THE USE OF DYNAMOMETER DATA FOR CALCULATING THE TORSIONAL LOAD ON SUCKER-ROD PUMPING GEARBOXES

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ABSTRACT

Knowledge of the magnitude of different components of mechanical torque acting on the gearbox is crucial for the design and analysis of sucker-rod pumping installations. Gearbox torques include the torque required to drive the polished rod and the torque used to rotate the counterweights. In addition to these, inertial torques arise in those parts of the pumping unit that turn at varying speeds. As shown in the paper, all torque components are functions of the crank angle, consequently their exact calculation necessitates the knowledge of the crank angle vs. time function. This circumstance, however, complicates torque calculations because contemporary dynamometers, used to acquire the necessary operating data, do not provide any information on the variation of the crank angle during the pumping cycle. The paper introduces a solution of the problem and presents an iterative calculation of the crank angle vs. time function from dynamometer data. Based on this function crank velocity, acceleration, as well as beam acceleration are easily found and all necessary gearbox torques can be evaluated. The paper describes the details of the developed calculation model and presents an example case.

INTRODUCTION

In a sucker-rod pumping unit the prime mover's rotary movement is transmitted through the V-belts and the gearbox to the crankshaft, i.e. the slow-speed shaft of the speed reducer. It is here where the energy input to the system by the prime mover is converted into useful power to lift well fluids to the surface. The power required at the crankshaft is primarily determined by the loads acting at the polished rod, which vary greatly during the pumping cycle. The effect of the polished rod loads on the crankshaft is torque, which is defined as force acting on a lever arm. Thus, in order to analyze the operational conditions of surface pumping equipment, it is essential to accurately predict the torques occurring at the crankshaft.

Torques on the gearbox are found from the variation of the polished rod load during the pumping cycle; torque calculations, therefore, depend on the accurate measurement of those loads. These are obtained from a dynamometer survey which is the most valuable tool for analyzing the performance of the pumping system. Polished rod dynamometers, as the name implies, are instruments recording polished rod loads during the pumping cycle. The conventional dynamometer produces a continuous plot of polished rod load vs. polished rod displacement, the so-called dynamometer diagram or card. Modern dynamometers, on the other hand, are electronic devices that record the loads and displacements at the polished rod in the function of time. Because of the operating differences of the two dynamometer types different procedures must be used to derive gearbox torques depending on the device used, as is discussed in this paper.

FUNDAMENTALS OF GEARBOX TORQUE CALCULATIONS

When calculating gearbox torques on a pumping unit two basic cases can be distinguished depending on the angular **velocity** of the crankshaft: (a) those with a **constant** or nearly constant, and (b) those with **varying** crankshaft velocities. In the majority of pumping installations the crankshaft's angular velocity is **constant** during the pumping cycle and matches the measured pumping speed. These are the cases when the gearbox is properly counterbalanced and an electric motor with a low slip drives the pumping unit. [1] The **API Spec. 11E** [2] suggests that up to a speed variation of 15% over the average pumping speed, neglecting inertial effects does not introduce errors greater than 10% in torque calculations.

The API Torque Analysis

The **API Torque Analysis** model (published in the appendices of **API Spec. 11E** [2] as a recommended calculation procedure) was developed for cases with constant crankshaft velocities and can be applied to any class of pumping unit geometry. This calculation model uses the torque factor concept with the following basic **assumptions**:

- **Frictional** losses in the pumping unit structure are neglected, i.e. a torque efficiency factor of unity is used.
- **Inertial** torques are neglected.
- The **change** in structural unbalance, ***SU***, with crank angle is also disregarded.

Under these conditions the **net torque** acting on the gearbox is simply found from the sum of the **rod torque** and the **counterbalance torque**. For mechanically balanced pumping units the following expressions are used:

$$T_{net}(\theta) = TF(\theta)\left[F(\theta) - SU\right] - T_{CB\max}\sin\theta \qquad \text{Conventional geometry} \qquad \mathbf{1}$$

$$T_{net}(\theta) = TF(\theta)\left[F(\theta) - SU\right] - T_{CB\max}\sin(\theta + \tau) \qquad \text{Mark II geometry} \qquad \mathbf{2}$$

$$T_{net}(\theta) = TF(\theta)\left[F(\theta) - SU\right] - T_{CB\max}\sin(\theta - \tau) \qquad \text{Reverse Mark geometry} \qquad \mathbf{3}$$

where: $T_{net}(\theta)$ = net torque on the gearbox at the crank angle θ, in lbs,
$TF(\theta)$ = torque factor at the crank angle θ, in,
$F(\theta)$ = polished rod load at the crank angle θ, lbs,
SU = structural unbalance of the pumping unit structure, lbs,
T_{CBmax} = maximum moment of counterweights and cranks, in lbs,
θ = crank angle, degrees, and
τ = counterweight arm offset angle, degrees.

The basic requirement for the calculation of gearbox torques is the knowledge of polished rod **loads** in the function of crank **angle** since rod torque is found by multiplying the loads and torque factors belonging to the same crank angle. This condition, however, is not met if data recorded on **conventional** dynamometer cards are used to find polished rod loads because these cards record the **load** against polished rod **displacement**. Thus the load vs. crank angle function, $F(\theta)$, must be derived before the torques on the speed reducer can be calculated. The procedure introduced in **API Spec. 11E** [2] and widely used for this purpose is based on data obtained from a conventional dynamometer survey.

Cases with Variable Crank Speeds

When the pumping unit is driven by a multicylinder engine, a high-slip or even an ultra-high slip (UHS) electric motor the angular velocity of the crankshaft changes during the pumping cycle: the crank speeds up when the unit is lightly loaded and slows down as the load becomes heavier. The high accelerations/decelerations coupled with the heavy rotating masses gives rise to **inertial** torques because of the flywheel effect; gearbox torque calculations must be appropriately modified to include these effects. In such cases, in addition to the torques normally present on the gearbox and discussed so far, **articulating** and **rotary inertial** torques must be also calculated. [3]

Inertial Torques

Articulating inertial torque exists even if the prime mover speed is **constant** and the crankshaft turns with a constant angular velocity. This torque is caused by those structural parts of the pumping unit that move with varying **accelerations** during the pumping cycle like the beam, horsehead, equalizer, etc. Articulating torque primarily depends on the angular acceleration pattern of the beam, i.e. $d^2\theta_b/dt^2$; this can be derived from a kinematic analysis of the pumping unit as proposed by **Svinos** [4] and is found from the following formula:

$$T_{ia}(\theta) = \frac{12}{32.2}\, TF(\theta)\, \frac{I_b}{A}\, \frac{\mathrm{d}^2\theta_b}{\mathrm{d}t^2} \qquad \mathbf{4}$$

where: $T_{ia}(\theta)$ = articulating inertial torque on the gearbox at crank angle θ, in lbs,

$TF(\theta)$ = torque factor at crank angle θ, in,
A = distance between the saddle bearing and the polished rod, in,
I_b = mass moment of inertia of the beam, horsehead, equalizer, bearings, and pitmans, referred to the saddle bearing, lbm ft^2, and
$\mathrm{d}^2\theta_b/\mathrm{d}t^2$= angular acceleration of the beam, 1/sec^2.

Rotary inertial torque has a much greater importance than articulating torque. It can either increase or decrease the load on the gearbox. At times when crankshaft speed increases, the additional load (rotary inertial torque) on the gearbox is converted to kinetic energy and is stored in the rotating parts. On the other hand, if crankshaft speed decreases then energy previously stored in the cranks and counterweights is returned into the system and the torque load on the gearbox is reduced. This kinetic energy **interchange** happens in the following rotating masses of the pumping unit:

- The cranks with crank pins,
- The counterweights and auxiliary weights, and
- The slow-speed shaft and slow-speed gear of the speed reducer.

Since all these components rotate around the crankshaft, their combined moment of inertia can be found from simple **addition** of the individual moments. Using the net moment of inertia of the rotating system, the rotary inertial torque on the gearbox is found from the next formula:

$$T_{ir}(\theta) = \frac{12}{32.2}\, I_s\, \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \qquad \mathbf{5}$$

where: $T_{ir}(\theta)$ = rotary inertial torque on the crankshaft at crank angle θ, in lbs,
I_s = mass moment of inertia of the cranks, counterweights, and slow-speed gearing referred to the crankshaft, lbm ft^2, and
$\mathrm{d}^2\theta/\mathrm{d}t^2$ = angular acceleration of the crankshaft, 1/sec^2.

As seen, articulating inertia changes with the angular acceleration of the beam, $\mathrm{d}^2\theta_b/\mathrm{d}t^2$; while rotary inertial torque changes with the angular acceleration of the crankshaft, $\mathrm{d}^2\theta/\mathrm{d}t^2$. Both of these can be derived from the angular **velocity** of the crankshaft, which, in turn, is the derivative of the crank angle vs. time function, $\theta(t)$. The latter can be inferred from electronic **dynamometer** measurements using the calculation model developed in this paper and detailed later.

Net Torque on the Gearbox

In cases when the pumping system operates with varying crankshaft speeds the **net** torque on the gearbox must include the **inertial** effects as well and the formulas derived for a constant crankshaft speed (**Eqs. 1** – **3**) cannot be used. The proper formula for net torque is the algebraic sum of all possible torque components:

$$T_{net}(\theta) = T_r(\theta) + T_{CB}(\theta) + T_{ia}(\theta) + T_{ir}(\theta) \qquad \mathbf{6}$$

where: $T_{net}(\theta)$ = net torque on speed reducer at crank angle θ, in lbs,
$T_r(\theta)$ = rod torque at crank angle θ, in lbs,
$T_{CB}(\theta)$ = counterbalance torque at crank angle θ, in lbs,
$T_{ia}(\theta)$ = articulating inertial torque at crank angle θ, in lbs, and
$T_{ir}(\theta)$ = rotary inertial torque at crank angle θ, in lbs.

Upon substitution into this equation of the relevant formulae introduced earlier, except for the expression for counterbalance torque, a generally applicable formula is found:

$$T_{net}(\theta) = TF(\theta)\left[F(\theta) - SU + \frac{12}{32.2}\frac{1}{A}\, I_b\, \frac{\mathrm{d}^2\theta_b}{\mathrm{d}t^2}\right] + T_{CB}(\theta) + \frac{12}{32.2}\, I_s\, \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \qquad \mathbf{7}$$

where: $T_{net}(\theta)$ = net torque on speed reducer at crank angle θ, in lbs,
$TF(\theta)$ = torque factor at crank angle θ, in,
$F(\theta)$ = polished rod load at crank angle θ, lbs

SU = structural unbalance, lbs.
A = distance between the saddle bearing and the polished rod, in,
I_b = mass moment of inertia of the beam, horsehead, equalizer, bearings, and pitmans, referred to the saddle bearing, lbm ft^2,
$d^2\theta_b/dt^2$= angular acceleration of the beam, 1/sec^2.
$T_{CB}(\theta)$ = counterbalance torque at crank angle θ, in lbs,
I_s = mass moment of inertia of the cranks, counterweights, and slow-speed gearing referred to the crankshaft, lbm ft^2, and
$d^2\theta/dt^2$ = angular acceleration of the crankshaft, 1/sec^2.

The first term in this equation represents **rod** torque, corrected for articulating inertial effects, the second one stands for the **counterbalance** torque, and the last term gives the rotary **inertial** torque. The formula can be applied to any mechanically counterbalanced pumping unit after substitution of the proper expression for counterbalance torque, $T_{CB}(\theta)$.

DETERMINATION OF INFERRED CRANK ANGLES

Introduction

As already discussed, gearbox torques are normally calculated based on dynamometer measurements that provide the variations of polished rod loads and positions. Modern dynamometer systems register these data in function of **time** but give no information on the crank **angles** valid at the measured times. This circumstance, however, prohibits a **direct** calculation of gearbox torques because all torque components depend on the crank angle, θ. **Rod** torque changes with the torque factor that varies with the crank angle; **counterbalance** torque is a direct function of crank angle, see **Eqs. 1 - 3**. **Inertial** torques are found from the acceleration patterns of different components of the pumping unit, these also change with the variation of the time history of crank angle. Gearbox torque components, therefore, can only be calculated if the change of crank angle with time, $\theta(t)$, is determined from dynamometer measurements.

Since direct calculation of crank angles from the measured polished rod positions is not possible, crank angles are inferred from the pumping unit's kinematic parameters. This can be completed several ways but all methods are based on setting the measured polished rod positions equal to the positions determined from the kinematic analysis of the pumping unit:

$$s(t) = S\ PR(\theta) \qquad \textbf{8}$$

where: $s(t)$ = measured value of polished rod position at time t, in,
S = polished rod stroke length, in, and
$PR(\theta)$ = calculated dimensionless position of rods at crank angle θ, -.

For each measured polished rod position, $s(t)$, the corresponding crank angle, θ, is found when the above equation is satisfied; this procedure results in a series of crank angles in function of time, $\theta(t)$. This function, in turn, allows one to find the two important components of gearbox torque: rod and counterbalance torque. To calculate **Rod Torque** the torque factor (***TF***) is found for each crank angle from the kinematic analysis of the pumping unit; torque is the product of the torque factor and the measured polished rod load. **Counterbalance Torqu**e, on the other hand, varies simply with the sine function of the crank angle. Determination of the inertial torque components, too, necessitates the knowledge of the crank angle-time function, $\theta(t)$, since both the angular acceleration pattern of the beam, $d^2\theta_b/dt^2$, and the angular acceleration of the crankshaft, $d^2\theta/dt^2$, change with the variation of the crank angle.

Calculation Procedure

The determination of the crank angle vs. time function, $\theta(t)$, is accomplished according to the calculation model described on the flowchart in **Fig. 1**. After the input of the pumping unit's main data (geometry type, dimensions, direction of rotation) and assuming a sufficiently small crank angle increment, $\Delta\gamma$, the polished rod's stroke length, S, and the starting crank angles of the up-, and the downstroke, θ_u, θ_d, respectively, are determined. These variables are evaluated according to the formulas recommended in **API Spec. 11E** [2] for the calculation of the kinematic parameters of pumping units.

Based on the measured polished rod positions, $\boldsymbol{s(i)}$, one can calculate the appropriate dimensionless positions:

$$PR_m(i) = \frac{s(i)}{S} \qquad \textbf{9}$$

where: $\boldsymbol{PR_m(i)}$ = dimensionless position of rods for the i[th] measured point, -
$\boldsymbol{s(i)}$ = i[th] measured polished rod position, in, and
$\boldsymbol{S}$ = polished rod stroke length, in.

The objective of the main part of the calculation process is to find for each $\boldsymbol{PR_m(i)}$ value the crank angle, $\boldsymbol{\theta}$, at which the position of rod function, $\boldsymbol{PR(\theta)}$, a basic kinematic parameter of the pumping unit is equal to $\boldsymbol{PR_m(i)}$. In the calculation procedure described in **Fig. 1** the required iterative solution is executed by successive approximations using two auxiliary angles γ_1 and γ_2 being apart from each other by the assumed crank angle increment, $\Delta\gamma$. At these angles, position of rods functions, $\boldsymbol{PR(\gamma_1)}$ and $\boldsymbol{PR(\gamma_2)}$, are evaluated using the API kinematic model for pumping units [2]. Using successive increments of the crank angle, the solution will fall between two consecutive angles γ_1 and γ_2 if the following expression becomes negative:

$$diff = \left[PR(\gamma_1) - PR_m(i)\right] \times \left[PR(\gamma_2) - PR_m(i)\right] \qquad \textbf{10}$$

where: $\boldsymbol{PR_m(i)}$ = dimensionless position of rods for the i[th] measured point, -, and
$\boldsymbol{PR(\gamma_1)}$, $\boldsymbol{PR(\gamma_2)}$ = calculated dimensionless positions of rods at crank angles γ_1 and γ_2, -.

Because of the small crank angle increment used in the program ($\Delta\gamma = 0.1$ degrees), linear approximation can be applied to find the final crank angle: $\boldsymbol{\theta(i)} = (\gamma_1 + \gamma_2)/2$. The error committed by using this approach is less than half of the increment used, i.e. $\Delta\gamma/2$, which is more than sufficient for the purpose.

The calculation model just described solves the basic problem of calculating gearbox torques by providing the variation of crank angles with time during the pumping cycle based on measured dynamometer data. The crank angle vs. time function, $\boldsymbol{\theta(t)}$, thus determined is used for the calculation of the angular acceleration of the crankshaft as well as that of the beam. These, in turn allow the determination of inertial torques on the gearbox.

CALCULATION OF ANGULAR ACCELERATIONS AND GEARBOX TORQUES

As already discussed, inertial torques on the gearbox depend on the angular accelerations of the different components of the pumping unit. Since the determination of the angular accelerations of the crank and the beam from the crank angle vs. time function, $\boldsymbol{\theta(t)}$, necessitates several considerations the solutions applied in this paper will be illustrated through an example problem. The sample well is produced with a 1.5” pump set at 8,000 ft and using a two-taper API 76 rod string. The surface pumping unit is a conventional C-640D-365-168 unit running at an average pumping speed of 5.98 SPM using a polished rod stroke length of 168 in. Polished rod position, $\boldsymbol{s(t)}$, and polished rod load, $\boldsymbol{F(t)}$, were measured by an electronic dynamometer in function of time; the dynamometer card constructed from those data is presented in **Fig. 2**.

First the crank angle vs. time function, $\boldsymbol{\theta(t)}$, is determined with the use of the calculation procedure described previously. Were the pumping unit’s crank turning at a constant speed, crank angles would fall on a straight line in function of time. In the example case, however, this is not the case as shown in **Fig. 3** because pumping speed varies during the cycle. Angular velocity of the crank, $\mathrm{d}\boldsymbol{\theta}/\mathrm{d}\boldsymbol{t}$, is the derivative of this function and is found by a numerical differentiation model using a five-step stencil as given here:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} \approx \frac{-\theta(t+2\Delta t) + 8\,\theta(t+\Delta t) - 8\,\theta(t-\Delta t) + \theta(t-2\Delta t)}{12\,\Delta t} \qquad \textbf{11}$$

where: $\mathrm{d}\boldsymbol{\theta}/\mathrm{d}\boldsymbol{t}$ = angular velocity of the crankshaft at time $\boldsymbol{t}$, 1/sec,
$\boldsymbol{\theta(t)}$ = crank angle at time $\boldsymbol{t}$, radians, and
$\boldsymbol{\Delta t}$ = time increment of measurements, sec.

To apply this model two extra points (crank angles) at each end of the pumping cycle are required; these are estimated by straight-line extrapolations of the input data. This solution ensures that the derivative does not have peaks at the two ends of the cycle; calculation accuracy is provided by the small time increment. Results are shown in **Fig. 3** that includes the variation of the crankshaft angular velocity (crank speed) with time.

After a smooth curve for the crank speed is obtained by numerical differentiation, it is fitted by a truncated Fourier series that is easy to differentiate to get the angular acceleration of the crankshaft, $d^2\boldsymbol{\theta}/d\boldsymbol{t}^2$, also shown in **Fig. 3**. Now the angular acceleration of the beam, $d^2\boldsymbol{\theta_b}/d\boldsymbol{t}^2$, is calculated from either a kinematic analysis of the pumping unit as proposed by **Svinos** [4] or using a more direct calculation model as introduced by **Gibbs** [5]. This paper uses the Gibbs approach that results in the least amount of computation effort, as proved by **Takacs – Kis** [6], and calculates the beam acceleration as follows:

$$\frac{d^2\theta_b}{dt^2} = \frac{1}{A}\frac{d^2}{dt^2}s(t) \qquad \mathbf{12}$$

where: $d^2\boldsymbol{\theta_b}/d\boldsymbol{t}^2$ = angular acceleration of the beam, 1/sec^2,
$\boldsymbol{s(t)}$ = polished rod position vs. time, in, and
$\boldsymbol{A}$ = distance from the center bearing to the horsehead, in.

The formula permits the direct calculation of the beam's angular acceleration from the polished rod position vs. time data obtained from a dynamometer survey by taking the second derivative of that function, $\boldsymbol{s(t)}$. The easiest way to differentiate this function is to first fit the measured data with a truncated Fourier series and then find the second derivative of that. Because of the relatively smooth variation of polished rod position with time a maximum of ten terms in the Fourier series are recommended by **Gibbs** [5].

The calculated angular accelerations (crankshaft and beam) are depicted in **Fig. 3** in function of time. As described previously, these values are the prerequisites of calculating gearbox torques. The variation of calculated torques during the pumping cycle is presented in **Fig. 4** for the example case. As seen, articulating inertial torque (denoted Art. in the figure) is negligible if compared to other torque components. Rotating inertial torque (denoted Rot.), on the other hand, very substantially reduces the torque load on the gear reducer whenever its sign is negative. Negative rotary torque indicates that energy stored in the heavy rotating parts of the pumping unit is released and helps reducing the net torque. This behavior is caused by the great variation of crankshaft speed during the pumping cycle in the sample case.

CONCLUSIONS

The most important conclusions of this paper can be summed up as follows.

- When electronic dynamometers are used to analyze the operation of the sucker-rod pumping system then calculation of gearbox torques is only possible if the change of crank angle with time, $\boldsymbol{\theta(t)}$, is inferred from dynamometer measurements.
- The iterative procedure developed in the paper provides an accurate description of the variation of the crank angle with time from the data of a dynamometer survey.
- The combination of numerical differentiation and the use of Fourier series, as described in the paper, results in the necessary angular accelerations of the crankshaft and the beam; these are used to find gearbox torque components.

REFERENCES

1. Takacs, G.: Sucker-Rod Pumping Manual. PennWell Books, Tulsa Oklahoma, 2003.
2. "Specification for Pumping Units." API Spec. 11E, 18th Edition, American Petroleum Institute, Washington, DC 2008.
3. Gibbs, S. G.: "Computing Gearbox Torque and Motor Loading for Beam Pumping Units with Consideration of Inertia Effects." JPT September 1975 1153-9.
4. Svinos, J. G.: "Exact Kinematic Analysis of Pumping Units." Paper SPE 12201 presented at the 58th Annual Technical Conference and Exhibition of the SPE, San Francisco, California, Dec. 5-8, 1983.
5. Gibbs, S. G.: Rod Pumping. Modern Methods of Design, Diagnosis and Surveillance. BookMasters, Inc., USA, 2012.
6. Takacs, G. – Kis, L.: "Finding the Best Way to Calculate Articulating Torque on Sucker-Rod Pumping Gear Reducers." BKL Koolaj es Foldgaz, 147/3 17-20, 2014.

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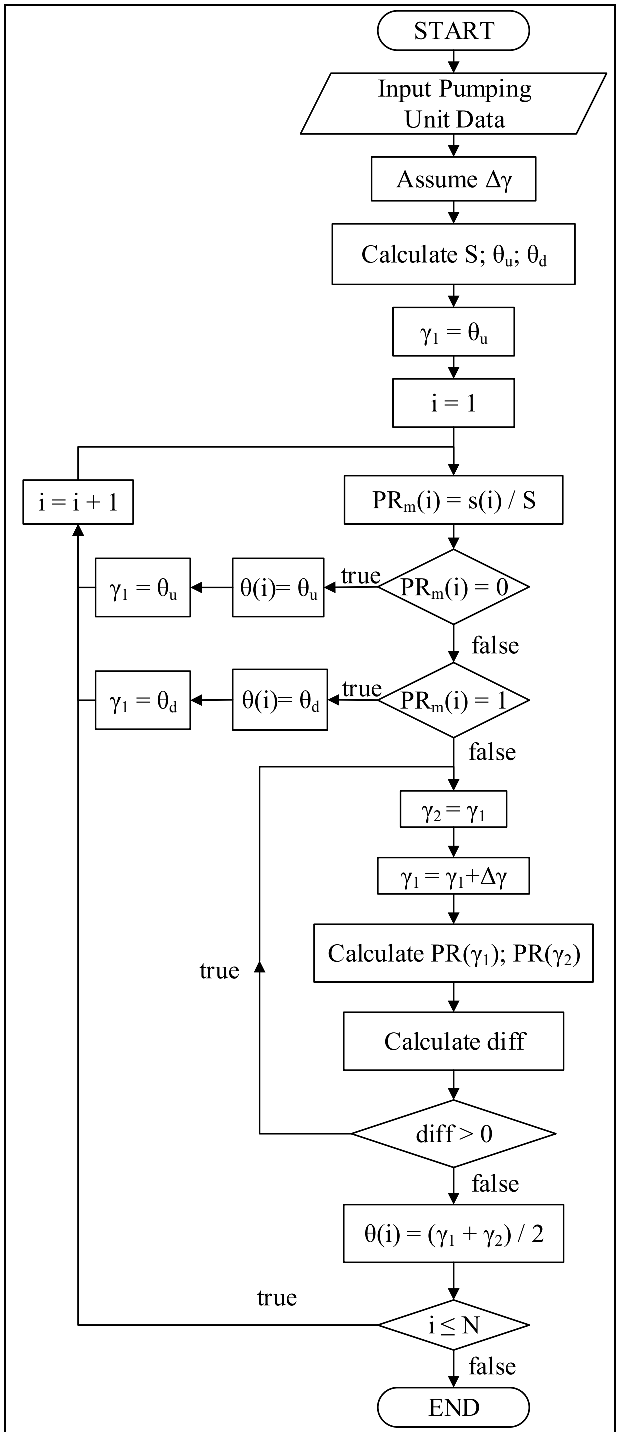

Fig. 1. Flowchart of inferred crank angle vs. time function calculations.

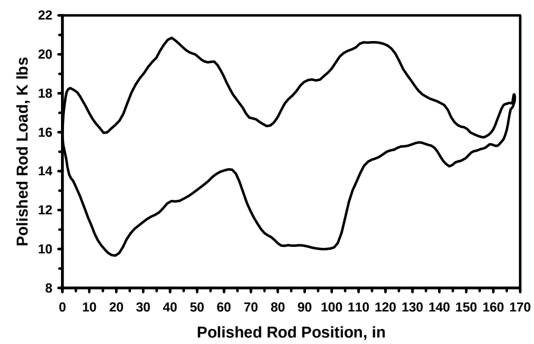

Fig. 2. Dynamometer card for the example problem.

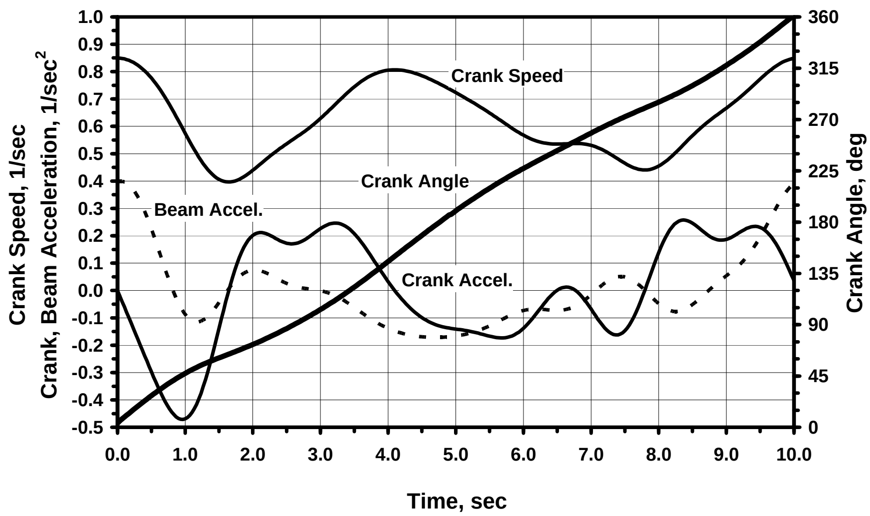

Fig. 3. Calculated angular velocities and accelerations for the example case.

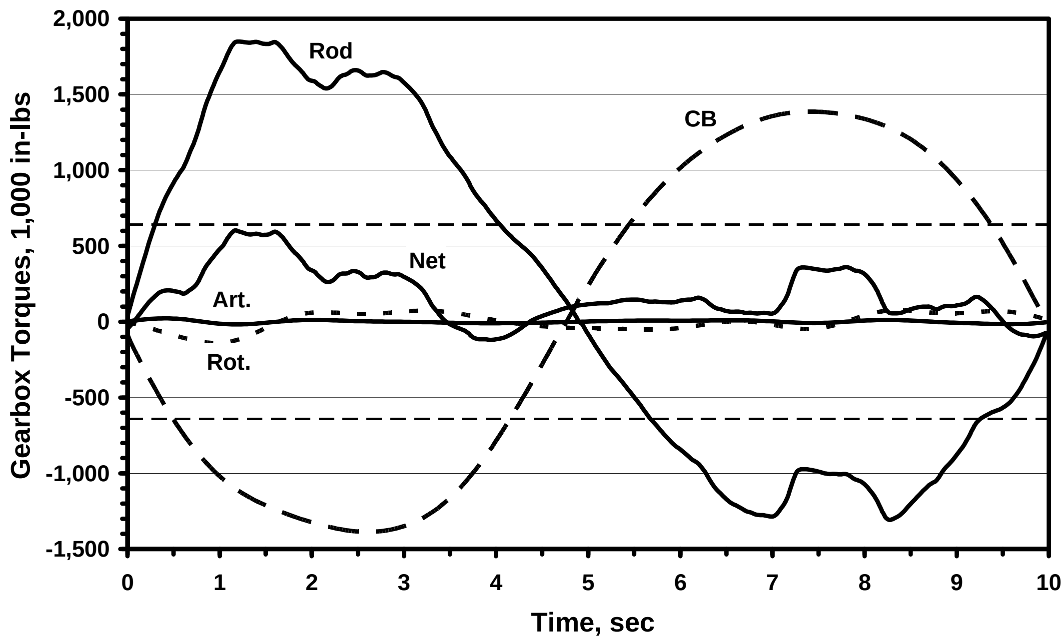

Fig. 4. Variation of gearbox torque components during the pumping cycle for the example case.