

# RELATING THE PHYSICAL PROPERTIES OF FRACTURING SLURRIES TO THE MINIMUM FLOW VELOCITY REQUIRED FOR PROPPANT TRANSPORT

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## ABSTRACT

Optimization of effective fracture area is among the principal tenets of fracturing design engineering. It is well understood that effective fracture area is a first order driver for well productivity, and that optimization of effective fracture area is often critical to economic exploitation of reservoir assets. Extensive testing in a large-scale slot apparatus was conducted to evaluate the relative effects of various component and treatment parameters on the proppant transport capability of various slurry compositions. The acquired data were utilized to determine the minimum horizontal slurry velocities necessary for proppant transport using the respective slurry compositions.

An 'index' to define the physical properties of a given proppant and fluid composition was defined. An empirical model was then derived to determine the minimum horizontal flow velocity required for suspension transport of a given slurry composition based upon its Slurry Properties Index. The minimum suspension transport velocity may then be compared to the flow velocity profiles from fracture design programs to estimate the propped fracture length likely be observed for those conditions.

Utilizing the new model, the most favorable combination of fracturing slurry component properties and pumping parameters can be identified and incorporated in fracturing treatment design and applications to optimize effective propped fracture length, and thereby well performance.

## INTRODUCTION AND BACKGROUND

Poor proppant transport can result in excessive proppant settling, often into the lower regions of the created fracture below the productive interval, yielding relatively short effective fracture lengths and insufficient coverage of the total height of the productive zone. Additionally, inadequate clean-up of the resultant propped fracture result in significant reduction of the conductivity of the propped fracture area. The cumulative effects of the afore-mentioned phenomena can result in a reduction of overall stimulation efficiency, yielding steeper post-stimulation production declines than may be desired. Post-frac production analyses frequently illustrate that the effective fracture area is less than that expected based upon the design, suggesting the proppant was not placed effectively throughout the designed fracture area or, existence of excessive proppant-pack damage. Optimization of effective fracture area is often critical to economic exploitation of reservoir assets, thus maximization of the propped fracture area is a key parameter for generating desired stimulated well performance. Efforts to improve effective fracture area have historically focused on the proppant transport capability of the fracturing fluid and the fracture clean-up attributes. A better understanding of the proppant transport process and the controlling variables was thought to have the potential for developing improved methodologies to maximize the effective fracture area.

The relative effects on proppant transport of the various proppant slurry component and treatment parameters were evaluated via extensive testing at the University of Oklahoma's Well Construction Technology Center. The techniques developed by Biot and Medlin to determine terminal settling velocities and suspension regimes were used to process and analyze the acquired data<sup>1</sup>.

The Biot-Medlin analysis is based upon evaluation of  $v_t/U$  versus  $U$ , where  $v_t$  is the terminal settling velocity calculated from transport observations for a given carrier fluid/proppant slurry, and  $U$  is the horizontal velocity. The quantity  $v_t/U$  is dimensionless. Utilizing the generated plot, the transport regimes of particulate slurries may be assessed using the criteria set forth by Biot-Medlin. These criteria state that the critical condition for particle 'pick-up' occurs at horizontal velocities in which the  $v_t/U$  value is 0.9. For  $v_t/U$  values of greater than 0.9, transport is by rolling or sliding. For  $v_t/U$  values  $< 0.5$  but  $> 0.1$ , a condition

of bed load transport occurs wherein at least some portion of the particles are moving in a traction carpet across the top of an immobile bed. Suspension transport, wherein there is a mobile bulk slurry, occurs only at horizontal velocity conditions such that the  $v_t/U$  value is  $<0.1$ . Thus, the minimum horizontal velocity for suspension transport can be defined as  $v_t / (0.1)$ . More simply, the minimum horizontal velocity for suspension transport is ten times the terminal settling velocity, i.e.  $(v_t) \cdot (10)$ .

The minimum horizontal velocities for suspension transport derived from the Biot-Medlin analysis were subsequently coupled with the horizontal flow velocities calculated from a simple geometric fracture model to estimate the propped fracture lengths for the various proppant slurry compositions. The effective fracture lengths estimated for the various slurries using these methods were reasonably consistent with lengths commonly derived from production history analysis of wells treated with sand/slickwater slurries and fracture geometries thought to be similar to those modeled.

An empirical proppant transport model has been in development using the Biot-Medlin minimum velocity data and the fracture velocity decay model to provide a design tool for effective fracture length optimization. The model has input parameters including fluid viscosity, proppant size, and both fluid and proppant specific gravity, injection rate and fracture geometry. The desired model output is the estimated propped fracture length of a given treatment design incorporating a fracturing fluid slurry having the variable input parameters. Iteration using such a model can be employed to define favorable combinations of fracturing slurry component properties and pumping parameters to optimize effective propped fracture length, and thereby, well performance.

## LARGE-SCALE SLOT FLOW TESTING

Three series of large-scale slot flow testing have been conducted at the Well Construction Technology Center. The WCTC slot apparatus is constructed of two plates of museum glass, with the slot having the dimensions of 16 ft (4.87 m) long and an internal height of 22 in. (0.559 m). A schematic diagram of the apparatus is shown in Fig 1. Detailed descriptions of the apparatus and testing procedures were presented previously<sup>2-3</sup>.

### **Slot Flow Test Series No.1**

The first set of tests was conducted on the slot flow apparatus configured with a ½-inch (12.7 mm) slot width, which combined with low pump rates, translated into very low lateral velocities in the slot. The results from the Series No. 1 testing showed that a 1.0 ppg slurry of 20/40 Ottawa sand in slickwater exhibited settling immediately upon initiation of the test even though the flow rate was 50 gal/minute (27.3 ft/min lateral velocity through the slot). The settling continued at each step change and bed height built rapidly. Subsequent tests were conducted to compare the behavior of ultra-lightweight proppants (ULW proppants) at conditions identical to those employed in the Ottawa sand tests. The concentrations using the ULW proppants were volumetrically the same as 1.0 ppg of sand. The ULW proppants exhibited dramatically reduced settling rates and equilibrium bed heights compared to sand. An average reduction in equilibrium bed height of over 80% was observed with the ULWP-1.25 (ultra-lightweight proppant having an SG = 1.25) was observed as compared to the tests with sand. The Series No. 1 slot flow tests confirmed the proppant transport benefits which may be gained from reducing the density of the propping agent.

Prior to initiating the Slot Flow Test Series No. 2, the slot width was reduced to 0.25 in. (6.35 mm) and a flow diffuser was added to the slot entrance<sup>4-5</sup>. The reduction in slot width to ¼" was intended to provide more a realistic testing condition for the flow of slickwater-type slurries. The flow diffuser was designed to improve the injection flow pattern, providing enhanced utility of the slot flow cell and resulted in much more consistency in subsequent testing.

### **Slot Flow Test Series No. 2 & No. 3**

The initial battery of Series No. 2 tests sought to repeat the previous testing by incorporating the modified apparatus and utilizing the same slickwater slurries tested in Series No. 1. With minimum pump rates fixed at about 1-2 gpm (3.785 – 7.57 lpm), the ¼-inch (6.35 mm) slot yielded 100% faster lateral velocities at every given pump rate as compared to the first tests with the ½-inch (12.7 mm) wide slot. The narrower slot width made little noticeable difference in the overall outcome of these tests. The trends noted in the first series were confirmed by testing in the narrower slot configuration, and thus, no further baseline testing was deemed necessary.

The test matrices developed for Series No. 2 & Series No. 3 were designed to evaluate the effects of slurry component physical properties on transport, including testing to facilitate assessment of:

- (a) the effects of using a higher fluid density, expanding the range of data from 8.34 to 10.1 ppg;
- (b) the effects of higher fluid viscosity, expanding the range of data from 5 cP to 60 cP;
- (c) the behavior of lower specific gravity ULW proppants, extending the range of SGs from 1.08 to 2.65;
- (d) the impact of increased proppant size;
- (e) the effects of increasing the proppant concentration in the slurry; and,
- (f) the effects of viscoelastic surfactant gelled fluid at two different viscosities, used for comparison to the polymer-based gels.

#### Fluid Densification Effects on Transport

Weighting brine “closes the Delta SG gap” to a much more significant degree with the specific gravity of ultra-lightweight proppants than with sand, thus the observed effects of fluid densification on transport of the differing density proppants are somewhat intuitive in light of the teaching of Stoke’s Law.

Tests conducted to evaluate the transportability of proppants of differing specific gravity in densified 9.4 ppg brine were compared to the previous evaluations using 8.4 ppg slickwater. Densified brine benefited the transport of sand proppant minimally, but had a profound effect on the transport of the ULWP-1.25 proppant, with the settled bed height being reduced by 57% using fluid densified to 9.4 ppg compared to that observed in non-densified slickwater (8.4 ppg).

Tests were then conducted to evaluate the transportability of differing specific gravity proppants in a slickened, 10.1 ppg brine for comparison to the previous observations in 8.34 ppg and 9.4 ppg fluids. A synopsis of the testing results and subsequent data derived from the Biot-Medlin analyses for the tests is provided as Table 1. The increase of fluid density to 10.1 ppg provided limited benefit to the Ottawa sand transport, reducing minimum horizontal velocity for suspension transport ( $MHV_{ST}$ ) by about 15% compared to the 8.4 ppg case. The differential observed is very near that which would be expected based upon the “Delta SG” multiplier in Stokes Law. Delta SG for the sand/water case is 1.65, whereas that for the sand/10.1 ppg brine is 1.44, providing for a 13% difference in Delta SG.

The test of the ULWP-2.02 in 10.1 ppg brine did not demonstrate significant improvement in transport relative to observations in previous testing. The Delta SG suggests that transport should be improved by about 35% but only an 18% improvement was observed compared to the ULWP-2.02 in slickwater. The only apparent difference in the testing, other than the brine density, was that the viscosities of the 10.1 ppg tests were observed to be 5 cP compared to 7 cP in the previous tests. Unfortunately, the calibration of the Fann 35 viscometer used was not checked during the testing process, so the accuracy of those measurements is in question.

#### Fluid Viscosification Effects on Transport

An HEC-based polymer solution was blended with a final viscosity of 29 cP and used to perform the viscosified fluid transport tests. Significant improvement in proppant transport and reduction in bed height development were observed to be afforded by increasing fluid viscosity. Quadrupling the viscosity reduced the maximum bed height development of Ottawa sand by over 77% from the slickwater base line. Bed height development was reduced by 72% and 92%, respectively for the ULWP-2.02 and ULWP-1.25, when compared to the lower viscosity baseline fluid (7 cps slickwater).

A battery of the tests was conducted with an HEC-based polymer solution with a viscosity of 60 cP (@ 511  $\text{sec}^{-1}$ ). Increased fluid viscosity effects were consistent with expectations, with the 60 cP fluids being

observed to enhance transport for all proppants evaluated, compared to those tests employing fluids of lesser viscosity. The calculated  $MHV_{ST}$  (Minimum Horizontal Velocity required for Suspension. Transport) of Ottawa sand was 51 % lower than that observed for the 29 cP test and 85 % lower than that observed in the 7 cP slickwater baseline test. The  $MHV_{ST}$  was reduced by 89 % for both the ULWP-2.02 and ULWP-1.25 proppants when compared to the lower (7 cps) viscosity, 8.34 SG fluids. However, the  $MHV_{ST}$  was observed to be reduced by 73% when evaluated in the higher viscosity fluid. No bed height development was observed for the ULWP-1.08 in either fluid. The effects of fluid viscosification on transport are illustrated in Table 2.

#### Fluid Densification & Fluid Viscosification Cumulative Effects on Transport

Tests were also generated to evaluate the combined effects of fluid density and fluid viscosity on proppant transport. A KCl brine densified to 9.4 ppg was utilized as the base fluid. HEC was used to viscosify the densified brine to 29 cps. The bed height reduction for Ottawa sand was the same as was observed with 29 cps slickwater (77%), indicating that the increased brine density has essentially no effect upon the bed height development under this set of test conditions.

A 90% reduction in bed height development was observed for the mid-density ULWP-2.02 material slurried in the 29 cps brine, as compared to that in the 7 cps brine.

Lastly, a test was conducted to evaluate the effects of the combined increased fluid viscosity and density on the transport of the ULW-1.25 proppant. Remarkable differences in settling rates were observed by the combination of viscosity and density increases in the carrier fluid. Bed height development was reduced by over 97% compared to the base sand/slickwater case, and by 91% compared to the ULWP 1.25 in 7 cp, 8.4 ppg slickwater.

#### Fluid Viscosification Effects of Viscoelastic Surfactants vs. Polymeric Viscosifiers

Several tests were conducted with viscoelastic surfactant gelled fluids (non-polymer viscosified) to assess any differences in transport performance as compared to polymer viscosified fluids of similar viscosity. The data from these tests are provided above as Table 2. It was anticipated that the relatively large low-shear viscosities provided by viscoelastic surfactant fluids would provide better transport than polymer-viscosified fluids of similar viscosity (measured @  $170 \text{ sec}^{-1}$ ). The observed data provided mixed support for that theory with the transport using the VES fluids ranging from slightly better than the polymeric viscosifiers to roughly equivalent.

#### Median Proppant Size Effects on Transport

Tests were conducted to evaluate the effect of proppant size on transportability. As illustrated in Table 3, the effects of increased median proppant size were observed to be fairly significant. However, this phenomena is predictable given that the median proppant diameter term in Stoke's Law is squared, suggesting that the difference in fall rate is directly proportional to the square of the difference in median diameter of the proppants.

Testing was conducted with 8/16 mesh Ottawa sand for comparison to the 20/40 mesh Ottawa sand data. The median diameter of the 8/16 Ottawa sand was 1.44 mm compared to 0.663 mm measured for the 20/40 Ottawa sand, or 2.2 times larger than the 20/40. The bed height of the 8/16 sand was observed to be 3 times that of the 20/40 in a similar fluid. The  $MHV_{ST}$  calculated for the 8/16 sand was 5 times that of the 20/40 sand.

Testing was also conducted with 8/12 mesh ULWP-1.25 for comparison to the 20/40 ULWP-1.25 data. The median diameter of the 8/12 ULWP-1.25 proppant was 3.53 mm compared to 0.68 mm measured for the 20/40 ULWP-1.25, or 5.2 times larger than the 20/40 ULWP-1.25. The increased size of the 8/12 resulted in a bed height of 0.75" compared to 0.5" for the 20/40. However, the calculated minimum velocity for transport was nearly 25 times greater than that required for the 20/40 ULWP-1.25.

#### Proppant Concentration Effects on Transport

The effect of proppant concentration on slurry transport was evaluated by comparing 0.7 ppg and 1.4 ppg concentrations of the ULW-2.02 proppant in slickwater, as shown in Table 4. The concentrations of the

ULW-2.02 proppant employed were equivalent in volume to 1.0 and 2.0 ppg of sand, 0.7 and 1.4 ppg, respectively. The impact of doubling the proppant concentration was observed to be relatively small. The observed bed heights were identical and the calculated  $MHV_{ST}$  for the 1.4 ppg test was about 20% higher than that of the 0.7 ppg test. The authors suggest that tests at 4.0 and 8.0 ppg sand volume equivalents should be conducted to further define the effect of proppant concentration on slurry transport. Since the testing is conducted in a batch style operation, settled proppants reduce the amount of mobile proppant in the system, so the process does not simulate a “steady state” operation (constant proppant concentration) exactly like a real frac job would. The higher concentration tests would allow for evaluation of which process is causing the phenomenon; the actual concentration effects on proppant transport, or the way that transport is being measured by the test apparatus configuration.

#### PREDICTION OF PROPPED FRACTURE LENGTH USING THE MINIMUM HORIZONTAL VELOCITY REQUIRED FOR SUSPENSION TRANSPORT UTILIZING BIOT-MEDLIN ANALYSES

Quantification of the minimum horizontal velocity required for proppant transport within a given fracturing slurry composition gives rise to the question: At what point within a fracture does the velocity fail to meet the defined criteria? Identification of that point defines the distance from the wellbore to which the proppant is transported, i.e., the propped fracture length.

Biot-Medlin’s criteria define the Minimum Horizontal Velocity for suspension transport as the velocity ( $U$ ) at which  $v_t/U$  is  $< 0.1$ . Bed load transport ( $v_t/U > 0.1$ ) is not believed to be capable of providing sufficient lateral proppant transport for significant extension of propped fracture length. Therefore, the lateral distance at which the minimum velocity ( $v_t/U < 0.1$ ) for suspension transport is no longer satisfied is thought to be a reasonable estimate of the effective fracture length, especially in slick-water applications.

The horizontal slurry flow velocity within a hydraulic fracture is dependent upon the injection rate and the fracture geometry development (the aspect ratio of the fracture length growth to the fracture height growth). A very simple 3-D geometric fracture model, having the ability to generate fracture geometries ranging from radial (1:1 aspect ratio) to elliptical (3:1 or 5:1 aspect ratios) was used to generate the decay curves. The fracture width was predicted by the model using 0.05 psi/ft stress contrast between the bounding layers and the zone of interest. Fluid efficiency averaging about 50% over each of the three separate frac simulations was applied to account for fluid loss. Proppant slurry injection was modeled using an initial height of 10 feet and a 10 bbl/min fluid injection rate (i.e. 1 bpm/ft of injection height). These values resulted in horizontal velocity of 17.1 ft/sec at the wellbore. The fracture growth progression was monitored by the instantaneous change in the major radii of the ellipsoidal fracture shapes (the horizontal direction in the case of the radial fracture simulation). The instantaneous change in the major radii over the course of the simulation was used as a proxy for fluid velocity at the tip of the fracture. Since the fracture simulation used a slick water base fluid, frac width from wellbore to tip averaged about 0.25” (6.35 mm)

The decay in horizontal velocity versus lateral distance from the wellbore for evaluated fracture geometries is illustrated in Figure 2. The most severe velocity decay was observed with the radial geometry, wherein the horizontal velocity at a distance of 100 ft was reduced by over 99.9% to 0.02 ft/sec, compared to the 17.1 ft/sec velocity at the wellbore. The greater the length to height ratio, the less severe the velocity decay observed. For the 5:1 elliptical model, the velocity decay was observed to be 97% in the initial 100 feet, resulting in an average horizontal velocity of 0.47 ft/sec.

Power law fits were applied to the various decay curves, allowing for calculation of the horizontal velocity at any distance from the well bore. Similar methodology was utilized to generate the estimates of estimated propped fracture lengths to be expected from the various proppant slurries in the modeled fractures. The data from this exercise are provided in Table 5. The effective propped fracture lengths for Ottawa sand/slickwater slurries are estimated to be in the range of 15 ft (radial) to 60 ft (5:1 elliptical) (4.57 – 18.3 m) for the geometries and conditions modeled. These values are consistent with lengths commonly derived from production history analyses of wells treated with sand/slickwater slurries. Combined viscosification and fluid weighting produced propped fracture lengths of 30 ft (radial) to 105 ft (5:1 elliptical) (9.14 – 32 m).

The effective fracture length estimated for the ULW proppants using this methodology were on the order of

two to five times greater than those for sand in similar carrier fluids. For example, a slickwater slurry of ULWP-1.25 was estimated to yield a propped fracture length of 40 feet (12.2 m) using the radial model and 95 feet (29 m) using the 3:1 elliptical geometry. The combined viscosification and fluid densification used with the ULWP-1.25 produced propped fracture lengths of greater than 290 feet (88.4 m) for the 3:1 model and 480 feet (146.3 m) for the 5:1 elliptical geometry fracture model.

The effective fracture lengths estimated for the various slurries using these methods were reasonably consistent with lengths commonly derived from production history analysis of wells treated with sand/slickwater slurries and fracture geometries thought to be similar to those modeled. For comparison purposes, modifying the Biot-Medlin criteria to evaluate the distances reached by bed-load transport ( $v_t/U$  values of 0.5 or lower) more than doubles penetration distances into the reservoir in the first sand case above. This illustrates that bed-load transport is probably a large contributor to the overall fracture length growth experienced by many frac designs, most notably, those using slick fluids.

An empirical proppant transport model has been developed to predict propped fracture length from the fluid and proppant material properties, the injection rate, and the fracture geometry. The premise of the model is that for each set of proppant slurry physical property parameters, there exist associated Biot-Medlin 'constants' for minimum horizontal flow velocities required for transport within a given fracture geometry.

### MODELING OF THE MINIMUM HORIZONTAL VELOCITY REQUIRED FOR PROPPANT TRANSPORT

The objective was to define the relationship between the physical properties of a proppant slurry, the minimum horizontal velocity required for transport of that proppant slurry, and the lateral distance within a created hydraulic fracture (of a given geometry and injection characteristics) to which that minimum horizontal velocity could be satisfied. Thus, three separate caches of information are required:

- (1) an 'index' describing the proppant slurry physical properties,
- (2) the minimum horizontal velocity required for suspension transport of a proppant slurry, and
- (3) characterization of the horizontal velocity within the hydraulic fracture.

The model has input parameters including fluid viscosity, proppant size, both fluid and proppant specific gravity, injection rate, fracture geometry, and fluid leak-off efficiency. The model output is the estimated propped fracture length of a given treatment design incorporating a fracturing fluid slurry having the input parameters. The intent is to utilize the model for optimizing the fluid, proppant, and/or treating parameters necessary to achieve a desired propped fracture length.

The Slurry Properties Index,  $I_{SP}$ , is a term defined to capture the inherent physical properties of a given proppant slurry, as shown in Equation 1. The inherent physical properties of a given proppant slurry include the median diameter and specific gravity of the proppant and the apparent viscosity and specific gravity of the fluid. Increasing magnitude of the  $I_{SP}$  value corresponds with increasing proppant transport difficulty.

#### Equation 1.

$$I_{SP} = d_{prop}^2 * (1/\mu_{fluid}) * (\Delta SG_{PS})$$

Where,

$d_{prop}$  = median proppant diameter, in mm.

$\mu_{fluid}$  = apparent viscosity, in cP

$\Delta SG_{PS} = SG_{Prop} - SG_{fluid}$

$SG_{Prop}$  = Specific Gravity of the proppant

$SG_{fluid}$  = Specific Gravity of the fluid

Equation 1 reveals the effects of various parameters on the Slurry Properties Index,  $I_{SP}$ . The proppant size very strongly influences the  $I_{SP}$ . Since the median diameter of the proppant is squared, increasing proppant size results in a relatively large increase in the index,  $I_{SP}$ . The fluid viscosity,  $\mu_{fluid}$  is in the denominator of the  $I_{SP}$  equation; therefore, increasing fluid viscosity reduces the magnitude of the  $I_{SP}$ , resulting in a

proportional improvement in proppant transport capability. The last term is  $\Delta SG_{PS}$ , the differential in Specific Gravity between the proppant and the fluid. An increase in the  $\Delta SG_{PS}$  (i.e. heavier proppant and/or lighter fluid) results in a proportional decrease in the proppant transport capability. A discrete Slurry Properties Index for each proppant/fluid case investigated was calculated using Equation 1 and the respective  $I_{SP}$  indices are provided in Table 6.

The Biot-Medlin analysis technique states that the minimum horizontal velocity for suspension transport is defined as the point at which the  $v_t/U$  vs.  $U$  plot crosses 0.1 on the Y-axis. Therefore, the Minimum Horizontal Velocity for Slurry Transport,  $MHV_{ST}$ , for a given proppant slurry is 10 times the  $v_t$  value determined for that slurry, as illustrated in Equation 2.

**Equation 2.** 
$$MHV_{ST} = v_t * 10$$

A linear best fit of the measured Slurry Property Indices versus their respective  $MHV_{ST}$  ( $v_t$  times 10) was performed to determine the  $MHV_{ST}$  of any given proppant slurry composition having properties within the ranges of those tested (Equation 2). The individual test data are shown in Table 6. A linear regression of the data was utilized for the current efforts, and the coefficient of the data fit is defined as the Transport Coefficient ( $C_{TRANS}$ ), as shown below in Equation 3.

**Equation 3.** 
$$MHV_{ST} = C_{TRANS} * I_{SP}$$

Or,

$$MHV_{ST} = C_{TRANS} * d_{prop}^2 * 1/\mu_{fluid} * \Delta SG_{PS}$$

Where,

$MHV_{ST}$  = Minimum Horiz. Velocity for Susp. Transport  
 $C_{TRANS}$  = Transport Coefficient  
 $I_{SP}$  = Slurry Properties Index  
 $d_{prop}$  = Median Proppant Diameter, in mm.  
 $\mu_{fluid}$  = Apparent Viscosity, in cP  
 $\Delta SG_{PS}$  =  $SG_{Prop} - SG_{fluid}$   
 $v_t$  = Terminal Settling Velocity

From Equation 2,

**Equation 4** 
$$v_t * 10 = C_{TRANS} * I_{SP}$$

The equation for the linear best fit of the test data is shown in Equation 4. The linear regression data is illustrated graphically in Figure 3.

**Equation 5** 
$$y = (0.0117)x$$

Thus,  $C_{TRANS} = 0.0117$ . Insertion of the  $C_{TRANS}$  value into Equation 3 yields a simplified expression to determine the minimum horizontal velocity for any proppant/fluid slurry (within the scope of the parameters evaluated to date), as shown in Equation 6.

**Equation 6** 
$$MHV_{ST} = (0.0117) * I_{SP}$$

*Example A.* Determine the minimum horizontal velocity,  $MHV_{ST}$ , required to transport proppant in a slickwater slurry comprised of 20/40 sand in a 7 cP, 2% KCl brine.

$$I_{SP} = (1150) * (0.4032) * (1/7) * (2.65 - 1.01) = 108.63$$

$$MHV_{ST} = (0.0117) * (108.63) = \underline{1.27 \text{ ft/sec}}$$

Note that the 1,150 multiplier is a unit conversion factor.

Example B. Determine the minimum horizontal velocity,  $MHV_{ST}$  required to transport proppant in a slickwater slurry comprised of ULW-1.08 sand in a 29 cP, slick water.

$$I_{SP} = (1150) * (0.5810) * (1/29) * (1.08 - 1.00) = 1.84$$

$$MHV_{ST} = (0.0117) * (1.84) = \underline{0.022 \text{ ft/sec}}$$

### LATERAL VELOCITY OF SLURRY IN A FRACTURE

The horizontal slurry flow velocity at locations within a hydraulic fracture may be estimated by various means including complex fracture design models as used here. The lateral velocity is dependent upon the injection rate and the fracture geometry development (the aspect ratio of the fracture length growth to the fracture height growth) and, the volume of fluid lost to leakoff. For purposes of this paper an ellipsoidal model was utilized, with capabilities ranging from 1:1 aspect ratio to 5:1 aspect ratio.

A suitable geometric fracture model must incorporate inputs defining the slurry injection rate, an initial injection height at the wellbore (perforated height), characterization of the fracture growth geometry, the created fracture height, the created fracture width, and the fluid efficiency.

The model employed for this exercise utilized the following constant inputs:

- 10 bpm injection rate.
- 10 ft initial injection height.
- fracture width calculated by poroelastic model ~1/4"
- ~50% fluid efficiency over the simulation duration.

Based on the parameters discussed above, power law curve fits of Figure 2 were generated for each of the fracture geometries analyzed. Equation 7 was developed from that work to determine instantaneous slurry velocity at any distance from the wellbore during a fracture simulation.

**Equation 7**  $V_s = (A) * (\text{Distance})^B$

Where,

$V_s$  = Slurry Velocity, ft/sec

Distance = Lateral distance from the well bore, ft

A = multiplier from the Power Law equation describing the Slurry Velocity vs. Distance for the desired geometry.

B = exponent from the Power Law equation describing the Slurry Velocity vs. Distance for the desired geometry.

If the  $MHV_{ST}$  is known, for example from Equation 3, it can be inserted for  $V_s$  in Equation 7 and the equation solved for Distance, which is the distance of suspension transport,  $D_{PST}$ .

**Equation 8** Radial Geometry (1:1 aspect ratio)

$$A = 512.5 \quad B = -2.1583$$

$$MHV_{ST} = (512.5) * (D_{PST})^{-2.1583}$$



**Equation 9**      Elliptical Geometry (3:1 aspect ratio)

$$A = 5261.7 \quad B = -2.2412$$

$$MHV_{ST} = (5261.7) * (D_{PST})^{-2.2412}$$

Example C. Determine the distance a proppant slurry comprised of 20/40 ULW-1.08 proppant and 29 cP slickwater will be transported in a fracture having a 3:1 length to height geometry with a 1 bpm/ft injection rate.

From Equation 1,

$$I_{SP} = (1150) * (0.581) * (1/29) * (1.08 - 1.00) = 1.84$$

$$MHV_{ST} = (0.0117) * (1.84) = \underline{0.022 \text{ ft/sec}}$$

From Equation 7,

$$V_s = (5261.7) * (D_{PST})^{-2.2412}$$

$$(D_{PST})^{-2.2412} = (0.022) / (5261.7)$$

$$D_{PST} = \underline{251 \text{ ft}}$$

Example D. Determine the distance a proppant slurry comprised of 20/40 Ottawa sand proppant and 7 cP slickwater will be transported in a fracture having a 3:1 length to height geometry with a 1 bpm/ft injection rate.

From Equation 1,  $I_{SP} = 108.63$ , and  $MHV_{ST} = (0.0117) * (108.63) = 1.27 \text{ ft/sec}$

$$D_{PST} = \underline{41 \text{ ft}}$$

EQUATING PROPPED FRACTURE LENGTH TO THE PROPPANT SLURRY PROPERTIES.  
 $D_{PST}$  VERSUS  $I_{SP}$

It was established above that the  $MHV_{ST}$  is directly proportional to the Slurry Properties Index,  $I_{SP}$  :

$$MHV_{ST} = C_{TRANS} * I_{SP}$$

$D_{PST}$  is the lateral distance from the wellbore at which the horizontal velocity falls below that required for suspension transport. Thus, the slurry velocity associated with the  $D_{PST}$  is equivalent to the  $MHV_{ST}$  or the Biot-Medlin  $v_t$  @ 0.1. The  $MHV_{ST}$  is proportional to the  $D_{PST}$  via the slurry velocity decay vs. lateral distance from the wellbore. Consequently, a direct relationship between  $D_{PST}$  and  $I_{SP}$  can be defined.

$$MHV_{ST} = (A) * (D_{PST})^B = (C_{TRANS}) * (I_{SP})$$

Therefore,

**Equation 10**       $(A) * (D_{PST})^B = (C_{TRANS}) * (d_{prop}^2) * (1/\mu_{fluid}) * (\Delta SG_{PS})$

Rearrangement of Equation 9 allows one to solve for the lateral distance of suspension transport,  $D_{PST}$ , using the slurry properties and constants from the Equation 3 ( $MHV_{ST}$  versus Slurry Properties Factor) and Equation 5 (Slurry Velocity Decay vs. Distance) relationships, as shown in Equation 8.

**Equation 11.** 
$$(D_{PST})^B = (1/A) * (C_{TRANS}) * (d_{prop}^2) * (1/\mu_{fluid}) * (\Delta SG_{PS})$$

Thus, with the development of the relationships of  $MHV_{ST}$  to proppant slurry component properties and velocity decay to fracture geometry, the propped frac length may be derived from simply the constants and coefficients of those relationships and the properties of the slurry components.

Equation 11 provides the opportunity to define the propped fracture length based solely upon the properties of proppant slurry components, and the fracture geometry for a fracture having an injection rate of 1 bpm per foot of injection or perforated height.

The  $D_{PST}$  to  $I_{SP}$  relationship may be further refined to allow for incorporation of injection rates deviating from the 1 bpm/ft of injection height. Since Slurry Velocity Decay model is structured for the injection rate to injection height to be equal to 1 bpm /ft of injection height, this can be accomplished by incorporating a multiplier of the ratio of the desired injection rate to the base injection rate of 1 bpm/ft, as shown in Equation 12. (e.g. if an injection rate of 1.5 bpm/ft were desired, the multiplier would be 1.5.)

**Equation 12** 
$$(D_{PST})^B = (IR) * (1/A) * (C_{TRANS}) * (d_{prop}^2) * (1/\mu_{fluid}) * (\Delta SG_{PS})$$

Where,

IR = the injection rate per foot of injection height, bpm/ft

Rearrange Equation 10 to solve for  $D_{PST}$ ,

**Equation 13** 
$$(D_{PST})^B = (IR) * (1/A) * (C_{TRANS}) * (d_{prop}^2) * (1/\mu_{fluid}) * (\Delta SG_{PS})$$

Example E. Determine the propped fracture length ( $D_{PST}$ ) from the Proppant Slurry Properties:

Proppant diameter,  $d = 0.635$  mm

Proppant SG = 1.25

Fluid viscosity = 30 cP

Fluid SG = 1.01

Injection Rate = 5 bpm/ft

3:1 Geometry:  $A = 5261.7$        $B = -2.2412$

$$(D_{PST})^B = (IR) * (1/A) * (C_{TRANS}) * 1150 * (d_{prop}^2) * (1/\mu_{fluid}) * (\Delta SG_{PS})$$

$D_{PST} = 90.4$  ft.

Equation 10 also provides opportunities for fracturing treatment design optimization as the various parameters can be manipulated to define the most favorable combination of fracturing slurry component properties and pumping parameters to optimize effective propped fracture length, and thereby well performance. For example, Equation 12 can be rearranged to solve for any one of the slurry component properties or the injection rate necessary to achieve a specified propped fracture length, as illustrated in Equations 14, 15, 16 and 17.

**Equation 14.** 
$$\Delta SG_{PS} = (A) * (1/IR) * (D_{PST})^B * (1/C_{TRANS}) * (1/d_{prop}^2) * (\mu_{fluid})$$

**Equation 15** 
$$d_{prop}^2 = (A) * (1/IR) * (D_{PST})^B * (1/C_{TRANS}) * (1/\Delta SG_{PS}) * (\mu_{fluid})$$

**Equation 16** 
$$\mu_{fluid} = (1/A) * (IR) * (1/D_{PST})^B * (C_{TRANS}) * (\Delta SG_{PS}) * (d_{prop}^2)$$

*Example F.* Determine the viscosity necessary to transport proppant 100 feet from the well bore using a proppant slurry comprised of 20/40 ULW-1.25 proppant and viscosified water. Assume a fracture having a 3:1 length to height geometry with 5 bpm/ft injection rate.

$D_{PST} = 100$  ft.

Proppant diameter,  $d = 0.6350$  mm

Proppant SG = 1.25

Fluid SG = 1.01

Injection Rate = 5 bpm/ft

Stokes unit conversion = 1150

3:1 Elliptical Geometry:  $A = 5261.7$        $B = -2.2412$

Solve Equation 16 for viscosity,

$\mu_{fluid} = \underline{37.6 \text{ cP.}}$

**Equation 17** 
$$IR = (1/D_{PST})^B * (1/A) * (C_{TRANS}) * (d_{prop}^2) * (1/\mu_{fluid}) * (\Delta SG_{PS})$$

The techniques described above provide engineers with new information for design and evaluation of hydraulic fracturing treatments, particularly those seeking to employ ULW proppants and low viscosity fracturing fluids.

Future work is anticipated to include validation of the presented techniques with production history matching on wells in which alternative propped fracture length data is accessible. Once validated, collaboration with fracturing software developers to incorporate the new relationships in treatment design models should occur.

Further refinement of the presented models should include incorporation of the expression of fluid rheology in Power Law form to better account for the non-Newtonian viscoelastic behavior. Additionally, data to better characterize the effects of proppant size and concentration would be beneficial.

## **CONCLUSIONS**

Approximately seventy large-scale slot flow tests have been completed. The data from these tests have been utilized to develop algorithms descriptive of the proppant transport efficiency observed with the various slurry components.

A methodology has been developed and presented to equate the physical properties of proppant slurry components to the propped fracture length using derived from analysis of the slurry transport data from the large-scale slot flow testing.

MHV<sub>ST</sub>, the Minimum Horizontal Velocity for Slurry Transport, has been defined as ten times (10X) the Biot-Medlin Terminal Settling Velocity,  $v_t$ , for all proppant/slurry compositions evaluated.

I<sub>SP</sub>, the Slurry Properties Index, has been defined to describe any proppant/fluid combination by its inherent physical properties.

$C_{TRANS}$ , the Transport Coefficient, has been defined as the slope of the linear regression of the  $I_{SP}$  vs.  $MHV_{ST}$  for all proppant/slurry compositions evaluated.

A model describing the relationship of the minimum horizontal velocity required for suspension transport versus the Slurry Properties Index,  $I_{SP}$ , for the respective slurries tests has been developed and presented. The applicable limitations of the model, based upon the ranges of the parameters evaluated: up to 60 cp viscosity, up to 10.1 ppg brine, 20/40 mesh to 8/12 mesh proppant size, and proppant apparent specific gravities from 2.65 down to 1.08.

Utilizing a simplified, geometric Slurry Velocity Decay model, a methodology has been derived to estimate the propped fracture length,  $D_{PST}$ , from the mechanical parameters of the pumping treatment, the physical properties of the slurry components ( $I_{SP}$ ,  $MHV_{ST}$ ).

Via rearrangements of the same derived equations, treatment design optimization can be accomplished via definition of the most favorable combination of fracturing slurry component properties and pumping parameters to optimize effective propped fracture length, and thereby well performance.

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Table 1  
Comparisons of the Effect of Fluid Density on Proppant Transport for 20/40 Proppants

Proppant 20/40	G <sub>prop</sub>	G <sub>fluid</sub>	isc.	Max. Bed Ht.	V <sub>t/U</sub> @ 0.1
Sand	.65	.34		21	1.268
Sand	.65	.4		18	1.170
Sand	.65	.01		71	1.188
UWP	.02	.34		41	.720
UWP	.02	.4		31	.690
UWP	.02	.01		31	.592

Table 2  
Comparisons of the Effect of Viscosifiers on Proppant Transport for 20/40 Proppants  
(P=polymer gel, S=surfactant gel)

Proppant 20/40	SG <sub>prop</sub>	SG <sub>fluid</sub>	Visc.		Max. Bed Ht.	V <sub>t/U</sub> @ 0.1
Sand	2.65	8.34	7	P	17.5	1.268
Sand	2.65	8.34	10	S	4.5	1.107
Sand	2.65	8.34	26	S	4	0.402
Sand	2.65	8.34	29	P	5.5	0.357

Sand	2.65	8.3 4	6 0	P	2	0.17 3
ULWP	2.02	9.4	6	S	4.2 5	0.61 8
ULWP	2.02	8.3 4	7	P	14	0.72 0
ULWP	2.02	8.3 4	9	S	1	0.52 7
ULWP	2.02	8.3 4	2 6	S	3.2 5	0.18 2
ULWP	2.02	8.3 4	2 9	P	3.2 5	0.16 1
ULWP	2.02	8.3 4	6 0	P	1	0.07 8
ULW P	1.25	9.4	6	S	2.2 5	0.13 0
ULWP	1.25	8.3 4	7	P	6	0.24 8
ULW P	1.25	9.4	7	P	3	0.12 9
ULWP	1.25	8.3 4	1 1	S	7	0.14 3
ULW P	1.25	8.3 4	2 9	P	0.7 5	0.05 2
ULW P	1.25	8.3 4	6 0	P	0.7 5	0.02 7
ULW P	1.08	8.3 4	5	P	0	0.10 2
ULW P	1.08	8.3 4	8	S	0	0.06 3
ULW P	1.08	8.3 4	2 9	P	0	0.01 7

Table 3  
Comparisons of the Effect of Median Proppant Size on Proppant Transport

Proppant	Gravel Size, mm	Median Size, mm	Gravel Flow Rate	Injection Rate	ax. Bed Height	t/U @ 0.1
Sand	2.65	0.66 3	8.3 4		1	.2 6

						8
Standard	2 .65	0 .66 3	8 .3 4	9	. 5	. 3 5 7
Standard	2 .65	1 .44	8 .3 4	6	6 .5	. 8 6 5
Standard	2 .65	1 .44	8 .3 4	0	2 .5	. 0 1 8
ULW P	1 .25	0 .68	8 .3 4		. 0	. 2 4 8
ULW P	1 .25	0 .68	8 .3 4	9	. 7 5	. 0 5 2
ULW WP	1 .25	2 .18	8 .3 4		. 0	. 7 6 0
ULW WP	1 .25	2 .18	8 .3 4	9	. 5 0	. 2 9 5

Table 4  
Comparisons of the Proppant Concentration Effects on Proppant Transport.

Proppant 20/40	Proppant Conc.	SG <sub>p</sub> prop	SG <sub>f</sub> fluid	Visc. sc.	Max. Bed Ht.	Vt/ U @ 0.1
ULW 2.02	0.7	2.02	8.34	9	1	0.5 27
ULW 2.02	1.4	2.02	8.34	8	1	0.6 47

Table 5  
Horizontal slurry velocity in feet per second necessary to satisfy the respective  
Biot-Medlin particle transport criteria for the various proppant fluid systems evaluated.

Prop pant	l u i d S G	l u i d V is c. , c p s	Propped Fracture Length, feet		
			R adia l (1:1)	Ell iptica l (3:1)	Ell iptica l (5:1)
Sand	.4		15	40	60
ULW -2.02	.4		25	65	95
ULW -1.25	.4		40	95	150
Sand	.4		16	42	63
ULW -2.02	.4		30	75	120
ULW -1.25	.4		45	110	175
Sand	.4	9	25	65	95
ULW -2.02	.4	9	44	108	172
ULW -1.25	.4	9	60	150	235
Sand	.4	9	30	70	105
ULW -2.02	.4	9	45	110	175
ULW -1.25	.4	9	125	290	480



Table 6  
Slurry Factors and Calculated Minimum Horizontal Flow  
Velocity Required for Suspension Transport for Slot Flow Tests

$SG_{prop}$	$d_{prop}^2$ (mm <sup>2</sup> )	$SG_{fluid}$	Visc. Type	$\mu_{fluid}$ , cP	Slurry Index, $I_{SP}$	MHV <sub>ST</sub>
2.65	0.4032	8.34	PA	7	119.3	1.400
2.65	0.4032	8.34	VES	10	83.49	1.107
2.65	0.4032	8.34	HEC	29	26.27	0.5
2.65	0.4032	8.34	VES	26	32.11	0.402
2.65	0.4032	8.34	HEC	60	13.92	0.173
2.65	0.4032	9.4	PA	7	110.1	1.200
2.65	0.4032	9.4	HEC	29	26.57	0.400
2.65	0.4032	9.4	VES	6	128.4	1.390
2.65	0.4032	10.1	PA	5	104.0	1.188
2.65	2.070	8.34	VES	26	151.1	1.865
2.65	2.070	8.34	HEC	60	78.56	1.018
2.02	0.380	8.34	VES	9	48.56	0.647
2.02	0.380	8.34	VES	9	38.15	0.527
2.02	0.380	8.34	PA	7	63.68	0.500
2.02	0.380	8.34	VES	26	16.81	0.182
2.02	0.380	8.34	HEC	29	15.07	0.200
2.02	0.380	8.34	HEC	60	7.67	0.078
2.02	0.380	9.4	PA	7	55.74	0.300
2.02	0.380	9.4	VES	6	42.38	0.618
2.02	0.380	9.4	HEC	29	15.36	0.140
2.02	0.380	10.1	PA	7	49.25	0.592
1.25	0.4264	8.34	HEC	60	2.04	0.027
1.25	0.4264	8.34	PA	7	17.51	0.150
1.25	0.4264	8.34	VES	11	11.14	0.143
1.25	0.4264	8.34	HEC	29	4.23	0.070
1.25	0.4264	9.4	VES	8	7.53	0.130
1.25	0.4264	9.4	PA	7	8.61	0.140
1.25	0.4264	9.4	HEC	29	2.08	0.020
1.25	4.752	8.34	HECA	6	218.6	2.760
1.25	4.752	8.34	HECC	27	48.58	1.295
1.08	0.5810	8.34	PA	5	9.35	0.102
1.08	0.5810	8.34	VES	8	5.84	0.063
1.08	0.5810	8.34	HEC	29	1.61	0.017

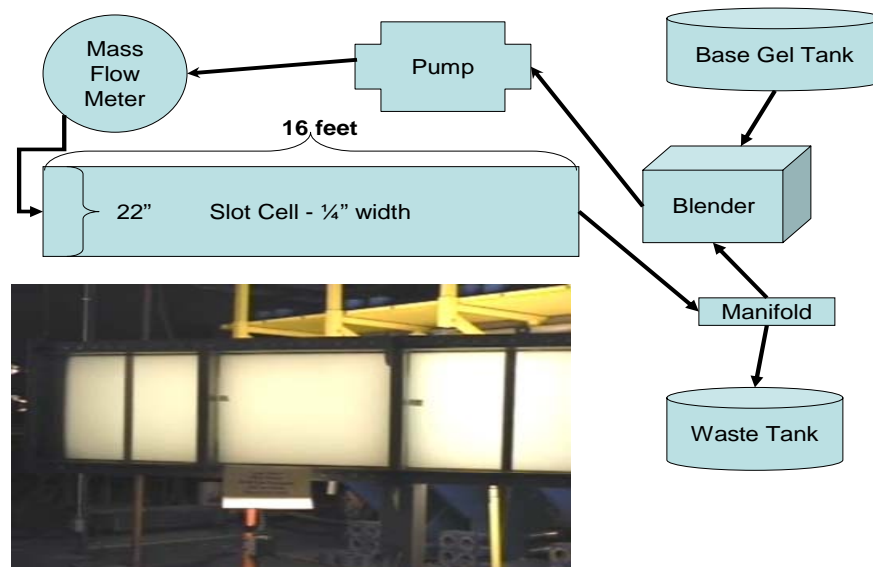


Figure 1- Schematic of the Slot Flow Cell Apparatus @ the Well Construction Technology Center

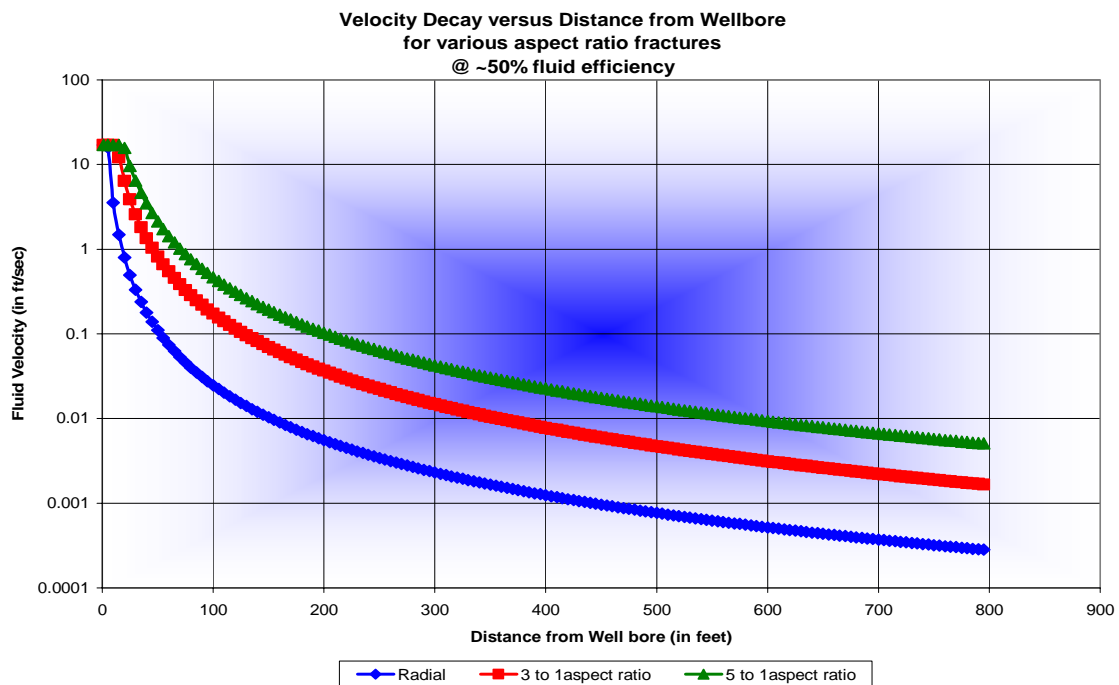


Figure 2 - Slurry Velocity Decay vs. Distance from Wellbore (10 bpm Injection Rate, 10 ft of Height @ Wellbore Velocity 17.1 ft/sec @ Wellbore)

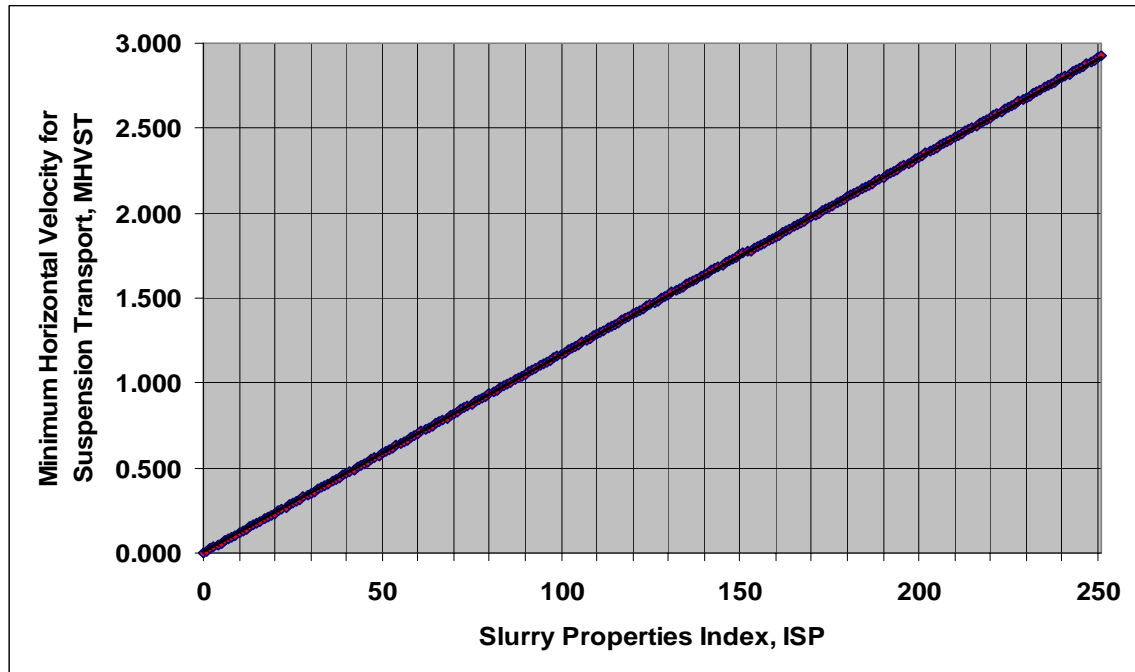


Figure 3 - Minimum Horizontal Flow Velocity for Slurry Transport as a Function of the Slurry Properties Index,  $I_{SP}$