RISK ANALYSIS

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INTRODUCTION

In the Oil and Gas Industry, risk analysis is the attempt by industry to quantify the range of outcomes of real world events by means of mathematical models of the real world. In the real world of multiple economic decisions that industry must contend with every business day, there are multitudes of externalities, known and unknown, that act upon real world events. These externalities are in general called risk and uncertainty. The Oil and Gas Industry tries to measure, quantify, and account for risk and uncertainty using stochastic and statistical methods.

The drilling of an oil or gas well is an uncertain venture--there are plenty of oil and gas producers that will attest to this fact. The usual method of evaluating an oil and gas drilling prospect is to first make a determination of the amount of hydrocarbon reserves that are to be found by the drilling of a well or a series of wells. If this prospect well is near other wells producing from the same target formation, one might be able to say, if the geology is similar, the prospect well will recover sellable hydrocarbons or reserves in an analogous quantity to the producing wells of the existing fields. If the above analogy can be assumed, a petroleum engineer can determine the estimated recoverable reserves (EUR) for each of the analogous wells or fields. Then using statistics the engineer can further determine the average EUR for the sample of analogous wells or fields along with a statistical property of the sample called the variance and standard deviation. The variance and standard deviation is a measure of the dispersion of the individual items in a distribution about the mean of the distribution. Table 1 is an example of how the average and standard deviation of a sample distribution of EURs can be determined.

In some analyses of drilling ventures, a simple statement of the expected reserves to be found and maybe a range (a high expected value and a low expected value) of reserves is sufficient. However, a more sophisticated analysis would say that the expected reserves is the average or mean of the sample analogous wells, that the sample had a range of reserves, and that the standard deviation of the sample is some calculated value (σ). At this point, the expected value of the reserves or mean and the low/high range quantities could be used to generate economic profiles of the proposed drilling venture. Very often, an economic profile of a drilling prospect based on the expected reserves is all that is presented to prospective participants. Something is still missing from the analysis: there has been no addressing of the risk associated with drilling the well with the expected reserves stated.

PROBABILITY

Generally project or prospect risk is quantified by probabilities, a mathematical statement of chance or risk. For instance, in the toss of a fair, unbiased coin, the probability of getting heads is one out of two (only one outcome, heads, out of two possible outcomes, heads or tails, is possible) or 50%. Using the sample of analogous well EURs shown in Table 2, it can be seen that each EUR is discrete. In this instance, an assumption can be made (note necessary in all cases) that each well had an equal chance of being discovered. Thus the well with an EUR of 40,000 barrels has a probability of occurrence of one in five or numerically a 20% probability of occurrence. Figure 1 is a stick graph of the discrete sample of five EURs versus their respective probability of occurrence.

Because each well in the sample has an equal probability of occurrence, the distribution of reserves vs. probability of occurrence in this case is said to be "uniform". In other words the probability of occurrence of the lowest value EUR is the same as the probability of occurrence of the highest value EUR. Uniform distributions are usually represented as shown in Figure 2.

As shown in the preceding examples, it should be apparent that both a discrete and a uniform distribution have statistical properties of mean, variance, and standard deviation. There are several other types of distributions commonly used in oil and gas risk analysis. The most commonly used are the triangular distribution, the binomial distribution and its approximation the normal distribution, and the lognormal distribution. The triangular distribution is shown in Figure 3. It is usually used to represent sample data for which an average or most likely value can be calculated, a high and a low value can be established, the sample size is small, and the true distribution shape is unknown.

The normal distribution (bell curve) and the binomial distribution is shown in Figure 4. It is usually used to represent very large samples; for instance, the distribution of the heights of all of the male adults in the United States is normally distributed. A particularly useful property of the normal distribution is that the area under the curve plus and minus one standard deviation from the mean is 68.25% of the entire area represented by the curve. The area plus and minus two standard deviations from the mean is 95.44% of the entire area and the area plus and minus three standard deviations from the man is 99.74% of the entire area. The binomial distribution is a sampling type of distribution and is very useful in oil and gas risk analysis. The assumption of the binomial is that the curve represents the complete universal population. The population is sampled one event at a time, and the probability of occurrence of each event is the same. The best example of a binomial process is the coin toss presented previously. Another example of a binomial process would be the probability of drawing a specific card from a deck of fifty-two playing cards. At each sampling of the deck of cards there is a probability of drawing a specific card of one over fifty-two or numerically 1.923%. There is also implied a probability of 98.08% at each draw of a card from the complete deck of not drawing the specific card. It should be obvious that in oil and gas risk analysis, if a company has drilled one hundred wells in a geologically similiar area and twenty of these wells were dry, the next well drilled in the area has a twenty percent chance of being a dry hole and an eighty percent chance of being a producer.

The lognormal distribution is very useful in oil and gas risk analysis in that oil and gas reserves for either individual well EURs in a field or total field reserves in a geologic providence tend to be lognormally distributed. A typical lognormal distribution is shown in Figure 5. A good layman's description of the lognormal distribution of oil and gas reserves is to say that there are a lot of wells in the sample with approximately the same reserves and a few wells in the sample with reserves that far exceed the reserves of the typical wells.

EXPECTED VALUE ANALYSIS

To this point there has been presented brief discussions of probability, statistics, and several types of distributions used in Oil and Gas risk analysis, but little has been mentioned as to how a prospect is actually analyzed. Most companies in their risk analysis of oil and gas projects use what is called an expected value analysis. The "expected value" of a prospect is a probability weighted or risk weighted expected outcome. It represents the average expected return in reserves or dollars per drilling attempt the company would realize over a series of repeated drilling trials.

As shown in Table 2, the expected value of the prospect reserves is 30,000 BBLS of oil for each commercially productive well. For simplicity, assume that oil sells for $30/barrel and the cost to drill and complete a well in this prospect is $500,000. Thus the company could expect to net $400,000 ($30 x 30,000 BBLS minus $500,000 cost to drill) of profit from the average well. Further, assume that if the well is dry, the out-of-pocket expenses is only $250,000, the cost to drill the well but not to complete it. It was stated previously that this particular company had a historical success rate of 80%, or in other words, the company had eighty successful wells and twenty dry holes out of a total number of one hundred wells drilled.

DECISION TREES

The risk analysis of the prospect can now be completed using a "decision tree" type of model. A decision tree is simply a mathematical model used to illustrate the decision process of arriving at the expected monetary value of drilling the prospect well. The computations of the Expected Value/Decision Tree Analysis of this prospect is illustrated in Figure 6.

As can be seen from Figure 6, the total expected value to the company from the drilling of the prospect is $270,000. The $270,000 of expected monetary value for this prospect represents the average monetary gain (could also be stated as reserves) the company would realize over a repeated number of drilling attempts of a prospect with the given assumed well cost, success ratios, and expected monetary gain per successful well.

The Decision Tree/Expected Value illustrated above is the most basic, most widely used, and only rigorous analysis procedure used by the Oil and Gas Industry today. This type of risk analysis procedure, when realistically performed for every drilling prospect/venture that a company has available to it, allows a company's management to rank and select those projects that will maximize the company's profit per dollar invested.

MONTE CARLO SIMULATION

The Decision Tree/Expected Value analysis illustrated in Figure 6, while being the most widely used type of risk analysis, still does not completely represent the real world of multiple outcomes. As stated before, the drilling for oil and gas is uncertain and risky at best and obviously the reserves found by any one drilling venture cannot be stated as a single average number. Anyone with any savvy in the industry will readily admit that the reserves to be found by the drilling of a well is better represented by a distribution of reserves and the actual reserves developed by the drilling of a single well is a matter of chance.

With the advent of modern computers, the ability of the industry to better model prospect outcomes has greatly improved through the use of a technique called Monte Carlo Simulation. This technique is a further sophistication of the Decision Tree/Expected Value Analysis. The Monte Carlo Simulation technique is basically to replace the successful outcome limbs of the decision tree analysis with a distribution of reserves and/or monetary gains. The Decision Tree/Expected Value analysis is then computed (by the computer) hundreds or even thousands of times. The Expected Value of each drilling trial can then be summed and averaged by the number of trials to arrive at a more realistic real world prospect Expected Value. Figure 7 is a graphic representation of how the Monte Carlo Simulation technique would be applied to a discrete probability distribution, such as that illustrated in Figure 1. As shown in Figure 7, the discrete probability distribution is converted into a continuous cumulative probability vs. value of reserves (X_i) plot. The computer then generates random number using an internal random number algorithm. The random numbers are themselves a uniform distribution. The random numbers are converted to percents and that value is entered into the cumulative probability vs. value of reserves (X_i) plot to arrive at a specific value of reserves (X_i) for each trial. The process is repeated many times by the computer, the results of each trial are summed, and the sum is averaged to give the expected value.

SELECTED REFERENCES

1. Dr. Paul Newendorp, Petroleum Exploration Economics and Risk Analysis, Permian Basin Graduate Center, p. 38.

2. Arthur W. McCray, Petroleum Evaluations and Economic Decisions, Prentice-Hall, Inc. 1975, p. 204, Figure 7.10.

Table 1
Calculation of Mean, Variance and Standard Deviation of a Sample

NUMBER OF WELLS	RESERVES, X_i EUR (MMBLS)	$(X_i-\bar{X})$	$(X_i-\bar{X})^2$
1	10	-20	400
1	20	-10	100
1	30	0	0
1	40	10	100
1	50	20	400
$\Sigma N = 5$	$\Sigma X_i = 150$		$\Sigma(X_i-\bar{X})^2 = 1000$

$$\text{MEAN} = \bar{X} = \frac{\Sigma X_i}{N} = \frac{150}{5} = 30 \text{ MBBLS}$$

$$\text{VARIANCE} = 6^2 = \frac{\Sigma(X_i-\bar{X})^2}{N} = \frac{1000}{5} = 200 \text{ MBBLS}$$

$$\text{STANDARD DEVIATION} = 6 = \sqrt{200} = 14.1 \text{ MBBLS}$$

Table 2
Calculation of Expected Values of Prospect Reserves

NUMBER OF WELLS	PROBABILITY OF OCCURRENCE	RESERVES PER OCCURRENCE (MBBLS)	EXPECTED VALUE PER OCCURRENCE (MBBLS)
1	.2	10	2
1	.2	20	4
1	.2	30	6
1	.2	40	8
1	.2	50	10
	Σ PROB = 1		Σ = 30

EXPECTED VALUE OF PROSPECT RESERVES = 30,000 BBLS

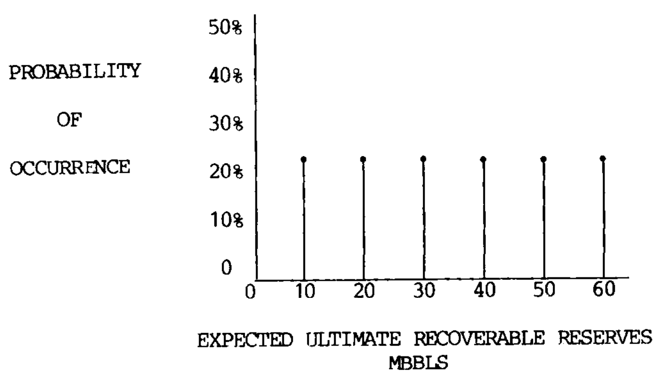

Figure 1 - Probability density function plot

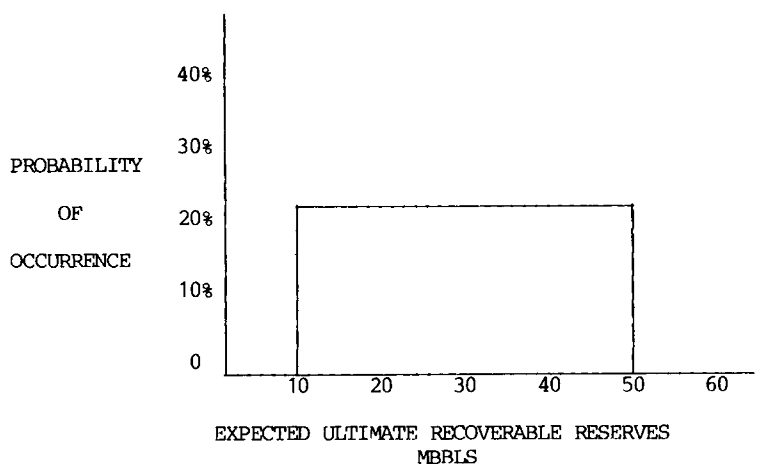

Figure 2 - Uniform distribution

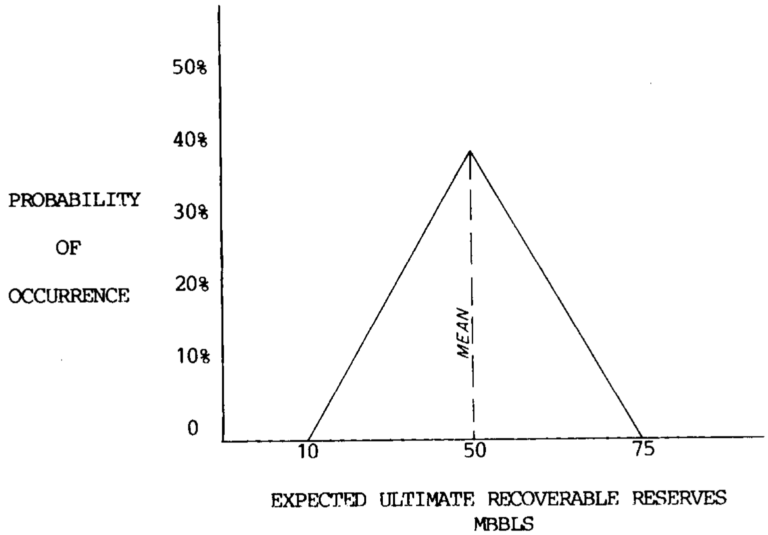

Figure 3 - Triangular distribution

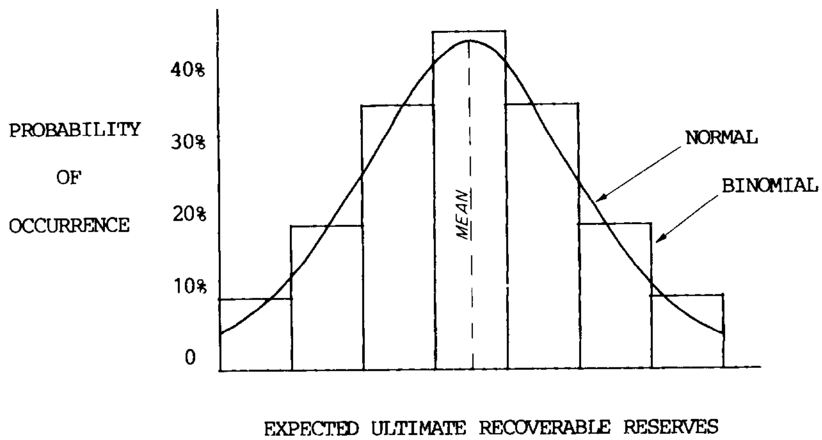

Figure 4 - Normal and binomial distributions

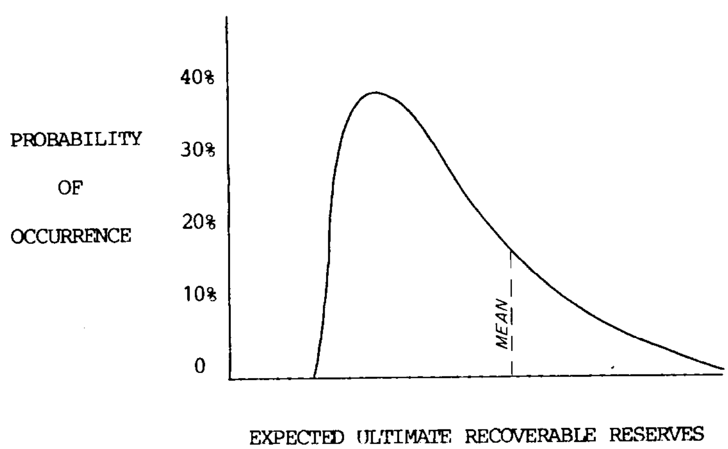

Figure 5 - Lognormal distribution

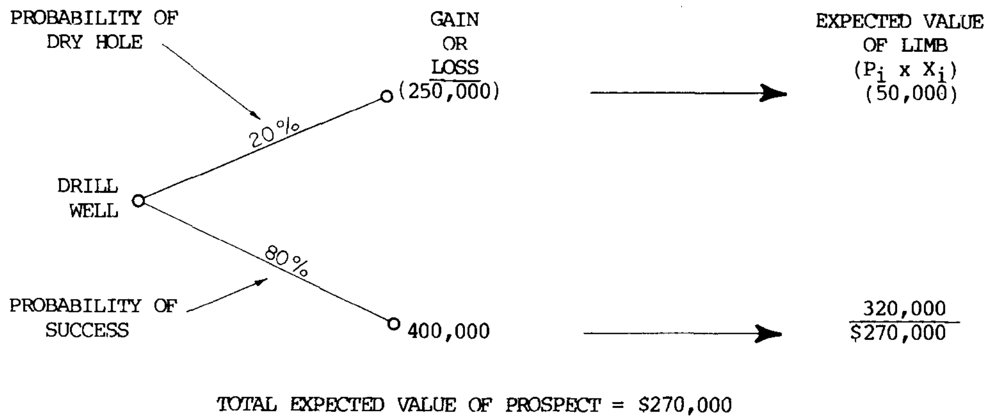

Figure 6 - Expected value/decision tree analysis of prospect

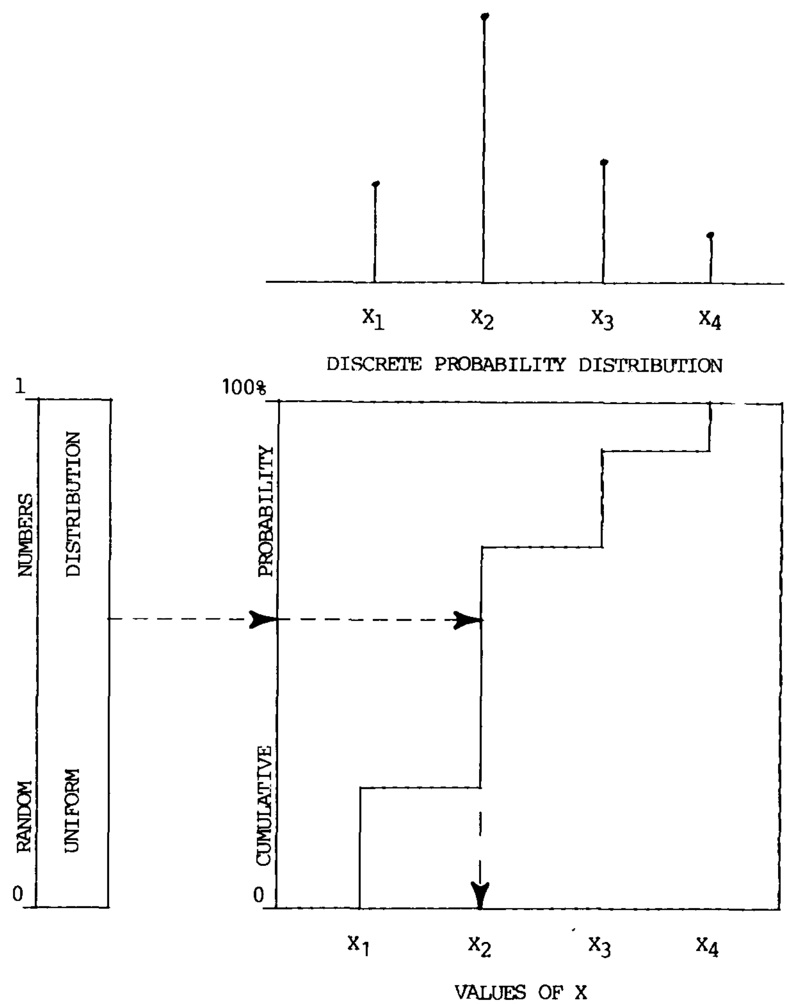

Figure 7[2] - Monte Carlo simulation technique
(random selection from a discrete distribution)