RHEOLOGICAL EVALUATION OF CEMENT SLURRIES: METHODS AND MODELS

by Charles E. Bannister
Dowell Division of Dow Chemical U.S.A.

ABSTRACT

The determination of the rheological behavior of cement slurries is essential for the proper evaluation of displacement pressures and flow rates for optimum cement placement. Several cement slurries have been examined, using pipe flow and concentric cylinder viscometers, in an effort to determine which method is better suited for determining such flow characteristics. Comparative analysis of the data indicates that the concentric cylinder viscometer may be inadequate for measurement of the rheological properties of cement slurries.

Studies using a pipe-flow rheometer indicate that an apparent "slip" at the pipe surface occurs during rheological evaluation of cement slurries. This wall "slip" is attributed to particle migration when cement slurries are sheared. Field evaluation of the rheological properties of cement slurries flowing in large-diameter pipes confirm the results of the pipe-flow rheometer.

Based on data obtained with a pipe-flow rheometer, a recommendation is offered as to which mathematical model most accurately describes the flow characteristics of cement slurries.

INTRODUCTION

Knowing what displacement pressure and flow rate will maintain a cement slurry in turbulent or plug flow in the wellbore annulus is essential in the design of primary cement jobs. Cement in plug or turbulent flow exerts a uniform displacement force against the mud in the wellbore annulus. In laminar flow, cement has a parabolic or "bullet-shaped" velocity profile across the area of flow. This results in cement "jetting" through the drilling fluid.[1] Incomplete mud removal can result in poor cement bonding, zone communication and ineffective stimulation treatments.

Characterization of the flow properties of fluids is determined by the relationship between the flow rate (shear rate) and pressure (shear stress) required for fluid movement. Extensive studies have resulted in the development of several mathematical models which describe the relationship between shear stress and shear rate. The three most commonly used models are the Newtonian, Bingham Plastic and Power Law models.[1,2,3] Most drilling fluids and cement slurries are non-Newtonian fluids and have been treated using a Bingham Plastic or Power Law type model.[3] Other models such as the Herschel-Bulkley and Robertson-Stiff models are not widely used at this time.[4]

Mathematical modeling of the flow behavior of cement slurries requires the accurate measurement of shear stress and shear rate. At the present time concentric rotational viscometers are extensively used for cement slurries. This type of viscometer permits the fluid placed in the annular space between a stationary and

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a rotating cylinder to be subjected to shear. The rate of shear is determined from the geometry and speed of the rotating cylinder. The shear stress is obtained by the measured torque induced by the fluid on the stationary cylinder (bob). With a small annular gap the shear rate is nearly constant through the fluid. The use of concentric cylinder viscometers is hindered by particle settling during measurement and the high shear rates the cement slurries experienced within the small annular gap. An investigation will compare the results of concentric cylinder viscometers with pipe-flow rheometers. The pipe-flow rheometer is similar to capillary-type viscometers. The rheological properties of a cement slurry are determined by measurement of the pressure drop across a length of pipe at a given slurry flow rate. The rheometer has been designed to permit rapid testing with minimal particle settling. The shear rates obtained by the pipe-flow rheometer resemble downhole conditions by being zero at the pipe axis and at a maximum at the pipe wall.[5] In addition, the rheological data can be easily compared with fluids flowing in large-diameter pipe.

EXPERIMENTAL PROCEDURE

Rheological measurements were obtained for a series of cement slurries using both a 12-speed Fann 35/SR12 viscometer and a pipe-flow rheometer. Each cement slurry was prepared according to API procedures using either Oklahoma or Longhorn Class H cement.[6] The amount of water needed to prepare 600-mL cement slurries was determined by weight, not volume. All solid additives were dry-blended with the cement prior to addition to water.

The 12-speed Fann 35 viscometer was calibrated using two weights (the calibration fixture was supplied by Fann Incorporated).[7] The spring (factor = 1) was calibrated to give a Fann reading of 127 with a 50-g weight and 254 with a 100-g weight. Calibration of the viscometer was checked every 10 to 15 days.

Rheological measurements using the Fann 35/SR12 viscometer were obtained by API procedures.[8] After stirring the cement slurry in an atmospheric consistometer for 20 minutes at the test temperature, it was poured into a 350-mL Fann cup. The slurry was sheared at 600 RPM for 60 seconds prior to measurement of the shear stress. Fann readings at lower shear rates were taken at 20-second intervals in the following order: 300, 200, 100, 6, 3, 180, 90, 60 and 30 RPM. The Fann viscometer was turned off after the 3 RPM reading to change the two-position gear shift lever. The conversion of Fann readings to shear stress and RPM to shear rate is given by (R1-B1 Bob-Rotor, spring factor = 1)

$$\tau_\omega = \frac{1.066}{100}\,\Theta \qquad (1)$$

and

$$\dot{\gamma} = 1.703\,(\text{RPM}) \qquad (2)$$

where

τ_ω = shear stress, lb/ft^2,

$\dot{\gamma}$ = shear rate, sec^{-1}, and

Θ = Fann reading.

The pipe-flow rheometer is pictured in Figure 1. A syringe pump with variable flow rate is equipped with a 140-mL syringe. The syringe is connected to a 1/4-in. cross tee. To one side of the cross tee is connected interchangeable pipes ranging in both internal diameter (0.083-0.305 in.) and length (6-48 in.). A pressure transducer (0-20 psi) is attached to the other side of the cross tee.

The flow rate of the syringe pump was verified by measuring the time to dispense a standard Newtonian oil of known viscosity (103 cp at 80°F) from a 50-mL and 140-mL syringe.

The pressure transducer was standardized in the range of 0-5 psi using air pressure. The standardization of the pressure transducer was also verified by measurement of the hydrostatic pressure of a water head. The calculated pressure is given by[10]

$$P = 0.433\ H \tag{3}$$

where

P = hydrostatic water pressure, psi

and

H = height of water, ft.

The results of measurements using the pipe-flow rheometer were verified using a 103-cp (80°F) Newtonian oil supplied by Fann Incorporated. A 140-ml plastic syringe was filled with oil and placed in the syringe pump. The pressure, as a function of pump rate, was measured using several pipe diameters and lengths. The relationships between pressure and flow rate with shear stress and shear rate for Newtonian fluids are given by[11]

$$\tau_\omega = 36\ \frac{DP}{(L)} \tag{4}$$

and

$$\dot{\gamma} = \frac{96\ v}{(D)} \tag{5}$$

where

τ_ω = shear stress, lb/ft^2,

D = inside pipe diameter, in.,

P = pressure, psi,

L = pipe length, in.,

$\dot{\gamma}$ = shear rate, sec^{-1}, and

v = velocity, ft/sec.

The shear stress-shear rate data are given in Table I. As shown in Figure 2, agreement between the Fann 35/SR12 viscometer and the pipe-flow rheometer is obtained.

A series of rheological measurements was initially performed in each slurry using the Fann 35/SR12 viscometer. Subsequently, a 140-mL syringe was filled with the cement slurry and attached to the syringe pump. One syringe volume of cement slurry was hand pushed through the pipe prior to measurement to eliminate air bubbles trapped in the pipe. The plastic syringe was refilled and the pressure at several flow rates was measured. The pressure remained constant after 15 to 20 seconds of pumping time at a given rate. The pressure was measured for both increasing and decreasing pump rates. The rheological measurements using the Fann 35/SR12 were repeated after measuring the pipe flow.

RESULTS

Several shear rate-shear stress measurements (25-30) were obtained for each slurry using three different pipe diameters.[12] The sequence of testing the three pipe diameters was random. Fann data were taken before and after the pipe flow measurements.[12] Each test, using the Fann and pipe-flow measurements, took approximately 20 minutes to complete. The rheological behavior of each cement slurry (using Longhorn H Cement [40% water, 0.36% gelling agent]) is shown in Figures 3 and 4. A graphical log-log plot of shear stress versus 96 v/D is linear for all cement slurries tested. This suggests that a Power Law-type rheological model is followed for pipe-flow measurements. Data taken using the rotational viscometer gave a non-Power Law relationship.

POWER LAW MODEL

The functional dependence of shear stress with shear rate is described by empirical relationships. One such relationship is known as the Power Law function and is described by[13]

$$\dot{\gamma} = \left(\frac{\tau_\omega}{K}\right)^{1/n} \tag{6}$$

where

τ_ω = shear stress, lb/ft^2,

$\dot{\gamma}$ = shear rate, sec^{-1},

n = flow behavior index, and

K = consistency index, lb sec/ft^2.

This model is a two-parameter model. The parameter K is a measure of the consistency of the fluid and is called the consistency index. The constant "n" is the flow behavior index and measures the degree of non-Newtonian behavior. Fluids which have $n < 1$ are pseudoplastic; $n = 1$ are Newtonian; and $n > 1$ are dilatant fluids. The viscosity ( ) of a fluid is the ratio of shear stress to shear rate and is expressed by

$$\mu = \frac{\tau_\omega}{\dot{\gamma}} = K\dot{\gamma}^{n-1}. \tag{7}$$

For pseudoplastic fluids, the viscosity decreases with increasing shear rates. Therefore, pseudoplastic fluids are often referred to as shear thinning fluids. Newtonian fluids have viscosities which are independent of shear rate.

The relationship between shear rate at the wall of a pipe and 96 v/D is given by[14]

$$\dot{\gamma} = \frac{3n' + 1}{4n'} \quad \frac{96\ v}{D} \tag{8}$$

where

$$n' = \frac{d \log \tau_\omega}{d \log(96\ v/D)}. \tag{9}$$

The shear stress at the pipe wall is determined using Eq. 4. Substitution of Eq. 8 into Eq. 6 gives

$$\frac{96\ v}{D} = \left(\frac{4n}{3n+1}\right)\left(\frac{\tau_\omega}{K}\right)^{1/n}. \quad (10)$$

Metzner and Reed have simplified the Power Law relationship for pipe flow by setting[14]

$$n = n' \quad (11)$$

and

$$K' = K\left(\frac{3n+1}{4n}\right)^{n}. \quad (12)$$

Substitution into Eq. 10 gives the Metzner-Reed Power Law model for fluids flowing in a pipe.

$$\frac{96\ v}{D} = \left(\frac{\tau_\omega}{K'_D}\right)^{1/n'}. \quad (13)$$

Equation 13 can be rearranged to give

$$\log \tau_\omega = \log K'_D + n' \log 96\ v/D \quad (14)$$

where K'_D is the consistency index evaluated at a constant pipe diameter. A graphical log-log plot of ω versus 96 v/D should be linear if the fluid follows a Power Law relationship. The slope of the linear plot will be n' and the intercept

will equal log K'_D. The results, as illustrated in Figure 4, show that all pipe-flow data follow the Metzner-Reed Power Law formulation. Evaluations of n' and K'_D by a linear regression analysis of the pipe-flow data are summarized in Table II for each cement system. It is apparent from these results that the Power Law parameters are dependent upon the pipe's geometry.

PIPE LENGTH

The investigation of the effects of pipe length on cement rheology was conducted at constant shear rate on a Longhorn Class H cement slurry (40% water, 0.57% HEC gelling agent).

Substitution of Eq. 4 into Eq. 13 and solving for pressure gives

$$P = \frac{K'_D}{36\ D}\left(\frac{96\ v}{D}\right)^{n'} L. \quad (15)$$

At a constant value of 96 v/D, a plot of pressure versus pipe length should be linear with a zero intercept. The results are graphically plotted in Figure 5 for two different values of 96 v/D. The zero intercepts show the measured pressure is not altered by "end" effects caused by the entrance of the slurry into the pipe. The results also indicate that any time-dependent (thixotropic) rheological behavior of the cement slurry is insignificant during the pipe-flow measurements.[15] Therefore, the deviation of the Power Law parameters must be due to pipe diameter and not to length.

PIPE DIAMETER

For fluids flowing in a pipe, the shear rate varies from zero at the center of the fluid to a maximum value at the pipe wall.[5] The basic equation relating velocity to shear stress in a pipe is given by[16]

$$\frac{96\ v}{D} = \frac{4}{\tau_{\omega}^{3}} \int_{0}^{\tau_{\omega}} \tau^{2}\ F(\tau)\ d\tau \qquad (16)$$

where $F(\tau)$ is the shear rate as a function of shear stress. Substitution of Eq. 6 into Eq. 16 and integrating gives the Metzner Reed Power Law formulation (Eq. 10). Equation 16 was derived by assuming that the velocity of the fluid at the wall is zero. A velocity component at the pipe wall is called slip. If wall slip (u) is not zero, Eq. 16 becomes

$$\frac{96\ v}{D} = \frac{96\,\mu}{D} + \frac{4}{\tau_{\omega}^{3}} \int_{0}^{\tau_{\omega}} \tau^{2}\ F(\tau)\ d. \qquad (17)$$

Substitution of the Power Law model (Eq. 6) into Eq. 17 gives

$$\frac{96\ v}{D} = \frac{96\ \mu}{D} + \left(\frac{\tau_{\omega}}{K'}\right)^{1/n'}. \qquad (18)$$

This states that an imposed shear stress at the wall (τw) will give a shear rate that is reduced by the velocity component of the shear rate at the pipe wall. Substitution of Eq. 13 into Eq. 18 gives

$$\left(\frac{1}{K'}\right)^{1/n'} = \left(\frac{1}{K'}\right)^{1/n'} + \frac{96}{D}\,\tau_{\omega}^{\frac{1}{n'}} \qquad (19)$$

where K'_D is the consistency index at a given diameter. Substitution of Eq. 19 into Eq. 14 gives

$$\log \omega = -n'\log \left[\left(\frac{1}{K'}\right)^{1/n'} + \frac{96}{D\tau_{\omega}^{\frac{1}{n}}}\right] + n'\ \log \frac{96\ v}{D}. \qquad (20)$$

A Power Law plot of log τ versus log (96 v/D) will be nonlinear if the dependence of slip at the wall upon shear stress is not given by

$$u = C_S\ \tau_{\omega}\ \frac{1}{n'} \qquad (21)$$

where C_S is a slip coefficient. This may be the case for cement slurries evaluated using a rotational viscometer. The cement slurries evaluated in this study do follow a Metzner-Reed Power Law function. Substitution of Eq. 21 into Eq. 19 results in

$$\left(\frac{1}{K}\right)^{1/n'} = \left(\frac{1}{K}\right)^{1/n'} + \frac{96\ C_S}{D}\ . \qquad (22)$$

The consistency index, K'_D, is now dependent only upon the pipe diameter and is independent of shear stress at the wall. In large-diameter holes, K'_D approaches the consistency index K'. Plots of

$$\left(\frac{1'}{K'_D}\right)^{1/n'} \text{ vs } \frac{1}{D}$$

are shown in Figures 6 and 7. The slope of the line is equal to 96 C_s and the intercept is equal to

$$\left(\frac{1}{K'}\right)^{1/n'}.$$

The parameters K', n' and C_s completely describe the rheological properties for each cement slurry using (see Appendix)

$$\frac{96\ v}{D} = \left[\left(\frac{1}{K'}\right)^{1/n'} + \frac{96\ C_s}{D}\right] \tau_\omega^{1/n'} \quad . \qquad (23)$$

The dependence of the slip coefficient upon the consistency index K' is graphically shown in Figure 8. The slip coefficient decreases when K' increases (slurry becomes thicker). As shown by Figure 9, the slip coefficient decreases as the water content increases. The decreases in the slip coefficient in cement slurries with larger water content suggest that the pseudo wall slip is related to the number of particles in the cement slurry.

Apparent slip at the pipe wall has been observed for suspensions of paper pulp.[17] When pulp flows through a transparent pipe, a clear water layer is observed adjacent to the wall. The fluid in this region has a lower viscosity and gives rise to pseudo wall slip. Wall slip has recently been reported for pastes of iron ore containing 16% water flowing through a tube.[18] Cement slurries contain hydrated particles which contribute to the rheological properties of the slurry. Particle migration within the slurry can be expected to result in apparent wall slip during shearing. This principle was verified by observation of apparent slip at the pipe wall for high-particle-content drilling fluids.

DRILLING FLUIDS

Rheological measurements were made on drilling fluids prepared using bentonite and barite weighting agents. The results are summarized in Table III. No wall slippage was observed for drilling muds having densities less than 16 ppg (61% water). A slip coefficient was observed when the water content becomes less than 38%. A graphical plot of slip coefficient versus

$$\left(\frac{1}{K'}\right)^{1/n'}$$

is given in Figure 10. In this range of water content, the rheological properties of the drilling mud are dependent upon the pipe diameter. The same type of linear relationship between slip coefficient and

$$\left(\frac{1}{K'}\right)^{1/n'}$$

exists for both muds and cement slurries.

FIELD EVALUATION

Rheological evaluation of an Oklahoma Class H cement slurry (38% water, 0.1% retarder, 0.1% prehydrated bentonite) was conducted using large-diameter pipe. The pipe layout is schematically shown in Figure 11. Two 50-bbl batches of cement were mixed in a twin-tank ribbon blender. Cement was added to each tank until the density was 16.6 ppg. The slurries were mixed for 30 minutes prior to pumping. Pressure transducers were located approximately 50 ft from the triplex pump and at the end of the pipe. A surge chamber was used to prevent pressure surges caused by the triplex pump. Data at each pump rate were taken every 15 seconds for at least two minutes. Rheological data were obtained on location using both the Fann 35/SR12 viscometer and the pipe-flow rheometer. Water was first pumped through the 1.815-in. I.D. treating line to test the pressure transducers.

Field Evaluation - Water

The pressure drop for fluids flowing in a pipe is given by[19]

$$P = \frac{0.039\ L\rho\ v^2 f}{D} \tag{24}$$

where

P = pressure drop, psi,
L = pipe length, ft,
ρ = fluid density, ppg,
v = velocity, ft/sec,
D = pipe diameter, in., and
f = Fanning friction factor.

The Fanning friction factor for Newtonian fluids in turbulent flow can be estimated by[20]

$$f = \frac{0.079}{N_R^{.25}} \tag{25}$$

where the the Reynolds number is given by[19]

$$N_R = \frac{928\ v\,\rho\,D}{\mu\rho}. \tag{26}$$

Substitution of the pipe geometry, fluid density and viscosity for water (Table IV) into Eqs. 24-26 gives

$$P = 5.3\ Q^{1.75}. \tag{27}$$

A plot of P versus $Q^{1.75}$ is given in Figure 12 and shows excellent agreement with Eq. 27.[12]

Field Evaluation - Cement

The rheological results for the cement slurries are tabulated in Table V.[12] Graphical log-log plots of shear stress and 96 v/D are shown in Figures 13 and 14. A plot of

$$\frac{1}{K'_D}^{1/n'}$$

versus 1/D is linear (Figure 15). The slip coefficient is shown in Figure 8 and is in excellent agreement with what was found using the pipe-flow rheometer.

Field Evaluation - Turbulence

As seen in Figures 13 and 14, the shear stress becomes abnormally high at high values of 96 v/D. This is an indication of the onset of turbulence in the pipe. For non-Newtonian fluids in turbulent flow, the Fanning friction factor becomes (n' = 0.5)[21]

$$f = \frac{0.0712}{N'_R{}^{0.31}}. \tag{28}$$

The Reynolds number for non-Newtonian fluids is given by[19]

$$N'_R = \frac{1.86\ v^{2-n'} D^{n'} \rho}{K'_D\ 96^n}. \quad (29)$$

Substitution of the pipe geometry, cement density and rheological parameter given in Tables IV and V into Eqs. 28 and 29 gives

$$P = 19.7\ Q^{1.69} \quad (D = 1.815 \text{ in.}) \quad (30)$$

and

$$P = 9.5\ Q^{1.69} \quad (D = 2.441 \text{ in.}). \quad (31)$$

A plot of pressure versus $Q^{1.69}$ is given in Figure 16 and is in excellent agreement with Eqs. 30 and 31.

CONCLUSIONS

Several cement slurries have been examined using a Fann 35/SR12 viscometer and a laboratory-scale pipe-flow rheometer. Results of these tests lead to the following conclusions.

1. All slurries tested to date are best described by a Metzner-Reed Power Law type model. Rheological data obtained using a Fann 35/SR12 viscometer did not follow a Power Law type model.

2. Studies using the pipe-flow rheometer indicate that apparent slip at the pipe wall occurs.

3. This apparent slip is associated with particle migration within the cement slurry during pipe shearing. The apparent wall slip associated with rheological measurements makes the rheological behavior of the cement slurries dependent upon pipe diameter and not length.

4. The slip coefficient found in the laboratory-scale pipe-flow rheometer has been verified using large-diameter pipe.

5. A three-parameter model based on the Metzner-Reed Power Law model is proposed which contains a slip coefficient (Eq. 1a Appendix).

6. The Fann 35 viscometer did not accurately predict the rheological properties of cement slurries flowing in a pipe (Appendix).

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APPENDIX
Rheological Evaluation of Cement Slurries

A 3-parameter model is required to relate shear stress with shear rate for a cement slurry flowing in a pipe. The mathematical relationship for rheological evaluation of cement slurries is expressed by

$$\tau_\omega = K'_D\left(\frac{96v}{D}\right)^{n'} \qquad (1a)$$

where

$$\left(\frac{1}{K'_D}\right)^{1/n'} = \left(\frac{1}{K'}\right)^{1/n'} + \frac{96\ C_S}{D}\ . \qquad (2a)$$

The term K'_D is the consistency index evaluated at a given pipe diameter. Experimental evaluation of the rheological properties of a cement slurry flowing in a pipe requires the measurement of shear stress at several pump rates and pipe diameters. Reduction of the rheological data obtained from a pipe-flow rheometer is obtained as follows.

1. A linear regression analysis of the log (shear stress) versus log (96 v/D) at each pipe diameter (Eq. 1a). (The slope is equal to n' and the intercept is equal to log K'_D.)

2. A linear regression analysis of a graphical plot of $(1/K'_D)^{1/n'}$ versus 1/D (Eq. 2a). (The slope is equal to 96 C_s and the intercept is equal to $[1/K']^{1/n'}$.)

The laminar flow pressure drops and Reynolds numbers may be calculated using

$$P = \frac{0.335\ L}{D[(\frac{1}{K'})^{1/n'} + \frac{96\ C_S}{D}]^{n'}} \left(\frac{96\ v}{D}\right)^{n'} \qquad (3a)$$

$$N'_R = \frac{1.86\ v^{2-n'}\ D^{n'}\ \rho}{96\ n'} \left[\left(\frac{1}{K'}\right)^{1/n'} + \frac{96\ C_S}{D}\right]^{n'} \qquad (4a)$$

where

P = pressure, psi,
L = pipe length, ft,
D = pipe diameter, in.,
v = slurry velocity, ft/sec,
n' = flow behavior index,
K' = consistency index, $lb/sec/ft^2$, and
N'_R = Reynolds number.

Table VI gives the calculated and experimental pressure drops for a cement slurry flowing through a 1.815-in. I.D. pipe. The calculated pressures were based on data taken from both the Fann 35/SR12 viscometer and the pipe-flow rheometer. Data taken from the Fann viscometer were best correlated using the Bingham Plastic Rheological Model.[19]

NOMENCLATURE

τ_ω - shear stress, lb/ft^2

$\dot{\gamma}$ - shear rate, sec^{-1}

Θ - Fann 35 reading

P - pressure, psi

H - height, ft

D - inside pipe diameter, in.

L - pipe length, in.

v - fluid velocity, ft/sec

K - consistency index, lb/sec/ft^2.

n - behavior index

K' - Metzner-Reed consistency index

n' - Metzner-Reed behavior index

K'_d - consistency index at given pipe diameter

μ - viscosity, cp

C_s - slip coefficient

μ - slip velocity, ft/sec

f - Fanning friction factor

N_R - Reynolds number

N'_R - Generalized Reynolds number

ρ - fluid density

SI CONVERSION TABLE

To Convert From	To	Multiply By
Degree Fahrenheit	Degree Celsius	$\frac{T_{^\circ F} - 32}{1.8}$
Pressure (psi)	kPa	6.895
Shear Stress (lb/ft^2)	kPa	4.788 E-2
Length (in.)	cm.	2.54
Velocity (ft/sec)	m/sec	3.048 E-1
Consistency (lb/sec/ft^2)	Pa sec	4.788 E + 1
Flow Rate (bpm)	m^3/min	1.590 E-1

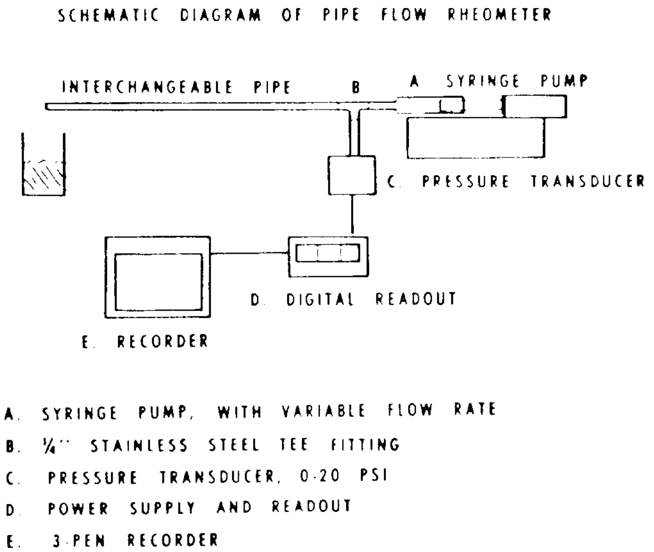

FIGURE 1—SCHEMATIC DIAGRAM OF PIPE-FLOW RHEOMETER.

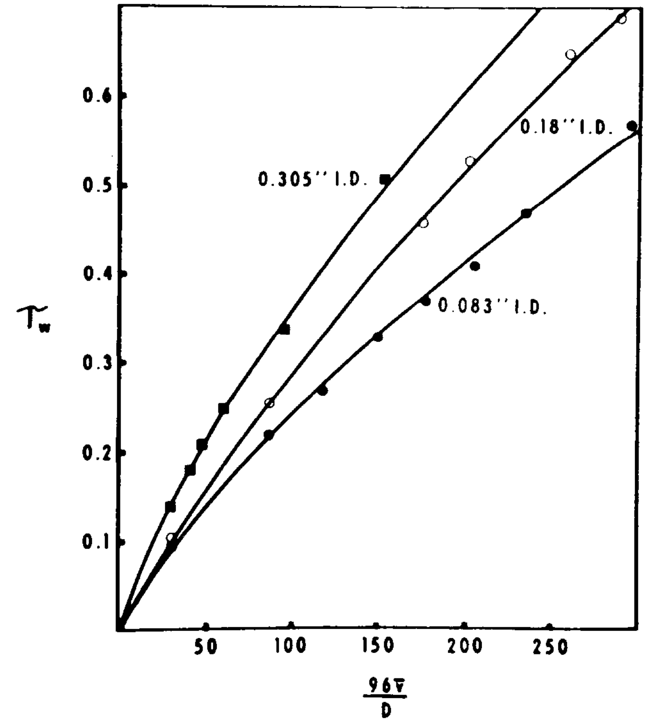

FIGURE 3—RHEOLOGICAL MEASUREMENTS USING THE PIPE-FLOW RHEOMETER DEPICTED IN FIGURE 1 (LONGHORN H, 40% WATER, 0.36% HEC, 80°F).

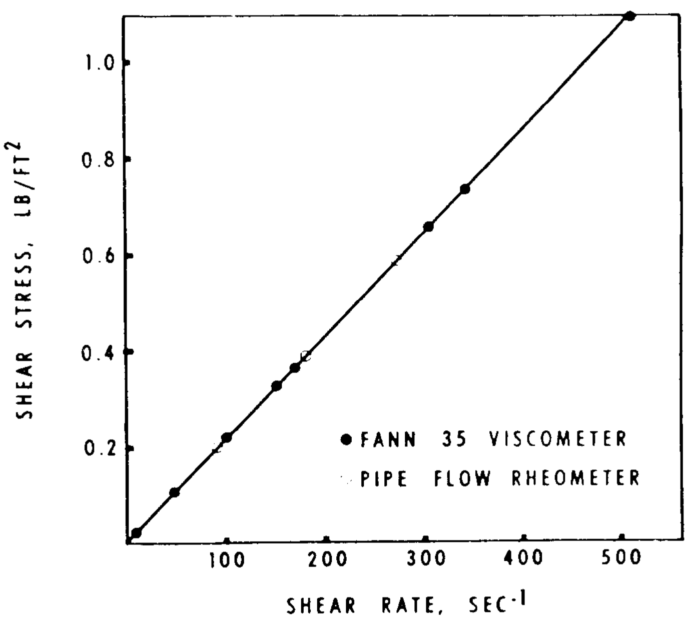

FIGURE 2—SHEAR STRESS-SHEAR RATE RELATIONSHIP OF A 103 CP NEWTONIAN OIL.

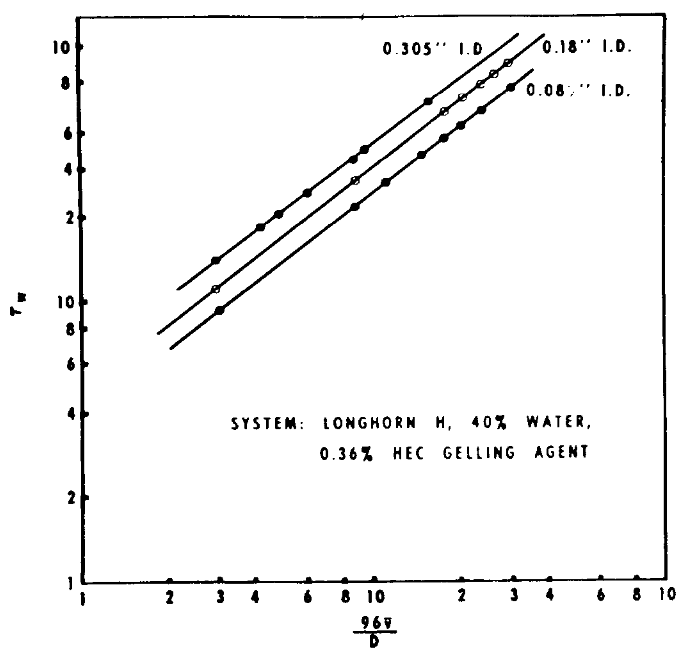

FIGURE 4—GRAPHICAL LOG-LOG PLOT EXEMPLIFYING THE POWER LAW BEHAVIOR OF PIPE-FLOW RHEOLOGICAL DATA.

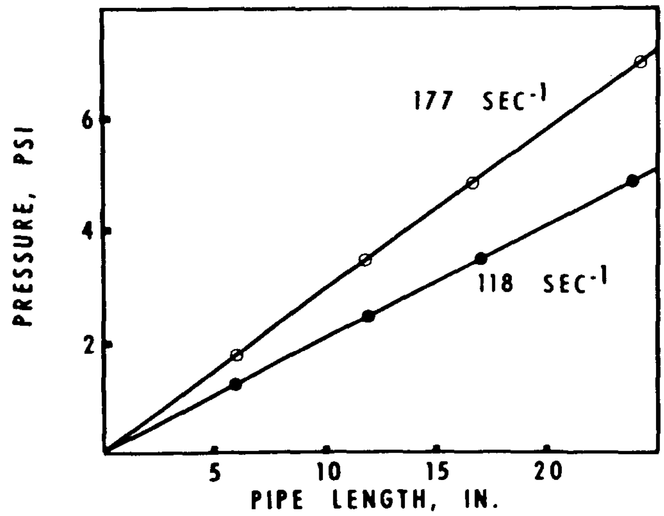

FIGURE 5—PRESSURE DROP AS A FUNCTION OF PIPE LENGTH, DEMONSTRATING THE VALIDITY OF EQ 15 (LONGHORN H, 40% WATER, 0.57% HEC, 80°F).

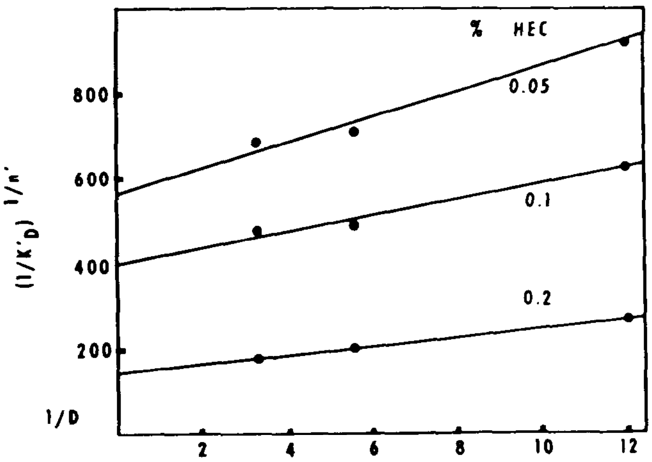

FIGURE 6—DEPENDENCE OF THE CONSISTENCY INDEX, K'D UPON PIPE DIAMETER (OKLAHOMA CLASS H, 42% WATER 80°F).

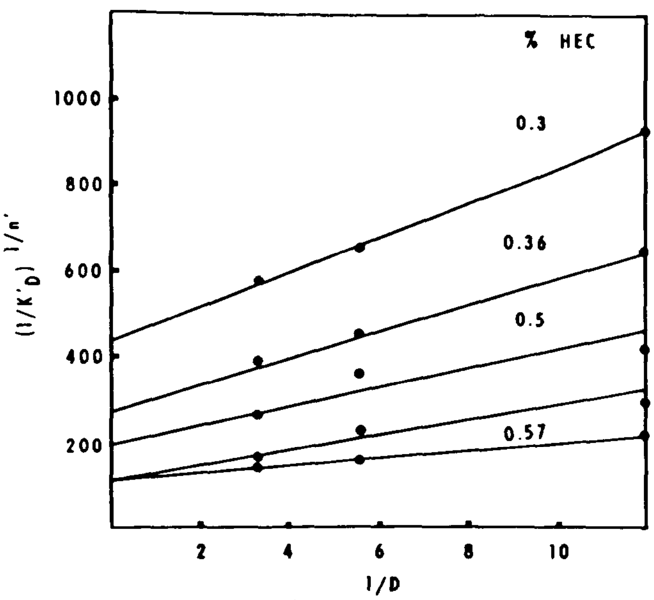

FIGURE 7—DEPENDENCE OF THE CONSISTENCY INDEX, K'D UPON PIPE DIAMETER (OKLAHOMA CLASS H, 40% WATER, 80°F).

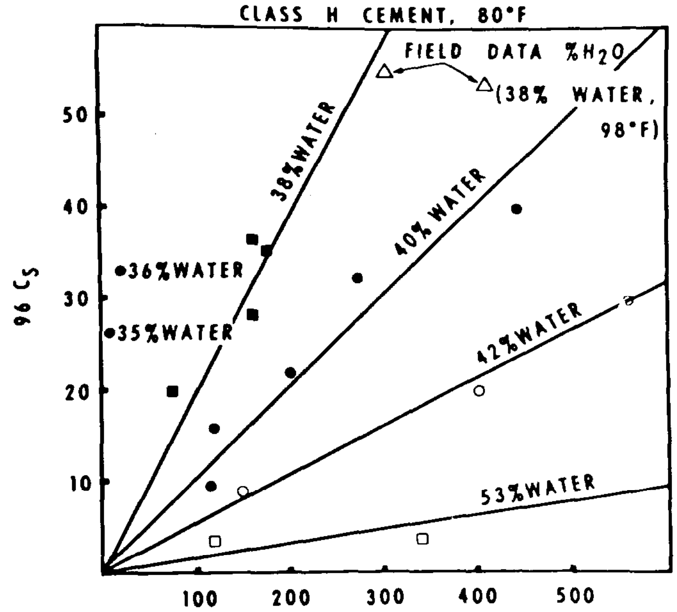

FIGURE 8—DEPENDENCE OF THE SLIP COEFFICIENT UPON THE CONSISTENCY INDEX, K'.

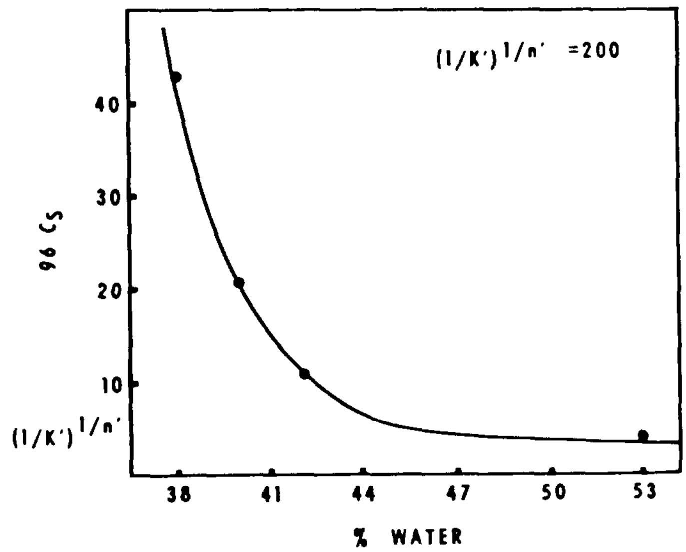

FIGURE 9—DEPENDENCE OF THE SLIP COEFFICIENT UPON CEMENT WATER CONTENT.

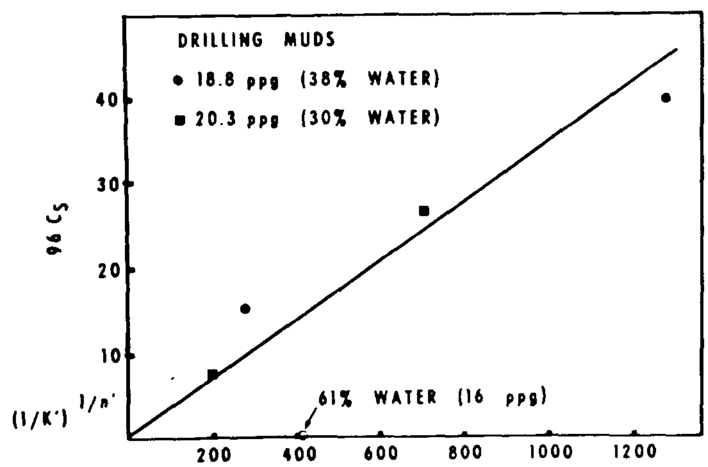

FIGURE 10—DEPENDENCE OF THE SLIP COEFFICIENT UPON CONSISTENCY INDEX (K') FOR HIGH DENSITY DRILLING MUDS.

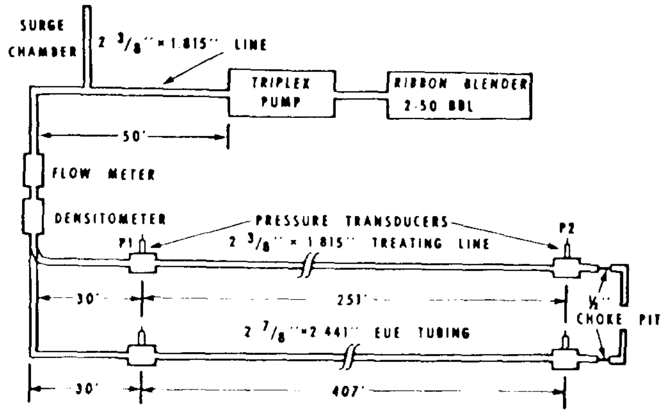

FIGURE 11—SCHEMATIC DIAGRAM OF RHEOLOGICAL TESTING AT DOWELL TRAINING CENTER KELLYVILLE, OKLAHOMA.

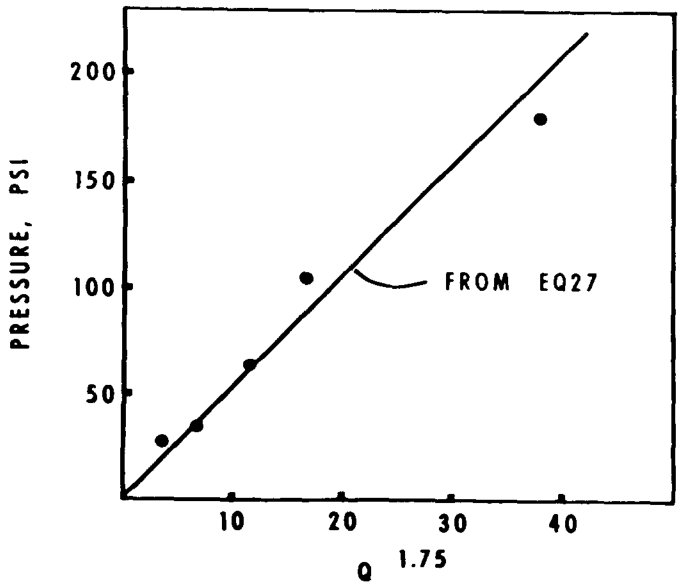

FIGURE 12—TURBULENT FLOW BEHAVIOR OF WATER FLOWING THROUGH A 1.815" I.D. PIPE. THE LINE WAS CALCULATED USING EQ 27.

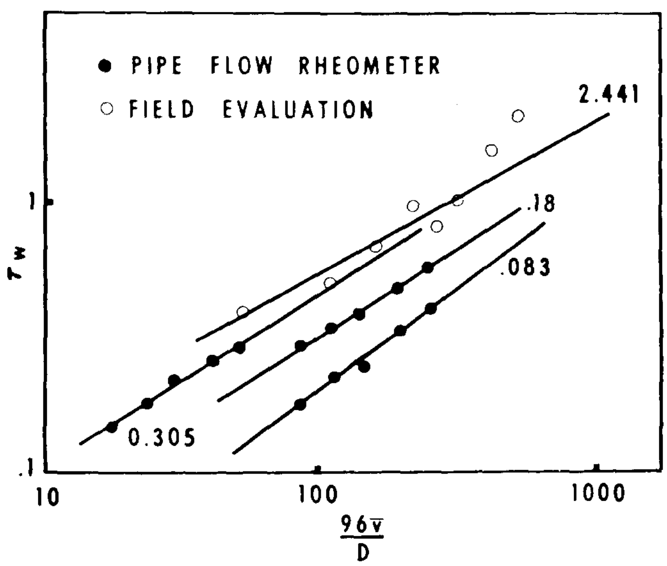

FIGURE 13—POWER-LAW PLOTS OF RHEOLOGICAL DATA OBTAINED IN THE FIELD.

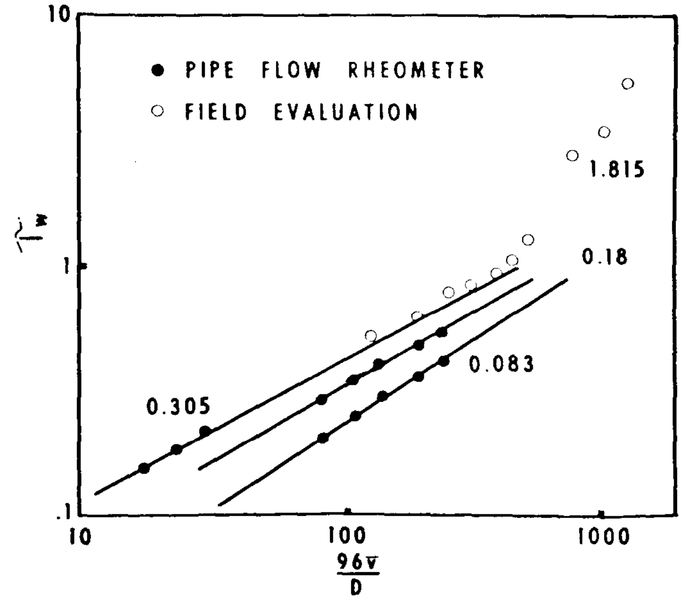

FIGURE 14—POWER-LAW PLOTS OF THE RHEOLOGICAL DATA OBTAINED IN THE FIELD.

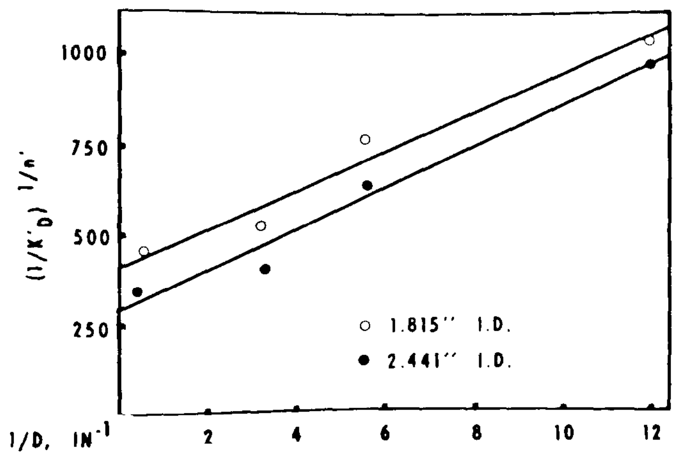

FIGURE 15—SCALE UP OF RHEOLOGICAL MEASUREMENTS (USING PIPEFLOW RHEOMETER) OF TWO SEPARATE MIXES OF OKLAHOMA CLASS H CEMENT SLURRIES (38% WATER, 0.1% RETARDER, 0.1% PREHYDRATED BENTONITE, 98°F.)

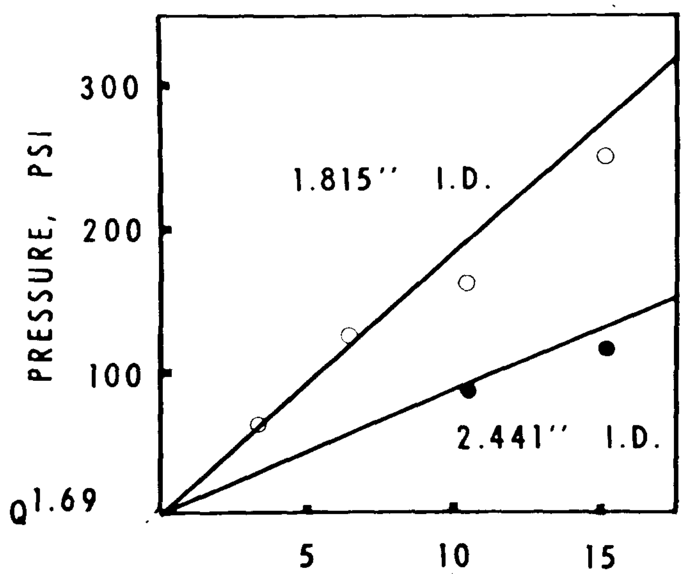

FIGURE 16—TURBULENT FLOW BEHAVIOR OF THE CEMENT SLURRY FLOWING THROUGH PIPE. THE LINES WERE CALCULATED USING EQ 30 AND 31.

TABLE 1—RHEOLOGICAL MEASUREMENTS OF A STANDARD NEWTONIAN OIL USING A FANN 35 VISCOMETER AND A PIPE-FLOW RHEOMETER (80°F.)

Fann 35		Pipe Flow			
RPM	Reading	Diameter, in	Length, in	Pump Rate, mL/min	Pressure, psi
600(1024)	204(2.17)	0.083	17	5.26(95.3)	1.15(0.20)
300(512)	103(1.10)			9.82(178)	2.30(0.40)
200(340)	69(0.74)			15.0 (272)	3.40(0.60)
180(304)	62(0.66)				
100(170)	34(0.36)	0.18	24	61.6 (109)	0.77(0.21)
90(152)	31(0.33)			108 (192)	1.50(0.41)
60(102)	21(0.22)			149.5 (266)	2.20(0.59)
30(51)	10(0.11)				
6(10.2)	2(0.02)	0.305	48	243 (88.7)	0.85(0.19)
3(5.1)	1(0.01)			476 (174)	1.75(0.40)
				714 (261)	2.60(0.60)
				163 (59.5)	0.55(0.13)

*Values given in parentheses are shear rates (sec^{-1}) and shear stress (lb/ft^2).

TABLE 2—RHEOLOGICAL PARAMETERS FOR CEMENT SLURRIES EVALUATED USING PIPE FLOW RHEOMETER (80°F)

Cement[1] System	Pipe Diameter, in.	K'_D	n'	$\left(\frac{1}{K'}\right)^{1/n'}$	$96\ C_s$
OK Class H 38% Water (41 cp, 54 lb/100 ft²	0.083 0.18 0.305	0.0155 0.0904 0.114	0.654 0.404 0.386	175	35
OK Class H 38% Water 0.1% HEC(2) (54 cp, 431 lb/100 ft²)	0.083 0.18 0.305	0.0179 0.0707 0.107	0.649 0.457 0.407	160	28
OK Class H 38% Water 0.2% HEC (85 cp, 52 lb/100 ft²)	0.083 0.18 0.305	0.0158 0.0732 0.126	0.721 0.502 0.416	77	20
OK Class H 42% Water 0.05% HEC (41 cp, 34 lb/100 ft²)	0.083 0.18 0.305	0.0180 0.0254 0.0610	0.589 0.560 0.430	560	30
OK Class H 42% Water 0.1% HEC (58 cp, 37 lb/100 ft²)	0.083 0.18 0.305	0.0240 0.0369 0.0954	0.578 0.533 0.381	397	19
OK Class H 42% Water 0.2% HEC (96 cp, 70 lb/100 ft²	0.083 0.18 0.305	0.0463 0.0874 0.190	0.551 0.458 0.319	150	9.5

Cement[1] System	Pipe Diameter, in.	K'_D	n'	$\frac{1}{K'}^{1/n'}$	96 C_s
Longhorn H 35% Water (73 cp, 119 lb/ 100 ft²)	0.083	0.0149	0.722		
	0.18	0.0603	0.573	10	26
	0.305	0.0854	0.582		
Longhorn H 36% Water (52 cp, 112 lb/ 100 ft²)	0.083	0.0156	0.694		
	0.18	0.0555	0.553	20	33
	0.305	0.0702	0.588		
Longhorn H 38% Water (32 cp, 110 lb/ 100 ft²)	0.083	0.0190	0.624		
	0.18	0.0658	0.459	160	36
	0.305	0.0953	0.434		
Longhorn H 40% Water 0.30% HEC (74 cp, 20 lb/ 100 ft²)	0.083	0.00848	0.697		
	0.18	0.00756	0.752	440	41
	0.305	0.0146	0.662		
Longhorn H 40% Water 0.36% HEC (94 cp, 29 lb/ 100 ft²	0.083	0.00777	0.749		
	0.18	0.00670	0.820	282	31
	0.305	0.0110	0.756		
Longhorn H 40% Water 0.5% HEC (109 cp, 41 lb/ 100 ft²)	0.083	0.00669	0.829		
	0.18	0.00702	0.844	200	22
	0.305	0.0108	0.811		
Longhorn H 40% Water 0.57% HEC (235 cp, 30 lb/ 100 ft²)	0.083	0.0173	0.752		
	0.18	0.0166	0.805	113	9
	0.305	0.0173	0.818		

TABLE 2—(CON'T)

Cement(1) System	Pipe Diameter, in.	K'_D	n'	$\frac{1}{K'}^{1/n'}$	$96\ C_s$
Longhorn H(3) 40% Water 0.57% HEC (218 cp, 19 lb/ 100 ft^2)	0.083 0.18 0.305	0.0130 0.0107 0.0147	0.859 0.846 0.741	120	17
Longhorn H 53% Water 0.85% HEC (110 cp, 33 lb/ 100 ft^2)	0.083 0.305	0.00322 0.00592	0.963 0.874	340	4
Longhorn H 53% Water 1% HEC (246 cp, 27 lb/ 100 ft^2)	0.083 0.18 0.305	0.0140 0.0154 0.0196	0.831 0.830 0.815	117	4.5

[1]Values given in parentheses are the plastic viscosity and yield point as determined using the Fann 35 viscometer and the Bingham Plastic Rheological model.

[2]Hydroxyethylcellulose (Hercules).

[3]Higher molecular weight HEC.

TABLE 3—RHEOLOGICAL PARAMETERS FOR DRILLING MUDS EVALUATED USING PIPE-FLOW RHEOMETER (80°F)

Cement[1] System	Pipe Diameter, in.	K'_D	n'	$\left(\frac{1}{K'}\right)^{1/n'}$	96 C_s
16 ppg[2] 61% Water 8% Bentonite (63 cp, 38 lb/100 ft^2)	0.083 0.18 0.305	0.0448 0.0537 0.0484	0.520 0.473 0.512	420	0
18.8 ppg 38% Water 7% Bentonite (74 cp, 38 lb/100 ft^2)	0.083 0.18 0.305	0.0435 0.0351 0.0904	0.514 0.566 0.418	280	15
18.8 ppg 38% Water 5% Bentonite (33 cp, 7 lb/100 ft^2)	0.083 0.18 0.305	0.00464 0.00438 0.00948	0.719 0.744 0.641	1280	40
20.3 ppg 30% Water 3% Bentonite (49 cp, 15 lb/100 ft^2)	0.083 0.18 0.305	0.0139 0.0131 0.0188	0.616 0.642 0.595	710	27
20.3 ppg 30% Water 5% Bentonite (87 cp, 46 lb/100 ft^2)	0.083 0.18 0.305	0.0516 0.0524 0.1134	0.525 0.523 0.398	200	8

[1]Values given in parentheses are the plastic viscosity and yield point as determined by using the Fann 35 viscometer.

[2]Mud systems weighted with barite to density indicated and percent additives based on barite.

TABLE 4—PARAMATERS FOR FIELD EVALUATION OF OKLAHOMA CLASS H CEMENT SLURRY (38% WATER, 0.1% RETARDER, 0.1% PREHYDRATED BENTONITE)

D = 1.815" I.D.

Cement Density = 16.6 ppg
Pipe Length = 251 ft.
Temperature = 98°F
Water Viscosity = 0.694 cp[1]
Plastic Viscosity (Fann) = 62 cp
Yield Point (Fann) = 12 lb/100 ft^2

D = 2.441" I.D.

Cement Density = 16.6 ppg
Pipe Length = 407 ft.
Temperature = 98°F
Plastic Viscosity (Fann) = 76 cp
Yield Point (Fann) = 27 lb/100 ft^2

[1]Cp "Handbook of Chemistry and Physics," 49th edition, 1968--1969, pp. F-37

TABLE 5—FIELD EVALUATION OF OKLAHOMA CLASS H CEMENT SLURRY (38% WATER, 0.1% RETARDER 0.1% PREHYDRATED BENTONITE, 98°F)

Pipe Diameter, in.	K'_D	n'	$\left(\frac{1}{K'}\right)^{1/n'}$	96 C_s
0.083	0.0127	0.628		
0.18	0.0262	0.548		
0.305	0.0345	0.540	408	54
1.815	0.0356	0.544		
0.083	0.00749	0.712		
0.18	0.0201	0.602		
0.305	0.0264	0.609	297	56
2.441	0.0497	0.510		

TABLE 6—CALCULATED PRESSURE DROPS FOR AN OKLAHOMA CLASS H CEMENT SLURRY 38% WATER, 0.1% PREHYDRATED BENTONITE) FLOWING THROUGH 1.815" I.D. PIPE (98°F)

Pump Rate, BPM	Pressure Drop, psi		
	Fann 35(1)	Pipe-Flow(2) Rheometer	Field Evaluation
0.5	16	25	24
1.0	24	36	37
1.5	32	45	43
1.75	36	49	48

(1)Rheological data analyzed using Bingham Plastic Model.

(2)Pressure drop calculated using eq. 3a in the Appendix.