SHALLOW SUCKER ROD WELLS AND FLUID INERTIA

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ABSTRACT

Fluid inertia forces can be large enough to be considered in sucker rod simulators. Fluid inertia forces are larger with shallow wells, large pumps, high stroke rates, and/or lower compressibilities. Texas A&M U has developed a fluid inertia model. This paper covers the primary assumptions, the equations, the boundary conditions, the initial conditions, a parameter study based on field data, the solution method, the discretized equations, the pump position, and the calculation sequence.

INTRODUCTION

Sucker rods are the most common artificial lift method used in the United States[1]. For most wells the forces due to fluid acceleration are small compared to the rod forces. Even so, fluid inertia forces can be important. The evidence includes the recent development of a downhole pulsation damper[2]. This paper is for rod pumping systems that see significant fluid inertia forces.

Sucker rod dynamics are usually modeled with Hooke's Law and Newton's Second Law. Such a model applies only the hydrostatic pressure at the pump. Fluid acceleration is excluded. Fortunately, the fluid inertia forces are often negligible compared to the forces covered by Newton's and Hooke's laws. Newton's and Hooke's laws give good results as long as the fluid inertia forces are negligible.

Shallow wells, large pumps, high stroke rates, and low compressibility increase the fluid acceleration forces relative to the rod forces. Production personnel have developed rules-of-thumb to cover such conditions. Indeed, API RP 11L[3] warns about the impact of fluid inertia.

The dynamic forces due to the acceleration of the fluid column are described by another series of equations. These include an equation of motion, a continuity equation, and an equation of state. This paper shows how to use the equations to develop a fluid inertia sucker rod simulator.

ASSUMPTIONS

The assumptions give a simple, one dimensional model for a Newtonian liquid. The rod is assumed to operate in the elastic range.

The rod element size, dX, is constant. The fluid element size, dZ, is constant.

The rod and the fluid equations are treated separately, except at the traveling valve. The pump velocity determines the fluid velocity at the pump. The fluid pressure at the pump determines the force on the pump.

Only viscous damping is included in the model. The rod damping is proportional to the rod velocity. The fluid damping is proportional to the fluid velocity. The rod viscous damping factor accounts for sliding friction and for structural damping.

The pump is assumed to be full and anchored.

The sign convention is positive up. Gravity is negative; g_z equals negative 32.1740 ft $/s^2$

The produced fluid is assumed to behave like a slightly compressible liquid. The composition is expected to be homogeneous. The gas anchor eliminates all free gas. The flow path is assumed to be a vertical, constant cross section, concentric annulus between the rod and tubing. Laminar flow is assumed.

These are much like the assumptions of Doty & Schmidt[4].

Other assumptions are listed in the body of the paper.

INTRODUCTION TO THE ROD EQUATIONS

Hooke's Law, Newton's Second Law, and the sonic velocity equation combine to give the classic, second order, partial differential wave equation (pde).

$$U_{tt} + C_r\, U_t = a_r^2\, U_{xx} \qquad (1)$$

Experience shows that a pair of first order pdes has numerical advantages over the classic wave equation. It is also easier to apply the force boundary condition at the pump. Newton's Second Law and Hooke's Law define the two, first order pdes.

Rod Equation of Motion

The rod equation of motion is derived from a force balance on an arbitrary element of rod[5]. This gives a second order pde when written in terms of rod stretch. Rod stretch is the same as rod displacement.

$$U_{tt} + C_r\, U_t - dF\, g_c/dM = 0 \qquad (2)$$

The rod mass term may be expanded for each element.

$$dM = \rho_r\, A_r\, dX/144 \qquad (3)$$

Substitution for the mass term gives

$$U_{tt} + C_r\, U_t - 144\, F_x\, g_c/(\rho_r\, A_r) = 0. \qquad (4)$$

The equation may be written in terms of rod stretch velocity. This gives the desired first order pde. Rod velocity is written as V_r to avoid confusion with fluid velocity, V_F.

$$V_{rt} + C_r V_r - 144 F_x g_c/(\rho_r A_r) = 0 \quad \text{(5)}$$

All the above forms exclude the static, buoyed weight of the rods. This is allowed by linear superposition because the static weight at each node is constant. The buoyed weight of the rod is added to the rod force at the end of the calculations. An alternative exists. The following form shows the gravity acceleration term. Note: up is positive, so g_z is negative.

$$V_{rt} + C_r V_r - 144 F_x g_c/(\rho_r A_r) - g_z (1 - \rho_F/\rho_r) = 0 \quad \text{(6)}$$

Equation of State

The equation of state for the rod is Hooke's Law[6]. Hooke's Law says that stress and strain are proportional to Young's Modulus.

$$F = E A_r U_x \quad \text{(7)}$$

The time derivative of Hooke's Law is the other first order pde.

$$F_t = E A_r V_{rx} \quad \text{(8)}$$

Rod Sonic Velocity

The sonic velocity is a material property. The same material properties appear in the classic wave equation. Thus, sonic velocity is part of the wave equation[7,8]. Sonic velocity is an important part of the stability requirement. This is because strain waves move through the rod at the sonic velocity.

$$a_r^2 = 144 g_c E/\rho_r \quad \text{(9)}$$

Summary of the Rod Equations

The wave equation is a combination of Newton's Second Law, the space derivative of Hooke's Law ($F_x = E A_r U_{xx}$), and the sonic equation.

This completes the rod equations. Another set of equations is needed to solve the fluid inertia problem. The other set is known as the fluid equations.

INTRODUCTION TO THE FLUID EQUATIONS

The fluid and the rod equations are similar in many ways. Both are vibration problems. Both see the transmission and reflection of sonic waves between the wellhead and the pump.

The pump motion makes density waves travel up the fluid column. The pressure goes up as the fluid is compressed. This gives the two part key to solving the fluid inertia problem. First, liquid compression implies a pressure increase. Second, the force on the pump is the product of fluid pressure times pump area. Significant, dynamic pressure changes must be added to the hydrostatic pressure of the fluid column. Shallow wells, wells with high strokes per minute, wells with large pumps, and wells with low compressibility fluids have higher dynamic fluid forces. This explains why fluid inertia is more significant in shallower, higher speed, higher rate wells and in wells with less compressible fluids.

The next sections summarize the equations that describe the motion of pressure waves through the fluid column.

Fluid Equation of Motion

The equation of motion comes directly from Bird, Stewart, and Lightfoot[9]. The derivation starts with the generalized equation in cylindrical coordinates. This is nearly one dimensional because the radial and angular velocity components are zero. However, the vertical stress tensor, τ_{Rz}, is two dimensional. The stress tensor is two dimensional because the laminar velocity distribution in the radial direction tends to be parabolic for laminar flow.

This simulator is forced to be one dimensional because we substitute a fluid damping term multiplied by the average fluid velocity for the stress tensor. This substitution is valid for laminar flow. The stress tensor for turbulent flow approaches zero. This is because the turbulent velocity distribution is nearly constant across the annulus. This model uses the laminar substitution.

The fluid damping term, C_{FP}, is derived from the power lost to friction. Power equals flow rate times pressure drop[10], and power equals mass times acceleration times velocity.

$$V_{Fav}\ a_F\ dP_f = (\rho_F\ a_F\ L/144)\ (C_{FP}\ V_{Fav}/g_c)\ V_{Fav} \quad \text{.................... (10)}$$

The friction pressure drop gradient up the annulus is

$$dP_f\ /dL = \mu_c\ V_{Fav}/(1000\ (d_o-d_i)^2). \quad \text{................................ (11)}$$

The fluid damping factor equation is

$$C_{FP} = 4.632\ \mu_c/(\rho_F\ (d_o-d_i)^2). \quad \text{.................................. (12)}$$

This simulator uses a single C_{FP} value for the entire fluid column.

$$(V_{Ft}+V_F\ V_{Fz})\ +144\ P_z\ g_c/\rho_F\ +C_{FP}\ V_{Fav}\ +\rho_F\ g_z/g_c = 0 \quad \text{................ (13)}$$

Each rod taper can have a different fluid damping factor. This is because the viscosity and density values can modify the damping term for each fluid element. The viscosity form of the fluid equation of motion becomes

$$(V_{Ft}+V_F\ V_{Fz}) + 144\ P_z\ g_c/\rho_F + 4.632\ V_{Fav}\ \mu_c/(\rho_F\ (d_o-d_i)^2) + \rho_F\ g_z/g_c = 0. \qquad (14)$$

If the viscosity form of the equation of motion is going to be used it makes sense to consider turbulent fluid damping as well.

Continuity Equation

The continuity equation is

$$d\rho_F/dT + d(\rho_F\ V_F)/dZ = 0. \qquad (15)$$

Equation of State

The equation of state for slightly compressible fluids is as follows[11,12]. Hewlett-Packard's Petroleum Fluids Pac is a quick source for the reference pressure and the reference fluid density[13]. Another quick source is to use the density at atmospheric pressure.

$$\rho_{F1} = \rho_{Fo}\ [1 + C_o\ (P_1-P_o)] \qquad (16)$$

The slightly compressible assumption has a limitation. A highly compressible fluid violates the assumption. An alternative is to use the exact equation of state.

$$\rho_{F1} = \rho_{Fo}\ e^{C_o\ (P_1-P_o)} \qquad (17)$$

Fluid Sonic Velocity

The speed of the compression waves is the fluid sonic velocity[14].

$$a_F^2 = 144\ g_c/(C_o\ \rho_F) \qquad (18)$$

Summary of the Fluid Equations

The solution to the fluid problem is a combination of the equation of motion, the continuity equation, the equation of state, and the sonic equation.

This completes the equations needed to solve the fluid compressibility problem. The next step covers how the rod and the fluid equations interact.

COUPLING

The rod and the fluid problems are solved independently, except at the traveling valve. This is called a lightly coupled system. Light coupling is in keeping with Gibbs' recommendation for using rod velocity to calculate viscous rod damping[15]. In a real well the fluid drag on the rod depends on the fluid velocity too. Fortunately, history matching validates Gibbs' approach. This supports the assumption that interaction between the rod motion and the fluid is significant only at the pump.

BOUNDARY CONDITIONS

The solution sequence is determined by the boundary conditions. The boundary conditions are introduced as they are needed. The finite difference method needs a boundary condition to start each of the calculation sequences.

Polished Rod Velocity

The polished rod velocity is the first boundary condition for the rod equation. This allows velocity calculations from the polished rod to the traveling valve to begin. The polished rod velocity may be calculated in different ways. Svinos[16] uses exact kinematics. Laine, Cole, and Jennings[17] use a Fourier series to accurately approximate exact kinematics. API RP 11L uses six percent second harmonic for conventional pump jacks.

Pump Velocity

The traveling valve velocity is the boundary condition for the fluid equations. Thus, the solution to the rod velocity equation provides the starting point for the fluid velocity equation.

Wellhead Pressure

Wellhead pressure is the second boundary condition for the fluid equations. This and the calculated fluid velocity are the starting points for the fluid pressure equation.

Rod Forces

Strictly speaking, most of the rod forces could have been found as soon as the rod velocities were known. However, the force at the pump would have remained unknown.

Pump Force

The force on the traveling valve depends on two pieces of information. One is the pressure at the pump. The other is the status of the traveling valve.

The pump force tensions the rod when the traveling valve is closed. The pump force lightly compresses the rod when the valve is open. Strictly speaking, three forces are at work. When the valve is closed most of the force is due to pressure times area. Flow through the open valve drags on the pump. Both directions cause drag between the moving and stationary parts of the pump. As a practical matter both drag forces tend to be small. This model assumes both drag forces are zero. Therefore the minimum and maximum pump forces are as follows.

$$F_{PN} = 0 \quad \text{(19)}$$

$$F_{PX} = A_p \left[-P_{ner} + P_{csg} - \frac{\rho_{Flner}}{144} (X_{ner} - X_{nl}) \right] \quad \text{(20)}$$

Note: a negative pump force pushes down on the traveling valve and puts tension into the rod.

Rod Force at the Pump

The rod force at the pump is a function of the pump boundary condition. The pump boundary condition is a function of the rod force at the pump. This means the boundary condition depends on the solution to the equations, and the solution depends on the boundary conditions.

This simulator resolves the dilemma by assuming the pump velocity is zero. The pump force is then estimated from Hooke's Law. If the estimated force is between the maximum and minimum pump forces, it then becomes the calculated force.

Otherwise, the pump force is set at the maximum or the minimum, whichever was exceeded. The pump velocity is then calculated from the rod velocity equation.

Doty & Schmidt present a clear discussion of the procedure for finding the pump motion. Gibbs[18] Schafer[19], and Bowlin[20] also discuss this topic.

INITIAL CONDITIONS

All the initial conditions are set at the static value. This says the system has been off long enough to stop moving. The initial rod and fluid velocities are all zero. The initial dynamic rod forces are all zero. The initial fluid pressures are all hydrostatic.

SOLUTION METHOD

Finite Differences

This model uses finite differences to solve the equations. The method substitutes a truncated Taylor series for each derivative. It is important to have a consistent sign convention. The sign convention is positive up. This means all positions and velocities that act in the upward direction are greater than zero. Rod tension is positive. Rod compression is negative. Here is an example.

$$\frac{dX}{dT} = \frac{X_1 - X_2}{T_2 - T_1} \quad \text{(21)}$$

The subscripts are correct. They are correct because time increases in the positive direction. The rod node numbers decrease in the positive direction. Node zero is at the polished rod. The highest node number is at the pump.

The equations are then solved algebraically. The unknown information is put on one side of the equal sign. The known values are put on the other side. The explicit method allows only one unknown per equation. This means some values must come from the previous time step. The explicit method is the simplest method because it avoids matrix solutions. The explicit method needs only algebra. The downside is that the solution can be unstable.

Stability and Accuracy

There are two stability requirements. Both limit the element length divided by the time step relative to the sonic velocity. Both criteria are for undamped waves. Damping improves stability. Experience shows that precision is best when the following equalities are satisfied.

$$a_{rj} = \text{rod sonic velocity} \le dX_j/dT \quad (22)$$

$$a_{Fk} = \text{fluid sonic velocity} \le dZ_k/dT \quad (23)$$

It is convenient, though not necessary, to let dX and dZ be constant. Precision is best when dX/dT and dZ/dT are close to the equalities.

This simulator reads the number of rod elements from the data file. The rod element size is calculated from the rod length. The maximum time step size is then estimated from the highest sonic velocity. This is usually a rod sonic velocity.

$$dX = X_{ner} / ner \quad (24)$$

$$dT_{max} = dX / a_{rj,max} \quad (25)$$

The actual time step size is picked so there are an integer number of time steps in one stroke. This meets the stability requirement as long as the selected time step is smaller than the maximum. The first of the next equations uses integer math. This forces nts to be an integer. One is added to guarantee that the time step is less than the maximum time step.

$$nts = 60 / (N\ dT_{max}) \quad (26)$$

$$dT = 60 / N / (nts + 1) \quad (27)$$

Once the time step size is known, the size and number of fluid elements are calculated. Integer math truncates nef to an integer. The fluid element size is smaller than the rod element size.

$$dZ \approx a_F\ dT \quad (28)$$

$$nef = X_{ner} / dZ \quad (29)$$

$$dZ = X_{ner} / nef \quad (30)$$

There is a trade off between speed and precision. Larger fluid element sizes honor the fluid sonic velocity stability criteria. The simulator runs faster with larger fluid elements. Accuracy goes down with larger fluid elements.

This was tested with the fluid element size equal to the rod element size. The loss of precision is on the order of ten percent or less.

DISCRETIZED EQUATIONS

The following summarizes the discretized equations. The reader can confirm them by substituting for the derivatives and completing the algebra. The subscript convention works like this. Subscripts are identified by the symbols "j" and "k". Rod node j-1 is above node j. Fluid node k+1 is below node k. Node zero is at the polished rod. The presence of the "1" symbol after a variable name and before the node subscript indicates data from the previous time step. V_{r1j} happens one time step before V_{rj}.

Polished Rod Velocity, Node j = Zero

$$V_{ro} = \text{polished rod motion} \quad (31)$$

Rod Equation of Motion, Nodes j = 1 to j = ner-1

Use this form of the equation for all but the bottom node.

$$V_{rj} = \frac{(2 - C_{rj}\ dT)\ V_{r1j} + \dfrac{144\ g_c\ 2\ dT}{\rho_{rj}\ A_{rj}}\ \dfrac{(F_{1j-1} - F_{1j+1})}{(dX_j + dX_{j+1})}}{(2 + C_{rj}\ dT)} \quad (32)$$

Rod Equation of Motion, Node j = ner

Use this form of the equation for the bottom rod node before the fluid velocities and pressures are updated. A later equation is used to find the pump velocity after the fluid velocities and pressures are updated.

$$V_{rj} = \frac{(2 - C_{rj}\ dT)\ V_{r1j} + \dfrac{144\ g_c\ 2\ dT}{\rho_{rj}\ A_{rj}}\ \dfrac{(F_{1j-1} - F_{1j})}{dX_j}}{(2 + C_{rj}\ dT)} \quad (33)$$

Equation of State, All Nodes

$$\rho_{F1k} = \rho_o\ [1 + C_o\ (P_{1k} - P_o)] \quad (34)$$

Fluid Velocity, Downstroke, Node k = nef

$$V_{Fnef} = \frac{V_{rner}\ A_{rner}}{A_{Tnef} - A_{rner}} \quad (35)$$

Fluid Velocity, Upstroke, Node k = nef

$$V_{Fnef} = \frac{V_{rner} \quad A_{pner}}{A_{Tnef} - A_{rner}} \qquad \text{(36)}$$

Fluid Equation of Motion, Nodes k = nef-1 to k = 1

$$V_{Fk} = \frac{V_{F1k}\left[\frac{1}{dT} - \frac{(V_{F1k-1} - V_{F1k+1})}{2\ (dZ_k + dZ_{k+1})} - \frac{C_{FPk}}{2}\right] - \frac{g_c}{\rho_{F1k}} \frac{(P_{1k-1} - P_{1k+1})}{(dZ_k + dZ_{k+1})} - g_z}{\frac{1}{dT} + \frac{V_{F1k-1} - V_{F1k+1})}{2\ (dZ_k + dZ_{k+1})} + \frac{C_{FPk}}{2}} \qquad \text{(37)}$$

Fluid Equation of Motion, Node k = Zero

$$V_{Fo} = \frac{V_{F1k}\left[\frac{1}{dT} - \frac{(V_{F1k} - V_{F1k+1})}{2\ dZ_k} - \frac{C_{FPk}}{2}\right] - \frac{g_c}{\rho_{F1k}} \frac{(P_{1k} - P_{1k+1})}{dZ_k} - g_z}{\frac{1}{dT} + \frac{(V_{F1k} - V_{F1k+1})}{2\ dZ_k} + \frac{C_{FPk}}{2}} \qquad \text{(38)}$$

Wellhead Pressure, Node k = Zero

$$P_k = P_{tub}$$

Fluid Continuity & Equation of State, Nodes k = 1 to k = nef-1

$$P_k = \frac{\frac{P_{1k}}{dT} - \frac{(V_{Fk-1} - V_{Fk+1})}{(dZ_k + dZ_{k+1})}\left[\frac{1}{C_o} + \frac{P_{1k}}{2} - P_o\right] - \frac{(V_{Fk} + V_{F1k})}{2} \frac{(P_{1k-1} - P_{1k+1})}{(dZ_k + dZ_{k+1})}}{\frac{1}{dT} + \frac{(V_{Fk-1} - V_{Fk+1})}{2\ (dZ_k + dZ_{k+1})}} \qquad \text{(39)}$$

Fluid Continuity & Equation of State, Node k = nef

$$P_k = \frac{\frac{P_{1k}}{dT} - \frac{(V_{Fk-1} - V_{Fk})}{dZ_k}\left[\frac{1}{C_o} + \frac{P_{1k}}{2} - P_o\right] - \frac{(V_{Fk} + V_{F1k})}{2\ dZ_k} (P_{1k-1} - P_{1k})}{\frac{1}{dT} + \frac{(V_{Fk-1} - V_{Fk})}{2\ dZ_k}} \qquad \text{(40)}$$

Rod Force, Nodes j = ner-1 to j = Zero

$$F_j = E_j \; A_{rj} \; \frac{dT}{dX} \; \frac{(V_{rj+1} - V_{rj-1})}{2} + F_{1j} \quad \ldots\ldots \quad (41)$$

Rod Force at the Pump

Rod force at the pump is one of the boundary conditions. This boundary condition is discussed in detail elsewhere.

Rod Force at the Pump, Stationary Pump

$$F_j = E_j \; A_{rj} \; \frac{dT}{dX} \; (0 - V_{rj-1}) + F_{1j} \quad \ldots\ldots \quad (42)$$

Rod Force at the Pump, Moving Pump

$$V_{rj} = \frac{(2 - C_{rj}\; dT)\; V_{r1j} + \frac{144\; g_c\; 2\; dT}{\rho_{rj}\; A_{rj}} \; \frac{(F_{j-1} - F_j)}{dX_j}}{(2 + C_{rj}\; dT)} \quad \ldots\ldots \quad (43)$$

PUMP STROKE

The pump stroke is the difference between the relative minimum and relative maximum pump positions during the final stroke. The pump positions at each time step come from the integral of the rod velocity during the last stroke.

$$U_j = \int_0^{nts\; dT} V_{rj} \; dT \quad \ldots\ldots \quad (44)$$

PUMP POSITION

Pump stroke can easily be extended to estimate the absolute pump position. The static stretch due to the buoyed weight of the rod hanging in the hole is needed. First get the buoyed weight for each rod taper.

$$W_{rbi} = W_{ri} \left(1 - \frac{\rho_{Fi}}{\rho_{ri}}\right) \quad \ldots\ldots \quad (45)$$

Find the static stretch for each rod taper.

$$U_{Si} = \frac{X_i}{E_i \; A_{ri}} \left[\frac{W_{rbi}}{2} + \sum_{n=i+1}^{nrt} W_{rbn} \right] \quad \ldots\ldots \quad (46)$$

Add the taper stretches together.

$$U_S = \sum_{1}^{nrt} U_{Si} \quad \text{(47)}$$

Modify the pump stroke integral to start at time zero. This modification keeps track of the pump relative to its initial, static position. Save all pump positions during the last cycle. Also keep track of the maximum and the minimum positions.

$$U_j = \int_0^{nts\ dT\ ncycl} V_{rj}\ dT \quad \text{(48)}$$

The pump position when the unit is off is the rod length plus the static stretch. Add the maximum and the minimum values of the pump stroke to estimate the range of pump motion.

$$U_{ner,static} = X_{ner} + U_S \quad \text{(49)}$$
$$U_{ner,max} = U_{ner,static} + U_{j,max} \quad \text{(50)}$$
$$U_{ner,min} = U_{ner,static} + U_{j,min} \quad \text{(51)}$$

This gives the ideal, absolute pump positions. Keep the vertical and the concentric assumptions in mind. Vertical and concentric means the rod hangs in the well without touching the tubing.

CALCULATION SEQUENCE

This section talks about how to use the discretized equations.

Initial Conditions

The initial rod and fluid velocities at all nodes are zero. This is because the system is at rest at the start of the first time step. Time equals zero at the start of the first time step.

The dynamic initial rod force at all nodes is zero. This is because there is no dynamic stretch when the system is at rest. The static force is superimposed on the dynamic forces at the end of the calculations.

The initial fluid pressure is the hydrostatic pressure at each node.

$$P_k \approx 0.433\ \gamma\ Z_k + P_{tub} \quad \text{(52)}$$

Rod Equation of Motion

Start at the polished rod. The polished rod velocity gives the needed boundary condition, V_{ro}. The following example gives polished rod velocity for a conventional pump jack. The Fourier coefficients come from Laine, Cole, and Jennings. (Note the similarity with API RP 11L's six percent second harmonic model.)

$$V_{ro} = \frac{S_{tk}\ \omega}{12} [.4973 \cos(\omega T) +.0631 \cos(2\omega T) +.0078 \sin(\omega T) +.0124 \sin(2\omega T)] \quad \text{.....................} \quad (53)$$

The other rod velocities are solved from the top down. Start at node one.

All the terms on the right hand side are known. A_{rj}, C_r, dT, dX, and ρ_{rj} are based on input data.

The rod damping factor, C_r, is the product of the dimensionless rod damping factor and the natural angular speed of the rod. The dimensionless factor is usually between 0.05 and 0.15. Heavy, viscous oils may need a higher rod damping factor. See Jennings and Laine[21], equations 3, 4, and 5, for composite rod sonic velocity.

$$C_r = C_d\ \omega_o \quad \text{..} \quad (54)$$

$$\omega_o = \pi\ a_{ro} / (2\ X_{ner}) \quad \text{..} \quad (55)$$

For the first time step V_{r1j}, F_{1j}, and F_{1j-1} are the initial conditions. After the first time step V_{r1j}, F_{1j}, and F_{1j-1} are known from the previous time step.

In general, the polished rod and the pump nodes require slightly different treatment. This means the rod velocity equation is different for the pump node. The difference is that there is no node below the bottom node. F_{1j} is used instead of F_{1j+1}.

In summary, the rod velocity at node j is a function of three independent variables. First is the previous rod velocity at node j, V_{r1j}. The other two are the previous rod forces immediately above and below node j, F_{1j-1} and F_{1j+1}.

Equation of State

The equation of state provides an explicit update of the fluid density. This is explicit because the updated density is based on the pressure at the end of the previous time step.

Fluid Velocity, Continuity Equation

Start at the traveling valve. A decision is required. Was the traveling valve moving up or down during the previous time step? Use the right equation. The upstroke uses the pump area to get fluid velocity at the pump. The downstroke uses rod and tubing areas at the pump to get the fluid velocity.

Fluid Equation of Motion

One fluid velocity equation applies to the intermediate nodes. The equation of state is used to get the fluid density for the previous time step, ρ_{Flk}. Three velocities and two pressures are needed from the previous time step. For the first time step V_{Flk-1}, V_{Flk}, V_{Flk+1}, P_{1k-1}, and P_{1k+1} are the initial conditions. After the first time step all five are known from the previous time step. Node k-1 is immediately above node k. Node k+1 is immediately below node k. All the other terms are based on input data. These are C_{FP}, dT, dZ, g_c, and ρ_{rj}.

The equation is discretized differently at the polished rod because V_{Flk-1} and P_{k-1} do not exist. Instead, the equation is discretized in terms of V_{Fk} and P_{1k}.

Wellhead pressure

The pressure at the wellhead, P_{tub}, is another boundary condition.

Fluid Continuity & Equation of State

Start at node one, and calculate pressures at each node. Remember to use the other pressure equation for the bottom node. Note that all the fluid velocities are for the current time step. Use the velocities just calculated for the current time step. Use the initial pressure conditions for time step zero. The next time steps will use the pressures calculated at this time step.

Rod Force

Most of the rod forces could have been calculated sooner. The discretized equation is straight forward. The exception is the pump force.

Rod Force at the Pump

At first glance, one may erroneously conclude that sucker rod simulation is a trivial problem. Actually, the pump boundary condition is difficult to formulate in mathematical terms. This is because the fluid load on the pump is a function of the pump velocity. The traveling valve may be stationary, moving up, or moving down.

The traveling valve is closed when the pump is moving up, and the maximum fluid pressure acts on the pump area. The traveling valve is open when the pump is moving down, and the net fluid pressure on the pump area is zero. Both the traveling and the standing valves are closed when the pump velocity is zero. Since the fluid load is transferred between the standing and traveling valves while both are closed, the pump force must be between the zero and the maximum value during the load transfer.

The first step in the procedure is to assume the pump is stationary, and use the appropriate equation to calculate the rod force at the pump. Physically, this means finding the rod force that satisfies the imposed rod elongation. If the calculated load is between zero and the maximum fluid load, then the pump velocity really is zero. If the calculated load exceeds the maximum fluid load, then the pump is moving up, the rod force is equal to the maximum fluid load, and the pump velocity may be calculated with the appropriate equation. If the calculated load is negative, then the pump is moving down, the rod force is zero, and the pump velocity may be calculated with the appropriate equation. Note: pump drag is assumed to be zero.

In summary, when the traveling valve is moving the pump force is bounded. Set the pump force at that limit, and calculate the traveling valve velocity. Be sure to use the current force values F_{j-1} and F_j in the rod velocity equation.

NEXT TIME STEP

This completes one time step. Add dT to the previous time to get the next time. Repeat the calculation sequence until the problem is solved.

CONVERGENCE AND ACCURACY

This simulator runs a predetermined number of cycles (or strokes) and stops. Five cycles is nominal. It usually takes more than four cycles to reach steady state. This is because the calculations start from a full stop. It takes a few strokes for the transients to die out. The actual number of cycles to reach convergence depends on the details of the case and on the required number of significant digits. More cycles give better convergence. More cycles also use more computer time. Past experience indicates that high spm and/or low viscous damping require extra cycles to approach steady state.

It would be relatively easy to let the model run until it reached a desired level of convergence.

Accuracy is related to the size of the rod elements. Smaller elements give higher accuracy than large elements. Smaller elements also use more computer time.

VALIDATION

Field data validates the simulator. Figures 1 and 2 show how much fluid inertia improves the history match.

Figure 1 concentrates on the pump card. A measured surface dynamometer card was used to calculate a pump dynamometer card. Figure 1 labels this the "measured" pump card, for brevity. The other two pump cards on figure 1 are calculated with predictive simulators developed at Texas A&M U. The worst history match results from a simulator that ignores fluid inertia. This is the rectangular card. The best match results from the simulator detailed in this paper. The improved quality of the history match speaks for itself.

Figure 2 is the surface equivalent of figure 1. The surface card calculated with the simulator detailed in this paper is clearly a better history match.

The calculated surface card that ignores fluid inertia is a poor match.

Figure 3 gives the results of a sensitivity study on the size of the rod elements used in the calculations. The three, six, and eight elements are 406, 203, and 153 feet long. The six and the eight element cases give the same answers at the end of five strokes. This meets the requirements for numerical convergence, provided five strokes give a suitable approximation of steady state. The three element case is nearly converged. These results support the rule-of-thumb developed for sucker rod simulators that ignore fluid inertia. The rule-of-thumb calls for rod elements to be 200 to 500 feet long. Use longer elements to reduce execution time; use shorter elements to finalize results.

Figure 4 studies the effect of pump efficiency on the calculated results. The peak polished rod load matches better for the 80% pump efficiency case. This might suggest that the measured 75% efficiency is only partly due to leakage at the pump. Some of the leakage may be in the tubing. This suggests an avenue for future development.

The overall improvement in the quality of the history match is impressive, and there is room for future development. A number of likely areas are worth considering. The slightly compressible fluid assumption can be improved. The fluid flow channel can be improved. The numerical solution could be more implicit. The rod damping could consider the fluid velocity. The fluid damping could be made density dependent. Plunger and traveling valve drag could be added to the pump boundary condition. Dynamic tubing pressure could be included. Each of these suggestions moves a step closer to modeling the actual, field conditions. Each suggestion, therefore, holds the potential for improving the quality of the history match.

RESULTS

Shallow wells have less rod mass than deeper wells. Shallow wells also have less hydrostatic pressure at the pump. This means the relative magnitude of the fluid inertia forces is closer to the magnitude of the hydrostatic fluid forces and the rod forces. Figures 5, 6, and 7 show that the ratio of peak pump load to hydrostatic pump load varies inversely with depth.

Higher spm wells have higher accelerations than slower wells. The fluid compression is related to the speed of the pump plunger. Higher spm increases the inertial forces of both the rod and the fluid. The static forces (rod weight and hydrostatic pressure) do not change. This means the relative magnitudes of the inertia forces are higher with higher spm. Figures 8, 9, and 10 show that the ratio of peak pump force to hydrostatic pump force varies with pumping speed.

Wells with larger pump plunger areas displace more fluid mass than wells with smaller pumps. Since inertial force equals mass times acceleration the fluid inertia forces are larger in wells with larger pumps. Figures 11, 12, and 13 show that the ratio of peak pump force to hydrostatic pump force varies with pump size.

Low compressibility fluids act more like solids than high compressibility fluids. A larger volume of low compressibility fluid is needed to accommodate a given amount of compression. The larger volume increases the mass of fluid that is accelerated. The extra fluid mass increases the fluid inertia forces. Figures 14, 15, and 16 show that the ratio of peak pump load to hydrostatic pump load varies inversely with fluid compressibility.

SUMMARY & RECOMMENDATIONS

The results of the sample calculations, figures 1 and 2, show that including fluid inertia can have a tremendous impact on the accuracy of the calculations.

The simulator detailed in this paper should be used to design sucker rod pumping systems with significant fluid inertia forces.

Significant fluid inertia forces are usually found in shallow wells, wells with large pumps, wells with high stroke rates, and/or wells with low compressibility fluids.

Additional development focused on better models of actual field conditions should lead to better history matching.

BASE CASE DATA

2.7	C_r, Viscous Fluid Damping, 1 /s	
6	ner, Number of rod elements	
10.66	N, Strokes Per Minute	
8	ncycl, Number of Cycles to run	
1	nrt, Number or Rod Tapers	
30.5E6 0.4418 1.634 490. 1220.	E_i, A_{ri}, W_{ri}, ρ_{ri}, X_i	
1.0	ω, Specific Gravity	
184.	A, Pump Jack dimension, in	(for exact kinematics)
96.047	C, Pump Jack dimension, in	(for exact kinematics)
114.	P, Pump Jack dimension, in	(for exact kinematics)
96.	I, Pump Jack dimension, in	(for exact kinematics)
151.344	K, Pump Jack dimension, in	(for exact kinematics)
37.	R, Pump Jack dimension, in	(for exact kinematics)
-1	J_R, Crank Rotation; 1 = CW, -1 = CCW	(for exact kinematics)
1	J_{CL}, Pump Jack Class; 1 = Conventional, 3= Reverse Geometry	
0	Crank to Counterweight Phase Angle	
146.4	S_{tk}, Actual Polished Rod Stroke	
7.051	A_p, Pump Area times Pump Efficiency, sq in	
843.	X_{nl}, Net Lift, ft	
114.7	P_{tub}, Tubinghead pressure, psia	
14.7	P_{csg}, Casinghead pressure, psia	
5.9396	A_T, Tubing area, sq in	
14.7	P_o, Equation of State Reference Pressure, psi	
2.6E-6	C_o, Fluid Compressibility, 1/psi	
1.40	C_{FP}, Fluid Damping Factor, 1/s	Between 0.5 and 2.5.

GLOSSARY

a_F	= fluid sonic velocity	= L/T	= ft/s
a_r	= rod sonic velocity	= L/T	= ft/s
a_{ro}	= pseudo rod sonic velocity	= L/T	= ft/s
A_F	= fluid area	= L^2	= in^2
A_p	= net pump plunger area	= L^2	= in^2
A_r	= rod area	= L^2	= in^2
A_T	= tube area	= L^2	= in^2
C_d	= dimensionless rod damping	= -	= -
C_{FP}	= fluid damping factor	= 1/T	= 1/s
C_o	= fluid compressibility	= L^2/F	= 1/psi
C_r	= rod damping factor	= 1/T	= 1/s
dF	= F(X) -F(X+dX)	= F	= lbf
d_i	= rod outside diameter	= L	= ft
d_o	= tubing inside diameter	= L	= ft
dM	= rod element mass	= M	= lbm
dP_f	= friction loss in annulus	= F/L^2	= psi
dR	= radial length	= L	= ft
dT	= time step	= T	= s
dU	= dynamic rod stretch	= L	= ft
dX	= rod element length	= L	= ft
dZ	= fluid element length	= L	= ft
E	= Young's Modulus	= F/L^2	= psi
f	= friction subscript	= -	= -
F	= fluid subscript	= -	= -
F	= F(T,X) = force in rod	= F	= lbf
F_j	= F(T,X) = previous force	= F	= lbf
F_{1j}	= F(T-dT,X) = prev force	= F	= lbf
F_{1j-1}	= F(T-dT,X-dX)	= F	= lbf
F_{1j+1}	= F(T-dT,X+dX)	= F	= lbf
F_P	= node ner force on the pump	= F	= lbf
F_{PN}	= miNimum force on the pump	= F	= lbf
F_{PX}	= maXimum force on the pump	= F	= lbf
F_x	= dF/dX	= F/L	= lbf/ft
F_x	= $E\ A_r\ U_{xx} = E\ A_r\ d^2U/dX^2$	= F/L	= lbf/ft
g_z	= gravity acceleration	= L/T^2	= ft/s^2
g_z	= -32.17404	= L/T^2	= ft/s^2
g_c	= gravity conversion factor	= ML/FT^2	= lbm ft/lbf/s^2
i	= taper number	= -	= -
j	= rod node index	= -	= -
k	= fluid node index	= -	= -
K	= rod spring constant	= F/L	= lbf/ft
M	= rod mass	= M	= lbm
ncycl	= number of cycles in a run	= -	= -
nef	= number of fluid elements	= -	= -
ner	= number of rod elements	= -	= -
nrt	= number of rod tapers	= -	= -
nts	= number of time steps	= -	= -
N	= stroke rate	= 1/T	= stk/min

p	= pump subscript	= -	= -
P	= pressure subscript	= -	= -
P	= fluid pressure	= F/L^2	= psi
P_k	= fluid pressure at node k	= F/L^2	= psi
P_{1k}	= previous pressure at node k	= F/L^2	= psi
P_o	= reference fluid pressure	= F/L^2	= psi
P_{csg}	= wellhead pressure, casing	= F/L^2	= psi
P_{tub}	= wellhead pressure, tubing	= F/L^2	= psi
P_z	= dP/dZ	= F/L^3	= psi/ft
r	= radial derivative subscript	= L	= ft
R	= rod subscript	= -	= -
R	= radial distance	= L	= ft
S	= surface subscript	= -	= -
S_{tk}	= pump jack stroke	= L	= in
t	= time derivative subscript	= T	= s
T	= time	= T	= s
T	= tubing subscript	= -	= -
U	= dynamic rod position	= L	= ft
U_S	= static rod position at pump	= L	= ft
U_{Si}	= static rod posit at taper i	= L	= ft
U_t	= $U_t(T,X)$, rod velocity	= L/T	= ft/s
$U_{t,j}$	= rod velocity at node j	= L/T	= ft/s
U_{tt}	= rod acceleration	= L/T^2	= ft/s^2
U_x	= dU/dX	= -	= -
U_{xx}	= d^2U+dX^2	= 1/L	= 1/ft
V_F	= fluid velocity	= L/T	= ft/s
V_{Fk}	= $V_F(X,T)$ = fluid velocity	= L/T	= ft/s
V_{F1k}	= $V_F(T-dT,X)$ at node k	= L/T	= ft/s
V_{Fav}	= average V_F	= L/T	= ft/s
V_{Fr}	= dV_F/dR	= 1/T	= 1/s
V_{Ft}	= dV_F/dT = fluid acceleration	= L/T^2	= ft/s^2
V_{Fz}	= dV_F/dZ	= 1/T	= 1/s
V_r	= U_t = rod velocity	= L/T	= ft/s
V_{rj}	= $V_r(T,X)$ = rod velocity	= L/T	= ft/s
V_{r1j}	= $V_r(T-dT,X)$ at node j	= L/T	= ft/s
V_{rt}	= U_{tt} = rod acceleration	= L/T	= ft/s
W_r	= rod weight in air	= F	= lbf
W_{ri}	= rod weight in air, taper i	= F	= lbf
W_{rb}	= buoyed rod weight	= F	= lbf
W_{rbi}	= buoyed rod weight, taper i	= F	= lbf
x	= depth derivative subscript	= L	= ft
X	= rod depth	= L	= ft
X_i	= rod length, taper i	= L	= ft
X_j	= rod depth at node j	= L	= ft
X_{ner}	= pump depth	= L	= ft
X_{nl}	= net lift	= L	= ft
z	= depth derivative subscript	= L	= ft
Z	= fluid depth	= L	= ft
Z_k	= fluid depth at node k	= L	= ft

GREEK

γ	= nominal fluid specific grav	= -	= -
1	= fluid viscosity	= FT/L^2	= lbf s/ft^2
μ_c	= fluid viscosity	= FT/L^2	= cp
ρ	= bulk density	= M/L^3	= lbm/ft^3
ρ_F	= fluid density (at T)	= M/L^3	= lbm/ft^3
ρ_{Fl}	= fluid density (at T-dT)	= M/L^3	= lbm/ft^3
ρ_{Fo}	= reference bulk density	= M/L^3	= lbm/ft^3
ρ_r	= rod bulk density	= M/L^3	= lbm/ft^3
τ_{Rz}	= stress tensor	= F/L^2	= lbf/ft^2
ω	= crank angular velocity	= 1/T	= rad/s
ω_o	= rod natural frequency	= 1/T	= rad/s

CONVERSION FACTORS

cp x 0.000,020,885,44 = lbf s $/ft^2$
g_c = 32.174,044 lbm ft /lbf $/s^2$

cp x 0.001 = Pa S
ft x 0.3048 = M
g_c = 1.0 Kg M /N $/s^2$
lbf x 4.448,221,55 = N
lbm x 0.453,592,4277 = Kg

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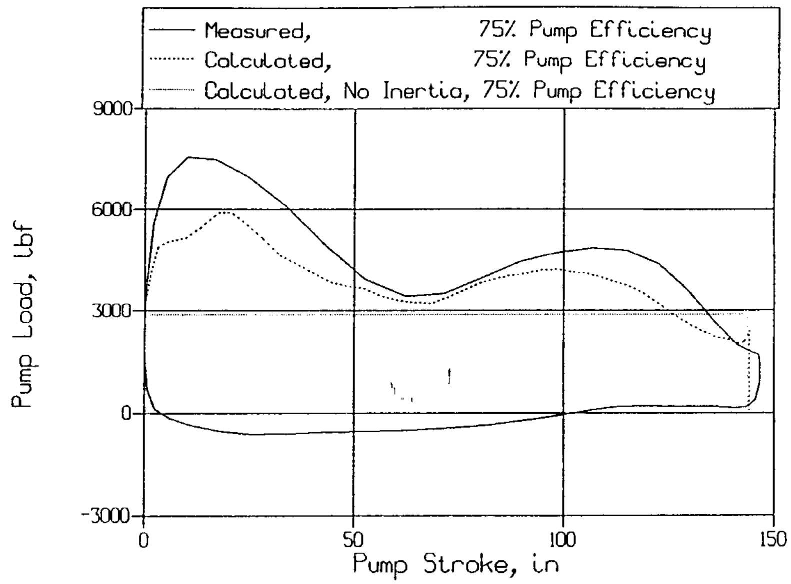

Figure 1 - Pump card, sensitivity to fluid inertia

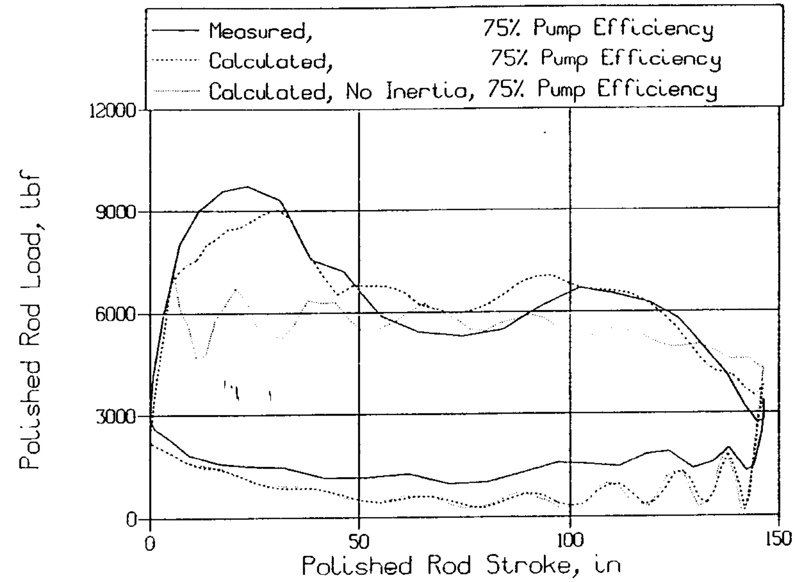

Figure 2 - Surface card, sensitivity to fluid inertia

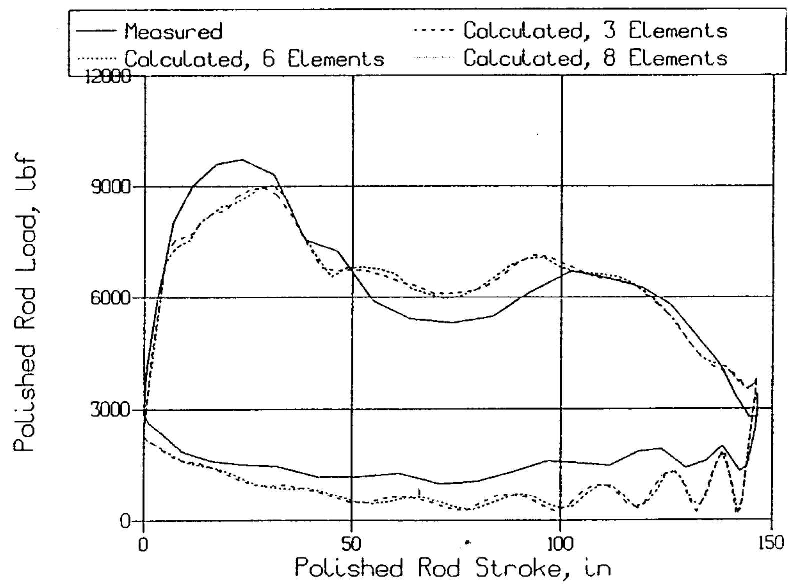

Figure 3 - Surface card, study number of elements

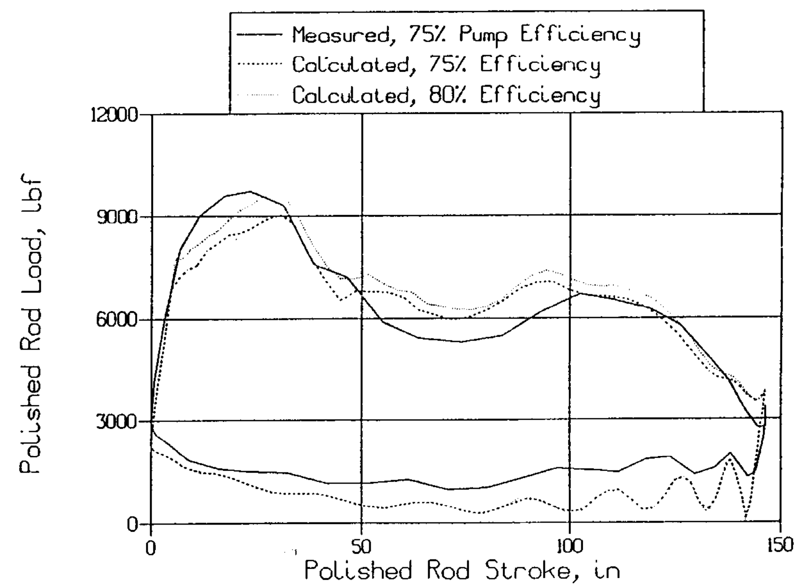

Figure 4 - Surface card, study of pump efficiency

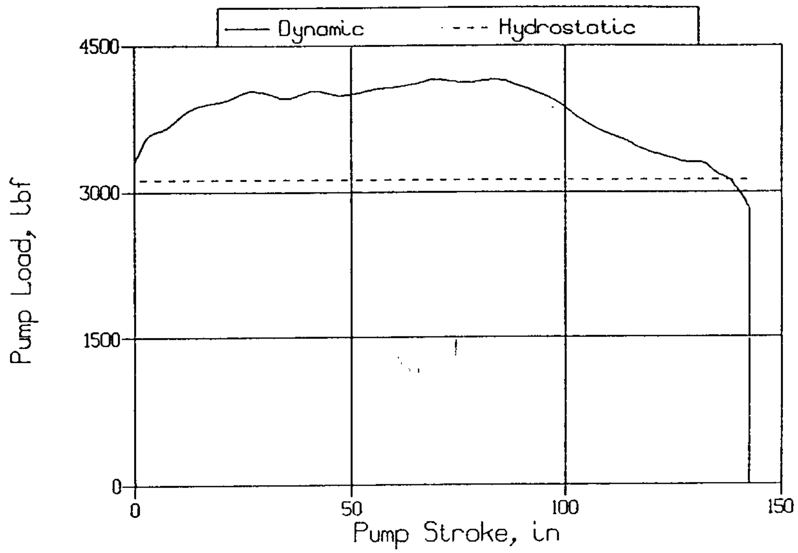

Figure 5 - Pump card, sensitivity to depth, 3000 ft: peak to hydrostatic load ratio = 1.3

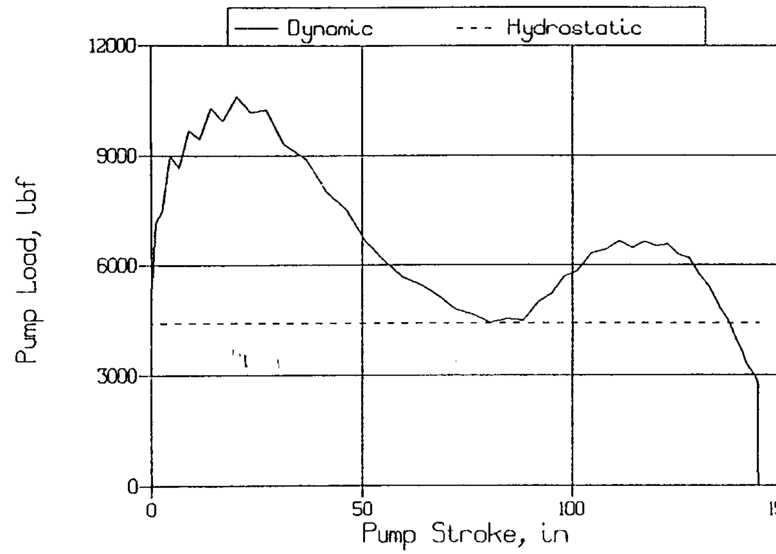

Figure 6 - Pump card, sensitivity to depth, 1220 ft: peak to hydrostatic load ratio = 2.4

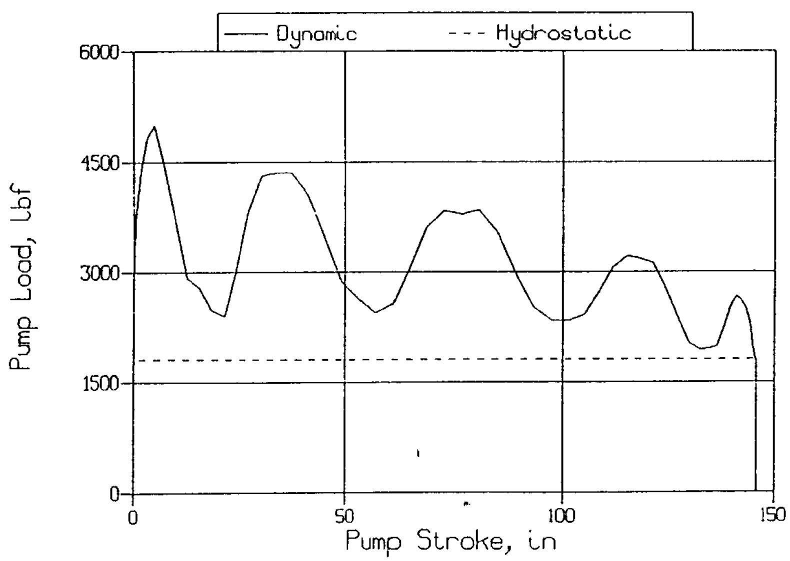

Figure 7 - Pump card, sensitivity to depth, 500 ft: peak to hydrostatic load ratio = 2.8

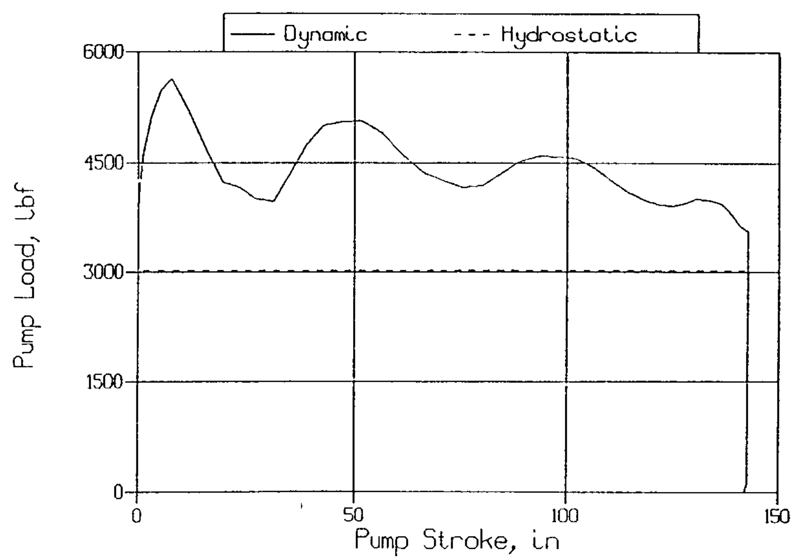

Figure 8 - Pump card, sensitivity to speed, 5.0 SPM: peak to hydrostatic load ratio = 1.9

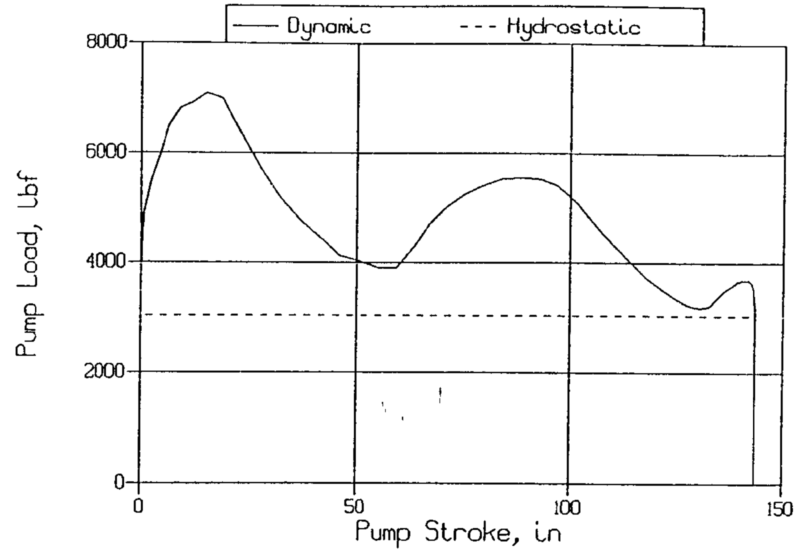

Figure 9 - Pump card, sensitivity to speed, 8.0 SPM: peak to hydrostatic load ratio = 2.3

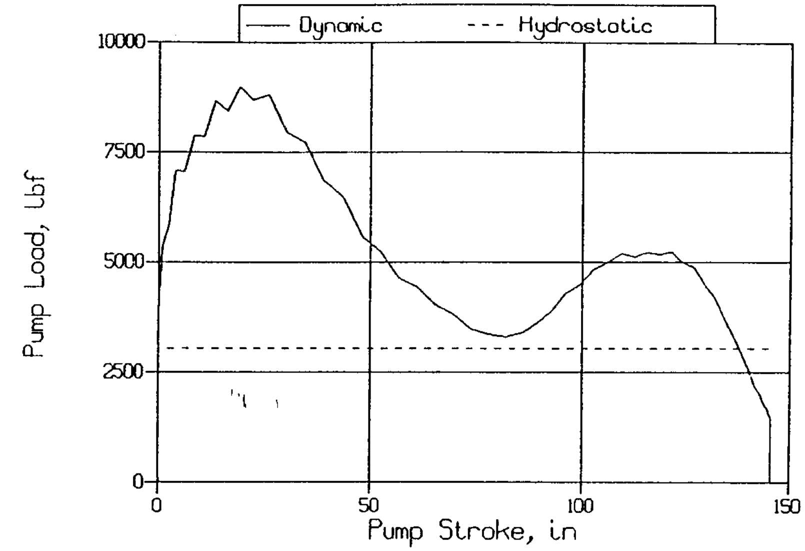

Figure 10 - Pump card, sensitivity to speed, 10.66 SPM: peak to hydrostatic load ratio = 3.0

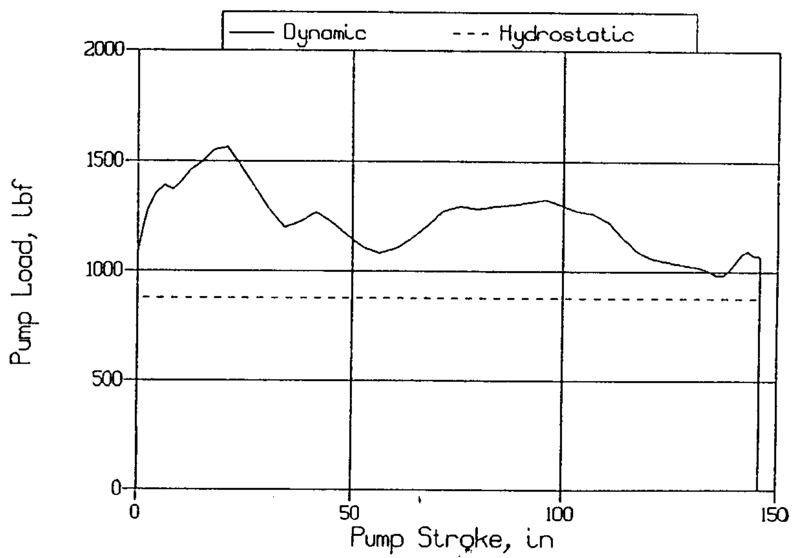

Figure 11 - Pump card, pump diameter sensitivity, 1.75 in: peak to hydrostatic load ratio = 1.8

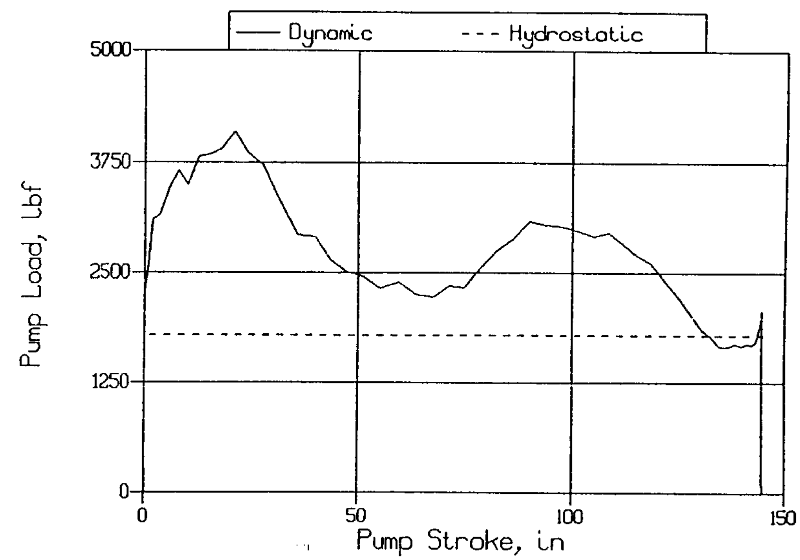

Figure 12 - Pump card, pump diameter sensitivity, 2.5 in: peak to hydrostatic load ratio = 2.3

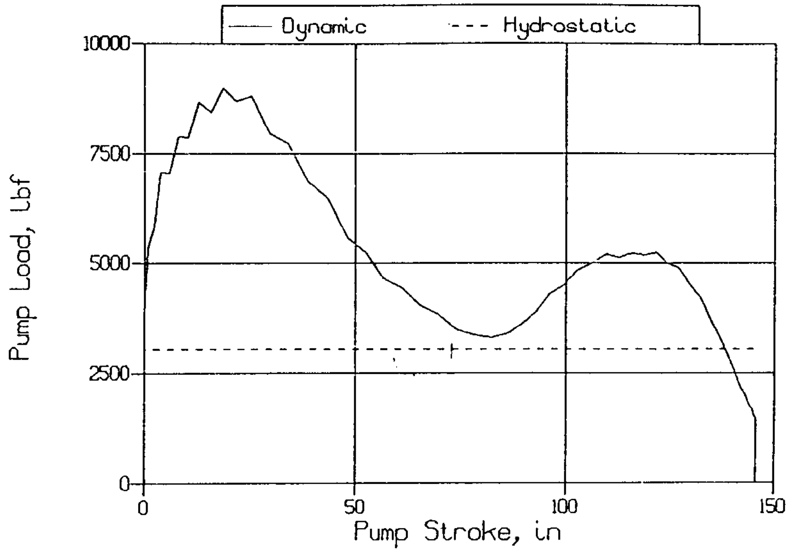

Figure 13 - Pump card, pump diameter sensitivity, 3.25 in: peak to hydrostatic load ratio = 3.0

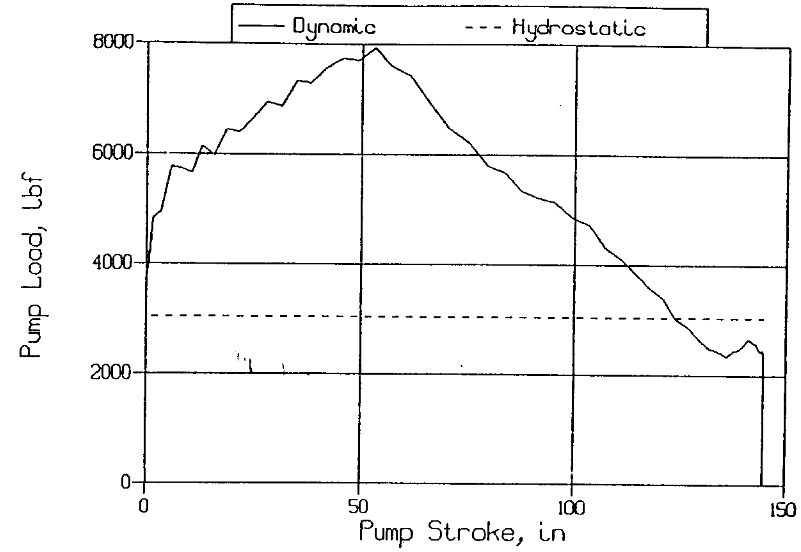

Figure 14 - Pump card, compressibility study, 10.E-6/psi: peak to hydrostatic load ratio = 2.6

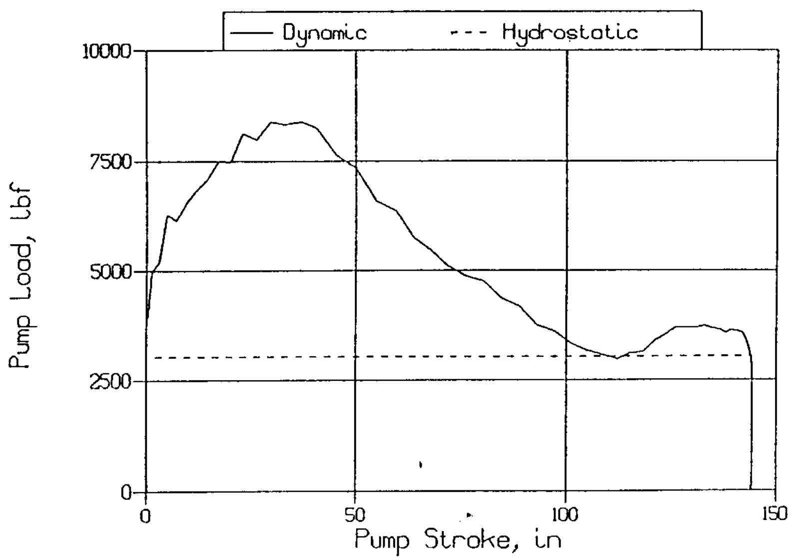

Figure 15 - Pump card, compressibility study, 6.0E-6/psi: peak to hydrostatic load ratio = 2.8

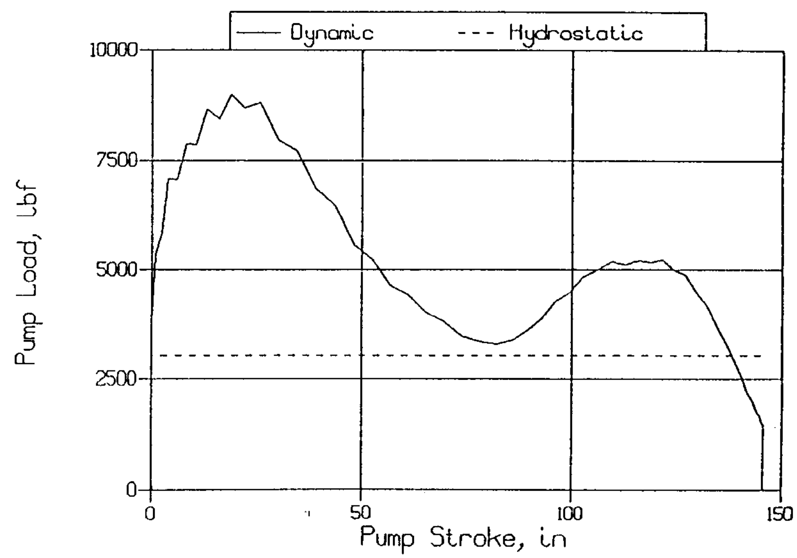

Figure 16 - Pump card, compressibility study, 3.0E-6/psi: peak to hydrostatic load ratio = 3.0