24.0	17.7	20.5	15.4	17.4
1-1/4	43.6	32.3	26.3	21.7	18.7	30.5	35.6	23.7	26.6	19.2	22.1	16.4	18.5
1-1/2	52.4	37.2	29.5	23.8	20.1	36.7	42.8	27.3	30.6	21.5	24.8	17.9	20.3
1-5/8	57.5	40.0	31.3	25.0	21.0	40.2	46.9	29.4	33.0	22.8	26.3	18.8	21.3
1-3/4	62.9	43.0	33.2	26.3	21.9	44.0	51.4	31.6	35.5	24.2	28.0	19.8	22.4
1-25/32	64.3	43.8	33.7	26.6	22.1	45.0	52.5	32.2	36.1	24.6	28.4	20.0	22.7
2	74.9	49.8	37.5	29.1	23.8			36.5	41.0	27.3	31.6	21.9	24.8
2-1/8	81.5	53.5	39.9	30.7	24.9			39.3	44.0	29.1	33.6	23.1	26.2
2-1/4	88.6	57.4	42.4	32.3	26.0			42.1	47.3	30.9	35.7	24.3	27.6
2-3/4		75.4	53.9	39.9	31.2					39.3	45.4	30.0	34.1
3-1/4		96.9	67.7	49.0	37.4							36.9	41.8
3-3/4			83.8	59.6	44.7							44.9	50.9
4-1/4				71.7	53.0								
4-3/4				85.4	62.4								

Supplement 1, presents a recommended method for determining the maximum allowable sucker rod load based on the minimum tensile strength of the rod, utilizing the well-known Goodman diagram in a modified form. Its application can be greatly simplified, by the use of two tables.[5] In developing this short cut we shall use the following nomenclature:

S_a = Maximum allowable stress for non-corrosive service, psi

S_{min} = Minimum downstroke stress, psi

P_a = Maximum allowable load, lbs.

P_{min} = Minimum downstroke load, lbs.

A_r = Area of top rod, sq. ins.

W_R = Total weight of rod string in air, lbs.

C = $Ln^2/70{,}500$

L = Length of stroke, ins.

n = Strokes per minute

T = Minimum tensile strength, psi

Sp.Gr.= Specific gravity

S_f = Service factor

Equation 2 in the above mentioned A.P.I. Supplement is:

$$Sa = (0.25\ T + 0.5625\ S_{min})\ S_f$$

This can be rewritten:

$$Pa = (0.25T\ A_r + 0.5625\ P_{min})\ S_f \quad \text{(a)}$$

P_{min} when Sp.Gr. is one is conventionally written[6]

$$P_{min} = W_r(1-C) - 62.5\ W_r/490$$
$$= W_r(0.8725-C)$$

This formula takes into account negative inertial and bouyancy effects of the downstroke. It does not consider rubbing and fluid friction which act in the same direction. A study of a great many dynamometer measurements of the minimum load indicates that actual minimum loads average about 80 per cent of the theoretical loads given by this formula. Applying this empirical factor we have:

$$P_{min} = 0.8\ W_r\ (0.8725-C) \quad \text{(b)}$$

Substituting (b) in (a) we have

$$P_a = [0.25\ T\ A_r + W_r(0.393 - 0.450C]\ S_f \quad \text{(c)}$$

Let $Y = 0.25\ T\ A_r$

Let $Z = (0.393 - 0.450C)$

Then where Sp. Gr. is one and P_{min} is unknown:

$$P_a = (Y + W_rZ)\ S_f \quad \text{(d)}$$

Where P_{min} is known

$$P_a = (Y + 0.5625\ P_{min})\ S_f \quad \text{(e)}$$

Y can be tabulated for each size and grade of rod as shown in Table 2, which was prepared for rods with 100,000 psi minimum tensile strength.

Z can be tabulated as shown in Table 3, for all rods.

The service factor S_f is to be applied to this basic rating as indicated and is to compensate for the corrosivity of the fluid. Since all well fluids are corrosive to some degree, and since corrosivity of well fluid varies greatly, it is recommended that each operator select his own values of S_f as his own experience dictates. It could be greater than one, although normally it will be less than one, varying inversely with the severity of the corrosion.

Thus, formulas (d) or (e) provide a simple, flexible means of arriving at the maximum allowable load based on a realistic stress-range diagram. One needs only to calculate the weight of the rod string in air, select two or three appropriate values from the tables, then simple multiplication and addition give the answer.

Tapered Strings: Where tapered strings are used, the above methods still apply if the strings are proportioned in such a manner that stresses at the top of each section are equal or the stress in the top section is greater than in the other sections. Where there are more small rods in the string than the above proportion requires, the smaller rods would be stressed higher than the top rod and misleading allowable loads would result.

Example:

100,000 psi minimum tensile strength rods
4,000 ft pump setting
2 1/4-in. pump
20, 74-in. strokes
1,700 ft - 7/8-in. rods and 2,300 ft - 3/4-in. rods
Specific gravity - 1.00
Fluid level at the pump

TABLE 2

Y Factor

(Y = 25,000 A)

Size of Top Rod	"Y"
1/2"	4908
5/8"	7670
3/4"	11045
7/8"	10532
1"	19635
1-1/8"	24850

TABLE 3

Z Factor

(Z = 0.393 - 0.450C)

S P M	STROKE LENGTH						
	54	64	74	86	100	120	168
4	.388	.387	.385	.384	.383	.381	.376
6	.380	.378	.376	.373	.370	.366	.354
8	.371	.367	.363	.358	.352	.344	.324
10	.358	.352	.346	.338	.329	.317	.286
12	.344	.334	.325	.314	.301	.283	.239
14	.326	.313	.300	.285	.268	.243	.183
16	.305	.289	.272	.253	.230	.197	--
18	.281	.261	.240	.215	.186	--	--
20	.255	.230	.204	.173	--	--	--
22	.226	.195	.164	--	--	--	--

Weight of rod string in air - 7,392 lbs.
Polished rod load - 18,310 lbs.
Fully inhibited against corrosion
$S_f = 1$)

Using formula (d):

$$P_a = (15{,}032 + .204 \times 7{,}392) = 16{,}540 \text{ lbs.}$$

The allowable load of 16,540 lbs. is less than the actual load of 18,310 lbs. The string is overloaded.

Refiguring the same string but with 16 strokes per minute and a measured polished rod of 16,780 lbs., we have:

$$P_a = (15{,}032 + .272 \times 7{,}392) = 17{,}040 \text{ lbs.}$$

This string is not overloaded since corrosion is not a factor but would be if a service factor of 0.90 for mild corrosion were applied.

$$P_a = (17{,}044 \times 0.90) = 15{,}340 \text{ lbs.}$$

Any possible controversy as to the validity of the above suggested methods, lies only in the percentages called out in Table 1, or in the 0.8 factor used in formula (b). The former could be resolved by a new table and by replotting the curves for the correction factor in Fig. 1. The question of the 0.8 factor could be resolved by using a different factor and refiguring Table 3, or by using formula (e) only.

REFERENCES

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