PRESSURE AND RATE TRANSIENTS IN COMMINGLED, LAYERED RESERVOIRS

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Abstract

Because of sedimentation processes over long geological times, hydrocarbon reservoirs are likely to be multilayered. For practical purposes, in the analyses of pressure and rate transient data the reservoirs are commonly treated as a single-layer model. Petroleum engineers better comprehend the models and analyses of single-layer systems than those of multilayer systems. As long as single-layer analysis yields acceptable results, the engineers tend to use single-layer models in analyzing multilayer data. If the multilayer responses are indistinguishable from the single-layer responses, then the single-layer analysis may be applicable to multilayer data.

Our research objectives are to study multilayer responses in pressure or rate transient data, estimate individual layer properties, and investigate the results of single-layer analysis on multilayer data. To achieve our objectives. we have revised and improved an analytical model called "Laysim." We used Laysim synthetic data in our study and limited the study to a radial and homogeneous model with isotropic layer properties and a well at center. The model contains layers with no-flow outer boundaries and a slightly compressible fluid with constant viscosity and compressibility.

In this research we used log-log diagnostic plots and semilog Horner plots to characterize multilayer pressure and rate transients, and to distinguish between multilayer and single-layer responses in both drawdown and buildup test data. We identified test types and layer properties that are likely to yield multilayer characteristics. We analyzed multilayer data using a single-layer model, and provided guidelines in interpreting the analysis results. We quantified layer properties by history matching methods using a simple, three-layer model; proposed a two-step procedure for history matching multilayer data; and provided guidelines in matching the data. The history matching methods are simple ways to estimate layer properties without having to do complicated layer testing and interpretation sequentially. We studied relative rate data that are used to allocate the total **flow** capacity and storativity obtained from a single-layer analysis to individual layer properties. We found their applicability and restrictions.

Introduction

Most oil and gas reservoirs are layered to various degrees because of sedimentation processes over long geological times. Layered reservoirs are likeiy composed of two or more layers that may have different formation and fluid characteristics. These reservoirs are usually divided into two groups: (1) crossflow systems where interlayer communication occurs in both the formation and the wellbore, and (2) commingled systems where layers communicate only through the wellbore. Accurate determination of permeability, skin factor, and pressure for each layer is necessary to understand and predict the reservoir performance.

In this research we limited our investigation to commingled systems (Fig. 1). Basic fluid properties used in the investigation are presented in Table I. We investigated two production modes, constant rate and constant pressure; two *test* types. drawdown and buildup; and the effect of varying reservoir properties:

permeability, drainage area, skin factor, porosity, initial pressure, and net pay. We also investigated multilayer models that have two, three. or six layers.

Analysis of transient test data from multilayer reservoirs is generally difficult and frequently ambiguous. At early times, multilayer characteristics in pressure or rate transients are indistinguishable from single-layer characteristics.' 'Therefore, to distinguish between the two systems we have **to** examine the characteristics of test data plots during boundary-affected flow. Diagnostic plots are probably good tools to delineate the outer boundary characteristics.

For some multilayer systems, the outer boundary characteristics may be the same or similar to those of single-layer. For those cases, single-layer analysis may be applicable to multilayer data. Consequently, we may be able to use a single-layer model to predict the performance of a multilayer reservoir. Therefore, we may need to find out which multilayer systems have responses indistinguishable from single-layer responses.

To estimate individual layer properties of a multilayer system, some investigators[2-5] proposed testing and interpretation techniques. The testing techniques are typically costly, tedious, and time consuming. Interpretations of the data acquired from the tests are usually complicated and not straightforward like the conventional, single-layer interpretation techniques. History matching test data offers an alternative in estimating individual layer properties. We may simplify the matching model by including only three layers, whereas the prototype can have more than three layers.

Petroleum engineers generally better comprehend the models and analyses of single-layer systems than those of multilayer systems. Furthermore, commercial software for single-layer analyses are easily obtained at affordable costs. Consequently, as long as single-layer analysis yields acceptable results, engineers tend to use single-layer models in analyzing multilayer data. Therefore, we need to have guidelines for interpreting the results of single-layer analysis on multilayer data.

Model Verification

Earlier versions of Laysim were developed by petroleum engineering students at Texas A&M. In 1990 Spath et al.[6] presented an algorithm to compute pressure distribution in commingled reservoirs that are produced with either constant or variable rate. Johnston and Lee' used a similar algorithm to develop an analytical solution for a low-permeability gas well, which is produced at a constant bottomhole pressure from commingled reservoirs. Gao and Lee[8] further developed Laysim to handle pressure buildup analysis for a gas well with constant bottomhole pressure production.

Laysim models were verified by comparing the data generated by Laysim to those generated from Eclipse™ numerical models for the same reservoir properties. In this research we used Eclipse 100™ version for single-phase models. The objectives of the work were to explore the capability and limitations of Laysim in modeling various multilayer models so that, we would understand Laysim and know with certainty in what cases that Laysim does or does not work. In the cases that Laysim did not work, we found solutions to its problems.

We compared and history matched Eclipse™ and Laysim data, which were generated from various two-layer models. For each comparison case the two-layer model has one varying parameter, such as drainage area, permeability, porosity, skin factor, net pay, or initial pressure. An example of the comparisons is shown in Fig. 2, which is a diagnostic plot for a drawdown model with varying layer permeabilities. In general, Laysim and Eclipse™ data compare well.

Multilayer rate and pressure characteristics

We used diagnostic plots (log-log graphs of pressure and pressure derivative vs. test time) and Horner plots (semilog graphs of pressure vs. a function of test time) to identify the multilayer characteristics and to distinguish between multilayer and single-layer responses in drawdown or buildup test data. Our objective was to identify multilayer models that exhibit characteristics distinctively different from single-layer models.

We found significant differences between multilayer and single-layer characteristics on buildup and drawdown diagnostic plots. Increasing pressure derivative following a transient period identifies the multilayer characteristics on buildup type curves. After the transient **and** prior to the increasing portion, the derivative curve sometimes dips. Increasing wellbore pressure following a transient period identifies the multilayer characteristics in a Horner plot. In drawdown type curves when one of the layers is still in transient flow, we can identify the multilayer characteristics from the slope of derivative curve that is less than one. Another characteristic is the time to reach pseudosteady state, which is one to several orders of magnitude greater in a multilayer system than in a single-layer system with identical properties.

To identify multilayer characteristics, we have to examine at least the first outer boundary effects and some data, preferably a log-cycle or more, beyond those boundary effects. Type curves are preferable to Horner plots in distinguishing between multilayer and single-layer characteristics. Buildup diagnostic plots delineate multilayer characteristics better than drawdown diagnostic plots do. Table 2 summarizes type of tests and layer property that yield multilayer characteristics. The results in the table are based on type curves of two-layer and three-layer models.

We generated diagnostic plots for multilayer models with two and three layers, constant-pressure and constant-rate production, and drawdown and buildup tests. We needed the diagnostic plots, which are good tools to identify the effects of outer boundary, because at early times the characteristics of multilayer reservoir are indistinguishable from those of single-layer reservoir. We also studied the characteristics of multilayer reservoirs in Horner plots, and compared the results to those of the previous studies.[9-12]

We characterized the inultilayer models by the following variables:

Flow capacity ratio $$\lambda_j = \frac{(kh)_j}{\text{[illegible]}} \quad \ldots\ldots (1)$$

Storativity ratio $\Omega_j = \frac{(\phi hA)_j}{(\phi hA)_n}$ ....(2)

Drainage area ratio $\gamma_j = \frac{A_j}{A_n}$ .... (3)

where j is the layer number, from 1 to n. Because the ratios above always refer to the last layer, **so** we write, for example, $A = 10/5$ for a three-layer model or $A = 10$ for a two-layer model. For the three-layer model, this means that the model has the flow capacity ratios of 10 and 5 for the first and the second layer, respectively. For the two-layer model, only the first layer is assigned a flow capacity ratio of 10. Single-layer models are identified by having all variables, A, Ω, and γ, equal to 1. The γ-variable is used to identify the models with varying layer drainage areas. It complements the Ω-variable. If γ equals one but Ω does not, all layers **in** a two-layer model have the same size drainage areas but differ in their storativities.

On constant-rate drawdown diagnostic plots (Fig. 3) multilayer characteristics are recognizable from the slopes of pressure derivative curve which are less than one when one layer in a multilayer model is still in the transient. Eventually, a unit-slope derivative curve will appear when all layers reach boundary-dominated flow. In real test data, the multilayer characteristics may not be obvious because of data noise. Besides that, the Characteristics also depend on the type and the contrast in values of layer properties.

The layer properties that do not identify multilayer characteristics on the constant-rate drawdown type curves are permeability, skin factor. and porosity. whereas layer drainage area (Fig. 4) causes a type curve shape that is different than that of single-layer reservoir. We observe that the larger the value of Ω (layer drainage area ratio), the more obvious the multilayer characteristics are. In this case, to have distinguishable multilayer characteristics the contrast in drainage area values should differ by an order of magnitude.

A combination of varying layer properties may or may not indicate multilayer characteristics. The combination of layer skin and drainage area yields a type curve shape that is characteristic of a multilayer reservoir if the Ω value (or the contrast in drainage area values) is sufficiently large. The combination of A and Ω (or layer penneability and drainage area) yields the characteristics of multilayer reservoir, when the layers with smaller drainage area have higher permeabilities. In these cases the layers quickly reach boundary-dominated flow. On the contrary, in the models where the layer with the highest permeability has the largest drainage area the combination of λ and Ω does not yield multilayer characteristics. The layer apparently dominates the outer boundary effects for some considerable time, consequently it exhibits only single-layer characteristics. Also, the combination of layer skin and permeability (or λ) does not yield multilayer characteristics.

In drawdown analyses the effects of super-positioning the layer solution prolong outer boundary characteristics. In Fig. **3** the start of complete depletion (or pseudosteady-state) takes approximately two log cycles from the end of the transient period. Number of layers affects the duration of boundary-dominated flow. The greater the number of layers, the longer the duration of boundary-dominated flow is. The greater number of layers also smoothes the outer boundary characteristics (see Fig. *5),* while the greater contrast in values of layer properties does not smooth but only prolongs the characteristics.

On a rate-time type curve for a constant-pressure drawdown case, an abrupt deflection on the curve is an indication of a multilayer reservoir (Fig. **6).** The deflection occurs when a layer following the others also reaches boundary-dominated flow. However, the deflection is not always obvious because it depends on the number of layers and the contrast in values of layer properties. The greater number of layers and the less contrast in values of layer properties smooth the deflection. Our experience suggests another indication of a multilayer reservoir is that Arps decline exponent (b) varies with time. According to Fetkovich ***et*** *al.*[13] the b-value for a multilayer system produced at a constant-pressure is between 0.5 and 1.

Similar to the constant-rate cases, varying layer skin or porosity does not yield multilayer characteristics on constant-pressure drawdown type curves. But, varying layer permeability in the constant-pressure cases yields the multilayer characteristics when λ is equal to or greater than 10. Varying layer drainage area also yields the characteristics when Ω is equal ***to*** or greater than 10. If one of the combined layer properties yields multilayer characteristics and is the dominant factor, then the combined properties will also yield the multilayer characteristics.

In Fig. 7 we compare the pressure and derivative Characteristics of two-layer model to those of a single-layer model on constant-rate buildup type curves. The pressure derivative curves of the two models indicate significantly different characteristics. The single-layer model has its derivative curve dip down to a zero value after a transient period. The derivative curve of the two-layer model bounces up after the transient period and a slight dip. During this period the total rate has been zeroed, but layer rates arc not null. **Fig. 8** indicates that interlayer crossflow occurs at the wellbore until the model reaches a final equilibrium (or pseudosteady-state). When the final equilibrium is reached, **all** layer rates become zero and the wellbore pressure equals to the average reservoir pressure of the model. On the type curves, the pressure curve becomes flat and the derivative curve dips down to zero.

When a multilayer model has one varying layer property, such as permeability, drainage area, skin factor, or porosity, then multilayer characteristics always appear on buildup type curves. Consequently, the combinations of these varying properties also yield rnultilayer characteristics. In drawdown cases, varying layer porosity or skin does not yield these characteristics. Depending on the contrast in values of layer properties, the combinations of varying layer properties may or may not yield the characteristics on drawdown type curves. *So* it is more likely that we can distinguish multilayer from single-layer characteristics in buildup analysis than in drawdown analysis.

In the cases where both layer permeability and drainage area vary, the multilayer characteristics are more likely to appear on the type curves of buildup data than those of drawdown data. In the drawdown cases the multilayer characteristics do not appear if Ω and λ have high values, i.e., equal or greater than 10.

Although a larger number of layers smoothes and prolongs outer boundary effects, multilayer characteristics are still easily recognizable on the buildup type curves (Fig. **9).** We also observe on the type curves that the time to reach the pseudosteady-state in multilayer models is two to several orders of magnitude longer than in single-layer models. The time depends on λ and Ω values or the contrast in values of varying layer properties.

In Fig. 10 we present the buildup type curves for the tests with constant-pressure production and compare the characteristics of two-layer and single-layer models. The two-layer model has similar pressure and derivative Characteristics **as** the model with constant-rate buildup tests. The increasing pressure derivative after a transient period, the derivative curve that dips down to zero, and the flat pressure curve when the model reaches the pseudo-steady state, are typical characteristics of a multilayer model in buildup analyses. These characteristics look very different from the single-layer characteristics.

As in the constant-rate cases, the buildup analysis of a multilayer reservoir produced at a constant pressure always yields type curves with multilayer characteristics, if the model has one varying layer property, such as permeability, drainage area, skin factor, or porosity. The combinations of these varying properties also yield multilayer characteristics. However, in the two-layer models with varying layer permeability and drainage area, the multilayer characteristics do not always appear and depend on the values of λ and Ω **.** This phenomenon is also observed in the constant-pressure drawdown cases.

In **1961** Lefkovits *et al.*[9] presented a hypothetical Horner plot (Fig. 11) for a two-layer model produced at a constant rate. The plot indicates a "flat" portion after a transient period. Later, Cobb *et al.*[10] and Raghavan ***et*** *al.*[11] showed and used similar Horner plots in their respective studies. Earlougher *et al.*[12] pointed out the typical "flat" portion in the hypothetical Horner plot does not exist in a depleted reservoir system nor in a rectangular reservoir. It is understood that in a depleted reservoir system the layer potentials (or reservoir pressures) may not significantly differ from layer to layer; therefore, the interlayer crossflow at the wellbore which occurs during shut-in is probably already weak and ineffective.

Our study indicates that only few cases exhibit the "flat" portion in Horner plots. In Fig. 12 we compare the Horner plots for two-layer and single-layer models with constant-rate production. The Horner plot for the two-layer model is likely to represent multilayer characteristics. After a transient period the wellbore pressure may or may not flatten. then it increases until it stabilizes at the average reservoir pressure of the multilayer model. The degree of pressure increase varies and depends on the contrast in values of layer properties. Thus, after the first straight line (or transient regime) we may see one or more straight lines. Obviously, the number of layers, as well as the contrast in values of layer properties, plays a role in the formation of additional lines.

Single-layer analysis of multilayer data

To check validity of the single-layer analysis of inultilayer data, we therefore need to investigate the analysis results. In this section we present the results of our investigation and offer some guidelines in interpreting the multilayer data using a single-layer model. In the investigation we used Welltest™ for constant-rate buildup and drawdown cases, Promat™ for constant-pressure drawdown cases, and Laysim itself **for** constant-pressure buildup cases. We studied the ambiguity in multilayer characteristics by analyzing multilayer data using Welltest™ radial models: composite and dual porosity, and Saphir™ rectangular models containing a well with arbitrary location.

We found that single-layer analysis of multilayer data probably yields a correct estimate of average permeability (or total flow capacity) of a multilayer reservoir. if the layers with negative skin do not control rate history, i.e., always produce 50% or greater of total well fluids. In this condition a drawdown analysis can provide a good estimate of pore-volume averaged drainage area of the layers that are in boundary-dominated flow. Single-layer composite and dual-porosity models have pressure transient characteristics similar to a multilayer reservoir with two or three layers. Single-layer rectangular models containing a well with arbitrary location may match multilayer data obtained from a multilayer low-permeability reservoir.

In the following cases we studied two-layer and three-layer models with either constant-rate or constant-pressure production. Their basic layer properties are presented in Table 3. In each case we varied only one layer property and generated multilayer data with Laysim. Fig. 13 is an example of type curve matches for a constant-rate drawdown case. The matches yield correct estimates of the system flow capacity and pore-volume averaged drainage area. The results of the single-layer analyses are summarized and presented in Table 4. In the table we compare the values of the single-layer properties obtained from the analyses to the average values of the respective properties of the multilayer models. Based on **Eqs. 4** through 7, we compute the average values of the input layer data.

Average permeability

$$\bar{k} = \frac{\sum_{j=1}^{n}(k.h)_j}{\sum_{j=1}^{n}h_j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Average drainage area

$$\bar{A} = \frac{\sum_{j=1}^{n}(A.h.\phi)_j}{\sum_{j=1}^{n}(h.\phi)_j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Average skin factor

$$\bar{s} = \frac{\sum_{j=1}^{n}(k.h.s)_j}{\sum_{j=1}^{n}(k.h)_j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Average porosity $$\bar{\phi} = \frac{\sum_{j=1}^{n}(A.h.\phi)_j}{\sum_{j=1}^{n}(A.h)_j}$$ ....(7)

Except for the cases with varying layer permeability (Cases 2.1 and 2.2) in Table 4, we can see that permeability and drainage area values: multilayer versus single-layer, are in good agreement. For the cases with varying layer skin (Cases 3.1 through 4.2), the estimated skin values obtained from single-layer analyses are less than the average skin values of the multilayer models. Because skin effects are near-wellbore phenomena, the layers with less damage or more stimulation naturally control the early production rate. Therefore, for the cases with negative skin values (Cases 3.1 and 3.2), the single-layer analyses indicate the values that are in the more negative end of the skin value range. For the cases with positive skin values (Cases 4.1 and 4.2), the single-layer analyses indicate a lesser degree of damage. In the cases with varying layer initial pressure (Cases 5.1 and 5.2) the multilayer and single-layer characteristics are identical. In these cases the initial pressure values input in the single-layer model were computed using Eq. **8.**

Average initial pressure $$\bar{p} = \frac{\sum_{j=1}^{n}(P_i.A.h.\phi)_j}{\sum_{j=1}^{n}(A.h.\phi)_j}$$ .... (8)

Fig. **14** is an example of type curve matches for a constant-rate buildup case. The results of the single-layer analyses are summarized and presented in Table 5. Except in the cases with varying initial pressure (Cases 11.1 and 11.2), the multilayer and single-layer characteristics in the buildup data are quite different. Therefore, in the buildup analysis we attempted to match only the early time data. The permeability values for multilayer versus single-layer are again in good agreement. However, the drainage area values obtained from single-layer analyses generally are not valid, because we only match a part of the data. In the cases with varying drainage area (Cases 7.1 and 7.2), the estimated areas from the single-layer analyses fall in the range of layer drainage areas. In the cases with varying layer initial pressure (Cases 11.1 and 11.2) the multilayer and single-layer characteristics in the buildup data. **like** in the drawdown data, are identical. The effects of varying layer skin on buildup data are the same as in the drawdown cases.

Fig. 15 is an example of Promat™ type curve match for a constant-pressure drawdown case. The results of Promat™ analyses are summarized acd presented in Table **6.** In these cases the two-layer and single-layer models generally have good agreement in the values of permeability and drainage area. Except in the case with varying layer permeability (Case 14) the drainage area is 35% smaller than the assigned individual layer of 130 acres. In Case 16 the Promat™ analysis yields a skin of –2.7, which indicates that the production is dominated by the stimulated layer (s = -3). Consequently, the estimated permeability (44 md) from Promat™ analysis is lower than the assigned layer permeability (50 md).

Fig. 16 is an example of type curve matches for a constant-pressure buildup case. In all cases both the two-layer and single-layer data were generated from Laysim models. Here, as in the constant rate buildup cases, we attempted to match only the early time data. Except in Case **18,** all drainage areas were set at **130** acres. The analysis results are presented in Table 7. In Case **18** the estimated single-layer drainage area again falls in the range of layer drainage areas. In Case 19 the single-layer permeability **(32** md) is close to the average permeability (27.5 md) of the two-layer model. The effects of varying layer skin (Cases 20 and 21) are the same as in the cases mentioned above. In Case 22 the single-layer analysis yields a porosity of 0.127, which is slightly greater than the average porosity (0.10) of the two-layer model. However, in this case we obtained different porosity values for the models.

There is a uniqueness problem with multilayer data. Some single-layer (radial) models, such as composite reservoirs and dual-porosity reservoirs with either transient or pseudosteady-state interporosity flow regime, have pressure transient characteristics similar to those of multilayer models. Using the composite and dual-porosity reservoir models, we analyzed the two-layer and three-layer data presented and discussed in the previous paragraphs. We limited our investigation to the constant-rate pressure buildup cases only. Our goal was to see how well the type curves match.

Fig. 17 is the match obtained when we used a pseudosteady-state dual-porosity reservoir model. Only the early portion of the data can be matched. When outer boundary effects dominate the transient characteristics, the matching falls apart. In this case only the permeability value is correct, and the values of other properties are either incorrect or open to interpretation. Fig. 18 is the match obtained when we used a composite reservoir model. Judging the quality of the matches among the three models, the composite reservoir model is likely to have characteristics the closest to the multilayer model's.

We also analyzed pressure transient data obtained from six-layer models with constant-rate production. The models were developed after tight sand (low-permeability reservoir) concepts: the less-permeable layers have larger drainage areas, and layer permeability and porosity have a logarithmic correlation. The logarithmic correlation is based on the characteristics **of** Upper Wilcox sand at Mercy in **Texas**[14] and is presented in **Eq. 9.** In Model 1 the layer drainage area and net pay increase in arithmetic series as the layer permeability or porosity decreases. In Model 2 the layer flow capacity (kh) is constant and set at 15 md.ft; therefore. the layer net pay is computed from **Eq.** 10. The layer drainage area **is** inversely proportional to the respective layer permeability (Eq. 11). Both Model **1** and Model 2 have zero layer skins. Model 1s and Model 2s have nonzero layer skins and the same properties as in Model 1 and Model 2. respectively. The layer properties and the skin values assigned to the six-layer models are shown in Table 8.

Layer permeability $$\log(k_l) = 35.026\,(\phi_l - 0.15) \quad \text{............................} \quad (9)$$

Layer net pay $$h_l = \frac{15}{k_l} \quad \text{..} \quad (10)$$

Layer drainage area $$A_j = \frac{c}{k_j} \quad , \qquad (11)$$

where: c is a constant. Here, $c = 15$.

In Model 1 (or Model 1s) the more-permeable layers have larger flow capacities but smaller drainage areas. In Model 2 (or Model 2s) the flow capacity is set constant and the same for each layer, but the less-permeable layers have significantly larger drainage areas and pore volumes. Consequently, in Model 1 and Model 1s the more-permeable layers (Layers 1 through 3) produce more than 50% of the total well fluids in early times and reach boundary-dominated flow earlier than the less-permeable layers (see Fig. 19). In Model 2 and Model 2s the less-permeable layers (Layers 5 and 6) dominate the rate history all the time and deplete later than the other layers (see Fig. 20). Inclusion of layer skin in Model 1s has minor effects on the layer rates at early times but does not affect the layer rate responses. In Model 2s the layer skins affect the rate responses considerably, and cause Layer 6 to become the major fluid contributor.

In Fig. 21 we present the type-curve matches for a drawdown case. In matching the six-layer data we obtained a good match in the early portion of the data only. In the later portion we only matched the pressure curve against the last data point to yield a reasonable value for the model's drainage area. For Model 1, Model 1s, and Model 2, the Welltest™ analyses yield the same permeabilities (or flow capacities) as the average permeabilities of the six-layer models. The Welltest™ drainage areas for the models are close to but less than the average drainage areas of the layers that have already reached boundary-dominated flow. The Welltest™ analysis of Model 1s data yields a skin value of 3.5, which is approximately the same as the average skin (3.9) of Model 1s.

Model 2s (Fig. 21) indicates typical low-permeability reservoir performance where the less-permeable layers (Layers 5 and 6) always control the rate history. Consequently, the Welltest™ yields a negative skin value that is the same as the average value of the skins of Layers 5 and 6. The negative skin affects the estimated permeability and drainage area. The Welltest™ permeability is less than the average permeability of the model; consequently, its estimated drainage area is larger than the average value of the drainage areas of Layer 1 through 4, which have already reached boundary-dominated flow. The Welltest™ results are presented in Table 9 and compared to the data from the six-layer models.

In all buildup cases we obtained good matches only in the early portion of the data. The later portion of the six-layer data exhibits completely different characteristics from the single-layer characteristics. Therefore, we made no attempt to interpret the Welltest™ estimated drainage area. As in the drawdown cases, for Model 1, Model 1s. and Model 2 the Welltest™ analyses also yield the same permeabilities (or flow capacities) as the average permeabilities of the six-layer models. For Model 1s, the Welltest™ analysis also yields a skin value that is approximately the same as the average skin of the model. In Model 2s the same results as in the drawdown case were obtained for the same reasons as explained in the previous paragraph.

We also studied the ambiguity in multilayer characteristics by analyzing multilayer data using a Saphir™ (single-layer) rectangular model that contains a well with arbitrary location. The outer boundaries of the model can be infinite-acting, sealing (no-flow), constant-pressure, or a combination of these boundaries. In Model 1 and Model 1s, Layer 1 through **3** reach boundary-dominated flow sequentially. The rectangular models match only the later portion of the Model 1 or Model 1s (Fig. 23) data, but miss the earlier portion of the data. The results are similar to those obtained from Welltest™ analyses for a radial model. The permeability and skin values are close to the average values of the respective six-layer properties, whereas the drainage areas are slightly less than the average drainage area of the depleted layers.

In Model 2 and Model 2s the less-permeable layers (Layers *5* and ***6)*** always produce at rates greater than those of other layers. The rectangular models match the drawdown data from the Model 2 and Model 2s (Fig. **23)** very well. For Model 2 the drawdown analysis yields the permeability value that is approximately the average permeability value from all layers. For Model 2s the estimated permeability value is less than the average because of the effects of negative skin. The Saphir™ analysis yields a skin value that is the same as the average value of Layers 5 and **6** only. The analyses also indicate that one of the outer boundaries is still infinite. It agrees with the layer rate responses, which indicate that the outer boundaries of Layers 4 through **6** in Models 1 and 1s, and **of** Layers 5 and **6** in Models 2 and 2s, are still infinite.

In buildup cases, when the beginning of layer outer boundary characteristics are separated by at least an order of magnitude in time, the derivative curve tends to dip deeply and form a "deep valley." The derivative characteristic is usually observed in a reservoir model that has two systems with contrasting properties, like composite, dual-porosity, and two- or three-layer models. Apparently, the rectangular model does not adequately match the "deep valley" derivative characteristics (Fig. **24).**

Estimating layer properties by history matching method

One of our research objectives is to estimate individual layer properties, such as flow capacity **(or** permeability), drainage area, skin factor, and pore volume or fluid-in-place. History matching appears to be a simple and quick method for estimating layer properties of a multilayer model. Although it is only an approximation method, history matching offers a means to estimate the layer properties without a need to sequentially conduct complicated layer testing and interpretation. In this research we study how to history match multilayer data using a simplified three-layer model.

Previously, we have shown the el'fects of super-positioning layer solutions. Superposition effects usually smooth and lengthen the duration of outer-boundary effects. The larger the number of layers, the smoother the outer-boundary effects are. We simplified the matching model by including only three layers. If the matching model is allowed to have only two layers, we expect that it will not adequately handle the superposition effects.

In our study we used the six-layer models--Model 1, Model 1s, Model 2, and Model 2s to represent multilayer models. The three-layer models are named after the six-layer models, which data are matched, plus a suffix indicating the ways in which the six-layer properties are combined into the properties of three layers (see Table 10). For example, if the matching model is called Model 1sm, it means the model is used to match against its counterpart: the six-layer Model 1s. The suffix "m" for the matching model tells us that the properties of Layer 1 and Layer **6** of Model 1s are the same as those in

Layer **1** and Layer 3 of Model 1sm respectively, and the properties of Layer 2 through Layer 5 of Model 1s are combined into the properties of Layer 2 in Model 1sm. Thus, the total properties of the two models, i.e., flow capacity and storativity, are essentially the same.

In combining the layer properties, we used Eqs. **4** through **6** for averaging permeability, drainage area, and skin factor. In averaging layer porosities we assume that net pay and porosity data are obtained from log analyses, and layer drainage areas are still unknown. Therefore, Eq. 12 is used to average layer porosities.

Average porosity $$\bar{\phi} = \frac{\sum_{j=1}^{n}(h.\phi)_j}{\sum_{j=1}^{n}(h)_j} \quad \text{(12)}$$

In Model **1** and Model 1s, flow capacity (or permeability) is assigned to the layers in order of magnitude, so Layer 1 has the largest flow capacity and Layer **6** has the lowest one. Layer 1 has the smallest drainage area (or pore volume) and reaches boundary-dominated flow the fastest, whereas Layer **6** has the largest area and reaches boundary-dominated flow the latest. The three-layer models, Model 1n and Model 1o. indicate that good matches in the early-boundary-affected portion of the data can be obtained, if Layer 1 contains the properties of Layer 1 of the Model 1 and Layer 2 has either the properties of Layer 2 only or the average properties of Layer 2 and Layer 3 of Model 1. When the late-boundary-affected portion of the data is matched, we obtain the total flow capacity, the total pore volume (also the total fluid in place), and the average properties of the least dominant group of layers. such as Layers **4** through ***6*** in Fig. 25. These layers have smaller flow capacities but larger areas (or pore volumes) than the others.

As expected, the layer skin affects only the early time data match and has minor effects on the late portion of the data. In general, the match qualities for both Model 1 and Model 1s are the same. The three-layer Model 1o and Model 1so appear to have the best match on the data of the respective six-layer models. Because the buildup pressure match depends on the match quality of the preceding drawdown data, in the buildup cases we also use Model 1o and Model 1so to history match the data from Model 1 and Model 1s (Fig. **26)**, respectively.

The buildup responses are more sensitive than the drawdown responses. Although both three-layer and six-layer models have the same pore volume and flow capacity of the total system, the distribution of the layer properties apparently affects the match quality. In this case, only the early-boundary-affected portion of the six-layer buildup data is perfectly matched. Inferring the type curve match, one might conclude that only the estimated properties of Layer 1 and perhaps, the estimated average properties of Layers 2 and 3 can be considered accurate.

In constant-pressure drawdown cases of Model 1 and Model 1s (Fig. 27), we obtain similar results as in the constant-rate drawdown cases. The three-layer models, Model 1n, Model 1o, Model 1sn, and Model 1so, also indicate good matches in the early-boundary-affected portion of the data. In these models

Layer 1 contains the properties of Layer 1 of the Model 1 and Layer 2 has either the properties of Layer 2 only or the average properties of Layers 2 and *3* of Model 1. The match quality of the late-boundary-affected portion of the data is not as good as in the constant-rate cases, because the outer boundary effects are more obvious. For comparison purposes we select the Model 1o and Model 1so (Fig. 28) to present the type-curve matches for buildup cases. Once again, in these cases only the early-boundary-affected portion of the six-layer buildup data is closely matched. The same inference as in the constant-rate cases may apply also.

Model 2 and Model *2s* represent typical low-permeability reservoirs, where the less-permeable layers (Layers **5** and **6)** have larger drainage areas (pore volumes) and they control rate responses. In constant-rate drawdown cases, Model 20 indicates good type-curve match on the early-boundary-affected portion of the data. It implies that Layer 1, which has the smallest drainage area, and the other layers with smaller areas, i.e., Layers 2 and 3, should be modeled **as** Layer 1 and Layer 2 in the three-layer models. respectively. When Layer 4 is included in Layer 1 of Model 2r, the match of early-boundary-affected data falls apart. The large drainage area of Layer **4** causes a delay in the early-outer-boundary effects.

When the layers with larger drainage areas, i.e., Layers 5 and 6, are modeled as Layer **3** in Model 2sp (Fig. 29), the match of late-boundary-affected data is reasonably good. Apparently, the late-boundary-affected data matches yield the properties of a layer or a group of layers, which has the larger drainage areas. It also yields the total flow capacity and pore volume (or fluid in place) of Model 2. The matches by Model 2p indicate that we may not need to have a perfect match of the early-boundary-affected data.

In Model 2s, Layer 6 has the lowest permeability but the largest drainage area. It is the most-stimulated layer with a skin value of –3. Layer 5 is the layer with the next-lowest permeability and a skin value of –1. These two layers always produce more than 50% of the total well fluids. 'Therefore, the early time data reflect the properties of the two layers. When Layer *6* is modeled as a layer (Layer 3) in Model 2sm and Model 2st, perfect matches of the early time data are obtained. When Layers 5 and 6 are combined and modeled as a layer (Layer *3)* in Model 2sp and Model 2sq, the matches of early time data are still acceptable. However, when the two layers are combined with other layers as in Model 2sn and Model 2so, then we do not obtain a good match of the early time data.

As in Model 2 the match quality of the early-outer-boundary portion is controlled by Layer 1, which has the smallest drainage area, and the other layers with smaller drainage areas (Layer 2, Layer *3).* The match quality of the late-boundary-affected portion is controlled by the layers with large drainage areas (Layers 5 and **6).** In Model 2sp (Fig. *29)* and Model 2sq, we have good matches of the late-boundary-affected portion.

Based on our observation of the match quality in the constant-rate drawdown cases, Model 2p and Model 2sp provide the best matches for the data from Model 2 and Model 2s, respectively. Therefore, we have used Model 2p and Model 2sp to history match the buildup data obtained from Model 2 and Model 2s, respectively. The match quality of buildup data very much depends on the match quality of the preceding drawdown data. Fig. **30** is the type-curve matches of Model 2s. We consider the matches good and acceptable.

A simple three-layer model can be used to estimate the layer properties of multilayer models by history matching. The history-matching method yields the properties of individual or grouped layers and of the total system. In setting up a three-layer model, if we have some ideas about layer properties, we should group the layers based on the same order or the closest order of flow capacity and drainage area (or pore volume) values. The early-time data indicates the properties of the layers that dominate production rate. These layers may have the largest flow capacity (or permeability) but positive skins and small drainage areas or the smallest flow capacity but negative skins and large drainage areas.

Matching early-boundary-affected data from a multilayer reservoir will yield average drainage area and pore volume of a layer or a group of layers with smaller areas (or pore volumes). The late-boundary-affected data from a multilayer reservoir indicate the properties of a layer or a group of layers which has the smallest flow capacity but the largest drainage area (or pore volume). Matching these late data will yield the total flow capacity (and average permeability) and pore volume (and the fluid in place) of a multilayer model. In some cases, to get a good match of the late data we may sacrifice a perfect match of the early-boundary-affected data, but we have to have the correct estimates of average flow capacity and pore volume of the layers that control the early portion.

In estimating layer properties by history matching using a three-layer model, we propose a two-step procedure. The first step is to match early data, which include the early time portion and the early boundary-affected portion, to estimate the properties of the more permeable layers or the layers with more negative skins that control early production. The second step is to match the late boundary-affected portion of the data to estimate the properties of the less-permeable layers, and the flow capacity and storativity of the total system.

History matching method for special cases

In special cases where the layers have only one property that varies significantly between layers, the three-layer model can be simplified such as is depicted in Fig. 31. In the model, Layer 2 contains the average properties of Layers 2 through 5. The layer properties are averaged based on **Eqs.** 4 through 7. Layer 1 and Layer 3 contain the same properties as Layers 1 and **6** in the six-layer model, respectively. The layer with the maximum value of the varying property is designated as Layer 1, while the layer with the minimum value is designated as Layer 3. The three-layer model is named after its counterpart six-layer model plus a suffix that indicates the varying property.

From our observations, we conclude that in special cases a simplified three-layer model can be used to history match multilayer data and obtain good estimates of layer properties. These properties are those of the total system and also, of either individual or grouped layers. The first and second layer of the model should contain the maximum and the minimum value of the varying property respectively, and the third layer should contain the average of the remaining values of the property.

The type-curve matches of constant-rate cases: buildup and drawdown, are presented in Figs. 32 and 33. In the drawdown case with varying layer permeability (Fig. **32)** we have excellent match. In the buildup case (Fig. 33), we see some deviations in the middle portion of the matches. Thus, outer boundary effects in the models with fewer layers are more obvious. and superposition effects of layer solutions do not significantly affect outer-boundary characteristics. Eventually, the derivative type-curves match and dip to zero simultaneously.

The type-curve matches of a constant-pressure case: buildup and drawdown, are presented in Figs. 34 and 35. In general, we have good matches in drawdown cases. In the drawdown case with varying layer permeability (Fig. 34) we also see some deviation from the match because of superposition (or layering) effects, Eventually, the match becomes better when the effects diminish. In general, the match qualities for buildup cases (e.g., Fig. 35) are also good.

Relative rate study

Relative rate (q_{Dj}) is defined as the ratio of layer rate (q_j) to the total rate (q_t) of a multilayer reservoir model (Eq. 13). In a production logging we estimate relative rates based on rate measurements across individual reservoir layers. The rate measurements are limited by the minimum threshold rate that a spinner in the production-logging tool can measure. The layer rate itself may vary from the bottom to the top of perforations (or a layer, in case of an open hole) where the measurement takes place. **As** a result, we obtain relative rate estimates based on layer and total production. Complex layer testing such as selective inflow performance, single-layer transient testing, multilayer transient testing, and vertical-interference transient testing, involve simultaneous layer-rate and pressure-data measurements.

Relative rate $$q_{D_j} = \frac{q_j}{q_t} \qquad (13)$$

When we analyze multilayer data using a single-layer model, we are likely to obtain the total properties of a multilayer reservoir. Hence, we estimate individual layer properties, i.e., flow capacity and pore volume, using relative rate data. Lefkovits *et al.*[9] relate relative rate to layer flow capacity (Eq. 14) and pore volume (Eq. 15). Eq. 14 is applicable when transient flow exists, whereas Eq. 15 is for pseudosteady-state conditions. The Lefkovits *et al.* study was limited to constant-rate cases and two layers having the same drainage areas. We expand the study of relative rate to include both constant-rate and constant-pressure drawdown cases; the models with two, three, and six layers; and the models with varying layer properties.

In transient flow, $$q_{D_j} \approx \lambda_j \qquad (14)$$

In pseudosteady-state, $$q_{D_j} \approx \Omega_j \qquad (15)$$

The lamda (λ_j) and omega (Ω_j) are defined in Eqs. 1 and 2, respectively. In the following drawdown cases only one layer property, either drainage area or permeability, varies, and the remaining layer properties are kept the same for each layer. We compared relative rate versus λ and Ω for a two-layer model with varying drainage area. The relative rates exactly equal to A's when all layers have the same permeabilities and to Ω's when all layers are in pseudo-steady state. In the case with varying layer permeability (Fig. 36) the relative rates approach λ's but never equal to the A's. If the relative rate data are used to estimate layer permeabilities, then we will underestimate the permeability of the more-permeable layer and overestimate that of the less-permeable layer. In Fig. **37** we compare relative rate to Ω for a two-layer model with constant-pressure drawdown. In this case, the relative rates are not equal to the Ω's when one of the layers is depleted and stops to produce. Thus, Eq. 15 is not applicable.

In Figs. **38** and **39** we compare relative rate to λ for six-layer models: Model-1 without layer skin, and Model-1s with layer skin. Apparently, skin effects further exaggerate the underestimation or overestimation of layer permeabilities. We also observed when the number of layers increases, the chances for all layers to be simultaneously in pseudosteady-state conditions become less likely. Consequently, Eq. 15 is never applicable. Thus, it is more difficult to allocate the pore volumes of individual layers.

In the study of relative rate versus λ and Ω relations, we get the same results for either constant-rate or constant-pressure cases. During transient flow, layer relative rates approach the respective layer flow capacity ratios but never equal them. Only when the layers have the same permeability are the relative rates equal to their respective flow-capacity ratios. If we estimate layer permeabilities from relative rate data - e.g., from flow-meter measurement - we will overestimate the permeabilities of the less-permeable layers and underestimate those of the more-permeable layers. Skin effects further exaggerate the underestimation and Overestimation of layer permeabilities.

Compared to constant-pressure drawdown, all layers are likely to be simultaneously in pseudosteady-state when they are produced with a constant total rate. Therefore, under constant-rate drawdown, the layer relative rates are more likely to be equal to the respective layer storativity ratios. **As** the number of layers increases, it is less likely that all layers are simultaneously in pseudosteady-state; thus, layer relative rates will never be the same as or close to the respective layer storativity ratios. Consequently, it is more difficult to allocate the estimated total pore volume obtained from a single-layer analysis to individual layers of a multilayer reservoir.

Summary and conclusions

There are significant differences between multilayer and single-layer characteristics on buildup and drawdown diagnostic plots. Increasing pressure derivative following a transient period and sometimes a dip in the derivative curve identifies the multilayer characteristics on buildup diagnostic plots. During this period interlayer crossflow at the wellbore still occurs, while the surface rate has already been zeroed. Eventually, as the total system approaches the final equilibrium (or pseudosteady-state) and the crossflow ceases, the pressure derivative decreases to zero and the wellbore pressure stabilizes at the average reservoir pressure of the total system. Another characteristic, previously observed by Lefkovits ***et al.***,[9] is the time to reach pseudosteady-state, which is one-to-several orders of magnitude greater in a multilayer model then in a single-layer model with identical properties. On a Horner plot, multilayer characteristics may be recognizable from increasing wellbore pressure after a transient period.

On constant-rate drawdown diagnostic plots, the multilayer characteristics are recognizable from the slope of pressure derivative curve, which is less then one when the layers begin to deplete and is one when all layers are already in boundary-affected flow. The number of layers determines the duration of the boundary-affected flow period and the smoothness of boundary-affected data on diagnostic plots. **As** layers begin to deplete, an abrupt deflection on a rate-time diagnostic plot is an indication of multilayer characteristics for constant-pressure drawdown cases. However, the deflection is not always obvious because it depends on the contrast of layer properties.

A simple three-layer model can be used to estimate the layer properties of multilayer models by history matching. History matching yields the properties of individual or grouped layers, and of the total system. In setting up a three-layer model, if we have some ideas about layer properties, we should group the layers by the closest order of flow capacity and drainage area (or pore volume) values.

To history match multilayer data. we propose a two-step procedure. The first step is to match early data, which includes the early-time portion and the early-boundary-affected portion, to estimate the properties of the more permeable layers or the layers with more negative skins that control early production. The second step is to match the late-boundary-affected portion of the data to estimate the properties of the less-permeable layers and the flow capacity and storativity of the total system. In special cases where the layers have only one property that varies significantly between layers, the three-layer model can be simplified. The first and the second layer in the model contain the maximum and the minimum value of the varying property, respectively. The third layer contains the average of the remaining values of the property.

Following are our conclusions:

1. Diagnostic plots are preferable to Homer plots in distinguishing between multilayer and single-layer characteristics. Buildup diagnostic plots delineate multilayer characteristics better than the drawdown diagnostic plots.
2. Single-layer analysis of multilayer data is likely to yield a reasonable estimate of the average permeability (or total flow capacity) of a multilayer model, provided the layers with negative skin do not dominate the rate history.
3. The drawdown analysis of multilayer data is likely to yield a good estimate of the pore-volume averaged drainage area of the depleted layers.
4. If the dominant layers have positive skin values, the estimated skin value obtained from single-layer analysis is generally less than but possibly close to the average skin value of the multilayer model.
5. If the dominant layers have negative skin values, the single-layer analysis of multilayer data yields a skin value that is approximately the same as the kh-averaged skin value of those layers. The effects of negative skin cause the analysis to underestimate the average permeability (or flow capacity) and overestimate the pore-volume averaged drainage area of the layers that are in boundary-affected flow.
6. **A** rectangular reservoir model that contains a well with arbitrary location can match the multilayer data obtained from a low-permeability reservoir, where one or two layers with low permeabilities and large drainage areas always produce greater than 50% of total well fluids.
7. Rectangular models may not be adequate to match the data from a multilayer reservoir that has only two or three layers and exhibits a "deep valley" in the derivative curve. Radial composite and dual-porosity models can match the "deep valley" portion better than the rectangular model.
8. Matching the early-boundary-affected portion of multilayer data will yield the average drainage area and pore-volume of a layer or a group of layers with smaller areas (or pore volumes).
9. Matching the late-boundary-affected portion will yield the total flow capacity (and average permeability) and pore volume (and the fluid in place) of a multilayer model. It also yields the

properties of a layer or a group of layers that has the smallest flow capacity but the largest drainage area (or pore volume).

10. In some cases, to get a good match of the late portion we may sacrifice a perfect match of the early-boundary-affected data, but we have to have the correct estimates of average flow capacity and pore volume of the layers that control the early portion.
11. If we estimate layer permeabilities from relative rate data, we will overestimate the permeabilities **of** the less-permeable layers and underestimate those of the more-permeable layers. Skin effects further exaggerate the underestimation and overestimation of layer permeabilities.
12. Only when all layers simultaneously are in pseudosteady-state are layer relative rates equal to the respective layer-storativity ratios. Hence, the relative rate data can be used to estimate layer pore volumes from the estimated total pore volume obtained from a single layer analysis.

Nomenclature

A	=	area, sqft
h	=	total net pay, ft
h_j	=	layer net pay, ft
k	=	permeability, md
P	=	pressure, psia
P_i	=	initial pressure, psia
P_{ij}	=	layer initial pressure, psia
q	=	rate, bbl/day
s	=	skin factor
ϕ	=	porosity, fraction
λ	=	flow capacity ratio
Ω	=	storativity ratio
γ	=	drainage area ratio

Subscript

j	=	layer index

Superscript

$\_$	=	average quantity

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Table 1 - Basic Fluid Properties

Property	Value
Oil viscosity, cp	0.5
Oil formation volume factor, rb/stb	1.1
Oil compressibility, 1/psi	5.0e-5
Total compressibility, 1/psi	5.4e-5

Table 2 - Multilayer Responses in Rate and Pressure Transients

Can we see multilayer-characteristics in these tests?

	Constant-rate		Constant-pressure	
Varying layer property	**Drawdown**	**Buildup**	**Drawdown**	**Buildup**
Drainage area (A)	Yes	Yes	Yes	Yes
Permeability (k)	No	Yes	Yes	Yes
Skin factor (s)	No	Yes	No	Yes
Porosity	No	Yes	No	Yes
k & A	Yes (1)	Yes	Yes (1)	Yes
A & s	Yes	Yes	Yes (2)	Yes
k & s	No	Yes	Yes (3)	Yes

Notes:
(1) It yields single-layer characteristics if a layer has large values in both properties.
(2) Multilayer characteristics are likely caused by boundary effects.
(3) Multilayer characteristics are likely caused by layer-permeability differences.

Table 3 - Basic Layer Properties of Two-Layer and Three-Layer Models

Permeability (k), md	50
Net pay (h), ft	50
Drainage area (A), acres	130
Skin factor (s)	0
Porosity (ϕ)	0.15

Table 4 - Analysis Results for Constant-Rate Drawdown Cases

Case		No. layers	Varyin layer property		Multilayer model				Single-layer model			
No.	**Name**		**Property**	**Value**	$\bar{k}$	A	$\bar{s}$	$\bar{\phi}$	k	A	s	ϕ
1.1	M1	2	A	130, 26	50	78	0	0.15	50	74	-0.1	0.15
1.2	TRD1	3	A	130, 65, 26	50	73.7	0	0.15	50	75	0.0	0.15
2.1	M2	2	k	50, 5	27.5	130	0	0.15	27.5	90	0.0	0.15
2.2	TRD2	3	k	50, 25, 5	26.7	130	0	0.15	26.7	106	0.3	0.15
3.1	M6	2	$s \leq 0$	0, -3	50	130	-1.5	0.15	50	128	-2.2	0.15
3.2	TRD5	3	$s \leq 0$	0, -2, -3	50	130	-1.7	0.15	50	131	-2.2	0.15
4.1	M5	2	$s \geq 0$	0, 5	50	130	2.5	0.15	50	129	1.6	0.15
4.2	TRD4	3	$s \geq 0$	0, 3, 5	50	130	2.7	0.15	50	131	1.9	0.15
5.1	**M13R**	2	P_i	2500,2000	50	130	0	0.15	50	130	0.0	0.15
5.2	TRD6	3	P_i	2500, 2250, 2000	50	130	0	0.15	50	130	0.0	0.15
6.1	M9	2	ϕ	0.15, 0.05	50	130	0	0.1	50	130	0.2	0.10
6.2	TRD3	3	ϕ	0.15, 0.10, 0.05	50	130	0	0.1	50	129	0.0	0.10

Table 5 - Analysis Results for Constant-Rate Buildup Cases

Case		No. layers	Varying layer property		Multilayer model				Single-layer model			
No.	**Name**		**Property**	**Value**	$\bar{k}$	$\bar{A}$	$\bar{s}$	$\bar{\phi}$	k	A^*	s	ϕ
7.1	M3	2	A	130.26	50	78	0	0.15	50		0.0	0.15
7.2	TRB1	3	A	130, 65, 26	50	73.7	0	0.15	27.5		0.0	0.15
8.1	M4	2	k	50, 5	27.5	130	0	0.15	26.7		0.2	0.15
8.2	TRB2	3	k	50, 25, 5	26.7	130	0	0.15	26.7		0.0	0.15
9.1	M8	2	$s \leq 0$	0, -3	50	130	-1.5	0.15	50		-2.0	0.15
9.2	TRB5	3	$s \leq 0$	0, -2, -3	50	130	-1.7	0.15	50		-2.1	0.15
10.1	M7	2	$s \geq 0$	0, 5	50	130	2.5	0.15	50		1.6	0.15
10.2	TRB4	3	$s \geq 0$	0, 3, 5	50	130	2.7	0.15	50		1.8	0.15
11.1	M14R	2	P_i	2500,2000	50	130	0	0.15	50	130	0.0	0.15
11.2	TRB6	3	P_i	2500, 2250, 2000	50	130	0	0.15	50	130	0.0	0.15
12.1	M10	2	ϕ	0.15, 0.05	50	130	0	0.1	50		0.0	0.10
12.2	TRB3	3	ϕ	0.15, 0.10, 0.05	50	130	0	0.1	50		0.2	0.10

***match early data only (except** in Cases 11.1 and 11.^)

Table 6 - Analysis Results for Constant-Pressure Drawdown Cases

Case		No. layers	Varying layer property		Multilayer model				Single-layer model			
No.	Name		Property	Value	$\bar{k}$	$\bar{A}$	$\bar{s}$	$\bar{\phi}$	k	A	s	ϕ
13	P1	2	A	130, 26	50	78	0	0.15	47.6	67	-0.3	0.15
14	P3	2	k	50, 5	27.5	130	0	0.15	26.7	84	-0.2	0.15
15	P5	2	$s \geq 0$	0, 5	50	130	2.5	0.15	51	125	1.3	0.15
16	P7	2	$s \leq 0$	0, -3	50	130	-1.5	0.15	44	130	-2.7	0.15
17	P9A	2	ϕ	0.15, 0.05	50	130	0	0.10	46.2	129	-0.5	0.10

Table 7 - Analysis Results for Constant-Pressure Buildup Cases

Case		No. layers	Varying layer property		Multilayer model				Single-layer model			
No.	Name		Property	Value	$\bar{k}$	$\bar{A}$	$\bar{s}$	$\bar{\phi}$	k	A^*	s	ϕ
18	P2	2	A	130, 26	50	78	0	0.15	50	112	0.0	0.15
19	P4	2	k	50, 5	27.5	130	0	0.15	32	130	0.0	0.15
20	P6	2	$s \geq 0$	0, 5	50	130	2.5	0.15	50	130	2.0	0.15
21	P8	2	$s \leq 0$	0, -3	50	130	-1.5	0.15	50	130	-1.5	0.15
22	P10	2	ϕ	0.15, 0.05	50	130	0	0.10	50	130	0.0	0.127

*set at 130 acres, except in Case 18

Table 8 - Layer Properties of the Six-Layer Models

Layer No.	Model 1, Model 1s				Model 2, Model 2s				s*
	h	k	ϕ	A	h	k	ϕ	A	
1	10	126	0.21	5	3	5.0	0.17	3.0	5
2	15	56	0.2	10	7.5	2.0	0.16	7.5	3
3	20	11	0.18	15	15	1.0	0.15	15.0	2
4	25	1	0.15	20	33.6	0.446	0.14	33.6	1
5	30	0.2	0.13	25	75.4	0.199	0.13	75.4	-1
6	35	0.02	0.1	30	168.5	0.089	0.12	168.5	-3

*Only for Model 1s and Model 2s

Table 9 - Comparisons: Welltest™ Results Versus Six-Layer Data

Model	Welltest™ Analysis					Six-Layer Model					
	Drawdown			Buildup			Index		Index		Index
	k	A	s	k	s	$\bar{k}_j$	j	$\bar{A}_l$	l	$\bar{s}_m$	m
1	17.42	9.6	0.37	17.42	0.5	17.42	1 - 6	10.9	1 - 3	0	1 - 6
1s	17.42	8.1	3.5	17.42	3.5	17.42	1 - 6	10.9	1 - 3	3.9	1 - 6
2	0.297	20.2	0.25	0.297	0.25	0.297	1 - 6	23.2	1 - 4	0	1 - 6
2s	0.209	29.8	-1.98	0.200	-2.03	0.297	1 - 6	23.2	1 - 4	-2	5 - 6

Table 10 - Six-Layer Model - Equivalent Layers

Model suffix	Three-Layer Model		
	Layer 1	Layer 2	Layer 3
m	Layer 1	Layer 2 thru 5	Layer 6
n	Layer 1	Layer 2	Layer 3 thru 6
o	Layer 1	Layer 2 and 3	Layer 4 thru 6
p	Layer 1 and 2	Layer 3 and 4	Layer 5 and 6
q	Layer 1	Layer 2 thru 4	Layer 5 and 6
r	Layer 1 thru 4	Layer 5	Layer 6
t	Layer 1 and 2	Layer 3 and 5	Layer 6
u	Layer 1 thru 3	Layer 4 and 5	Layer 6

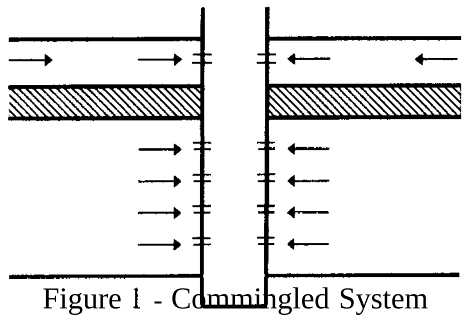


Figure 1 - Commingled System

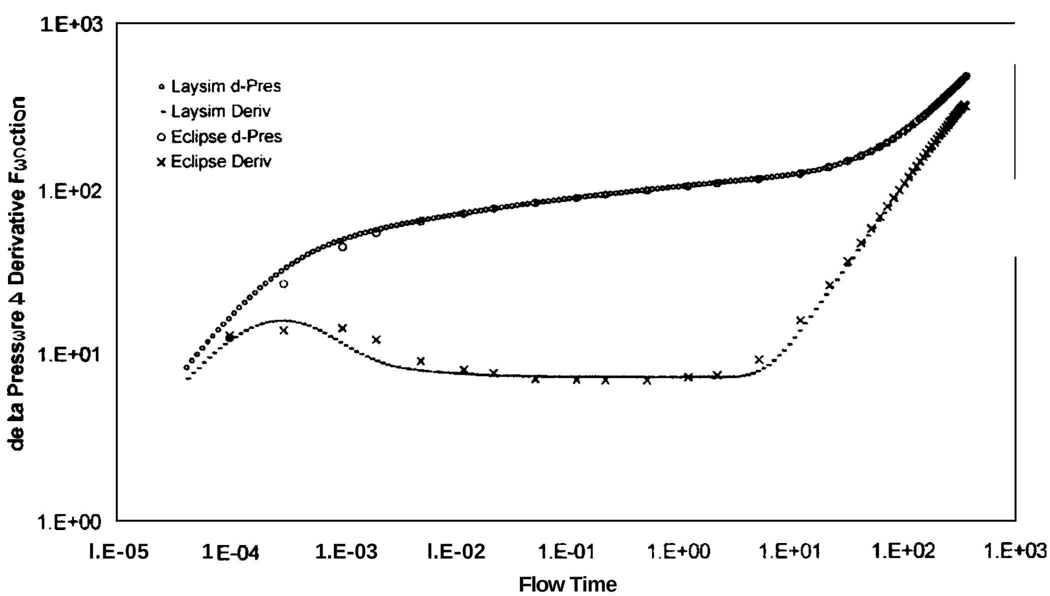


Figure 2 - Two-Layer, Constant-Rate Drawdown Case
Varying Layer Permeabilities: 5 and 50 md

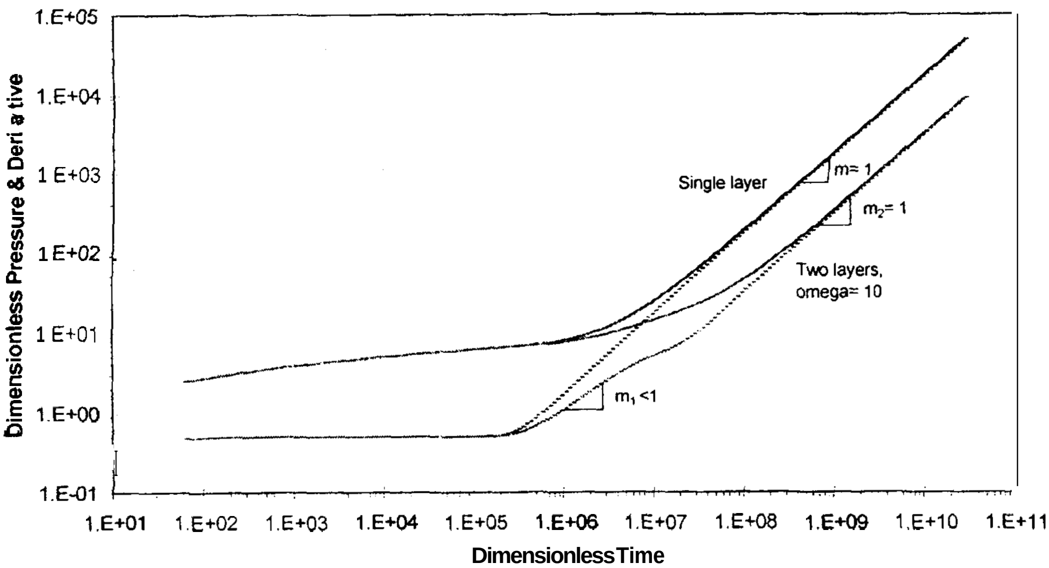


Figure 3 - Two-Layer vs. Single-Layer Characteristics
Constant-rate drawdown type curves $\lambda = 1$, $\gamma \neq 1$ $C_D = 0$, $s = 0$

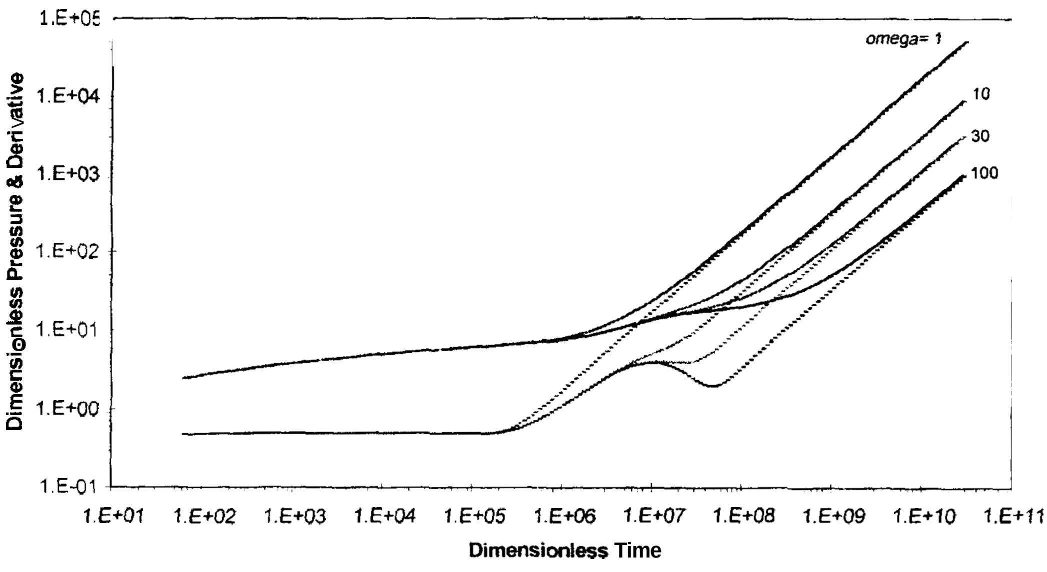


Figure 4 - Two-Layer, constant-rate drawdown type curves.
Varying layer drainage area $\lambda = 1$, $C_D = 0$, $s = 0$

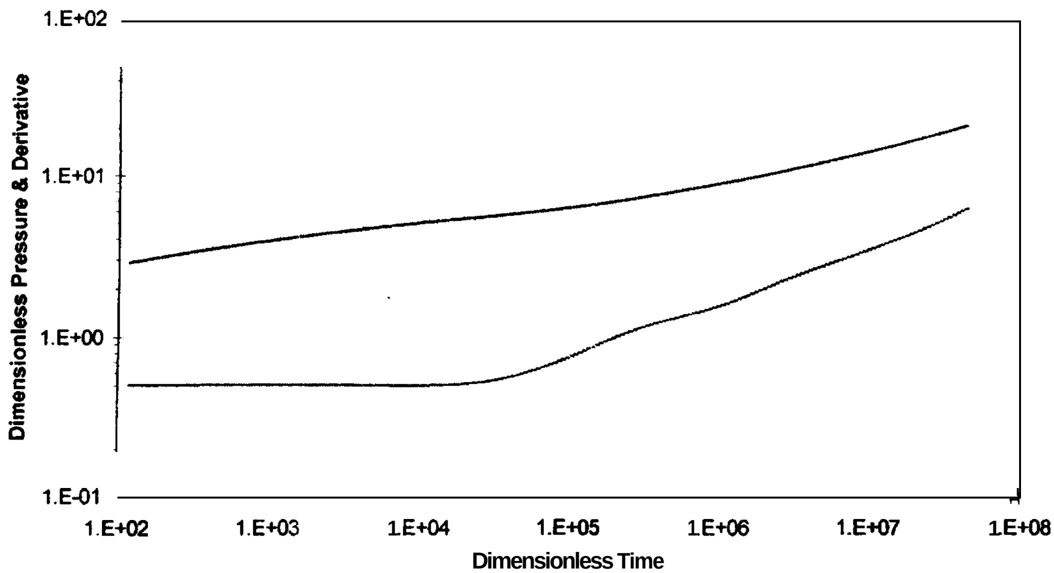


Figure 5 - Six-Layer, Constant-Rate Drawdown Type Curves
Varying Layer Properties $C_D = 0, s = 0$

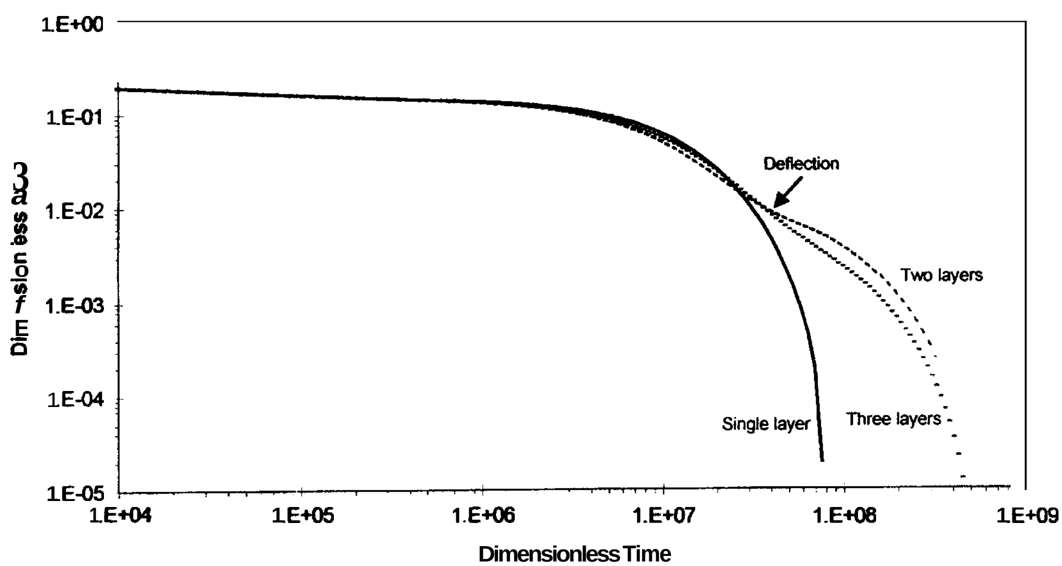


Figure 6 - Muitilayer vs. Single-Layer Characteristics
Constant-pressure drawdown type curves $\Omega = \gamma = 1/1$, $C_D = 0, s = 0$

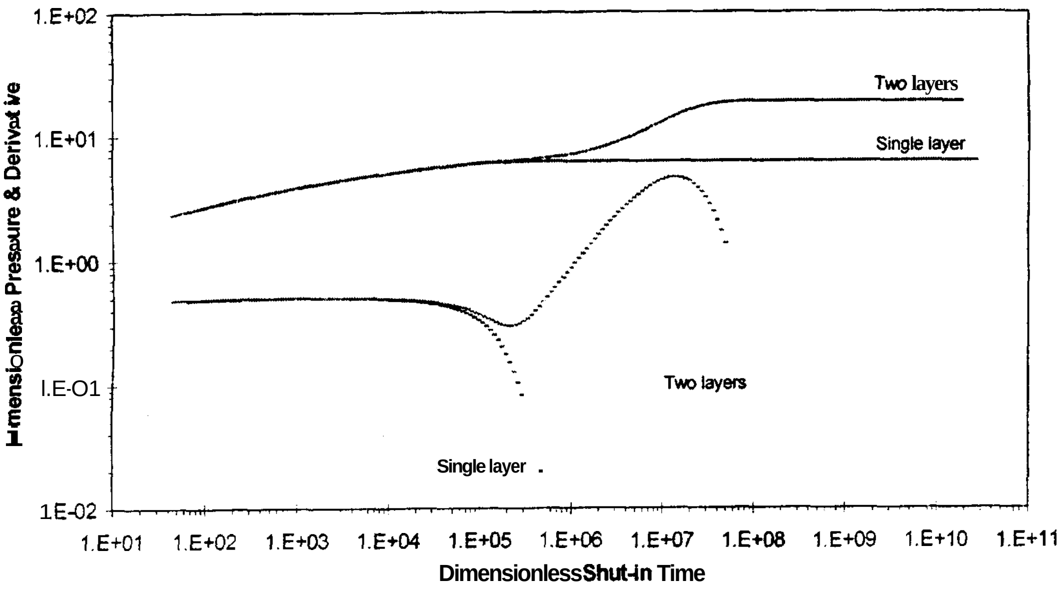


Figure 7 - Two-Layer vs. Single-Layer Characteristics
Constant-rate buildup type curves $\Omega = \gamma = 1$, $C_D = 0$, $s = 0$

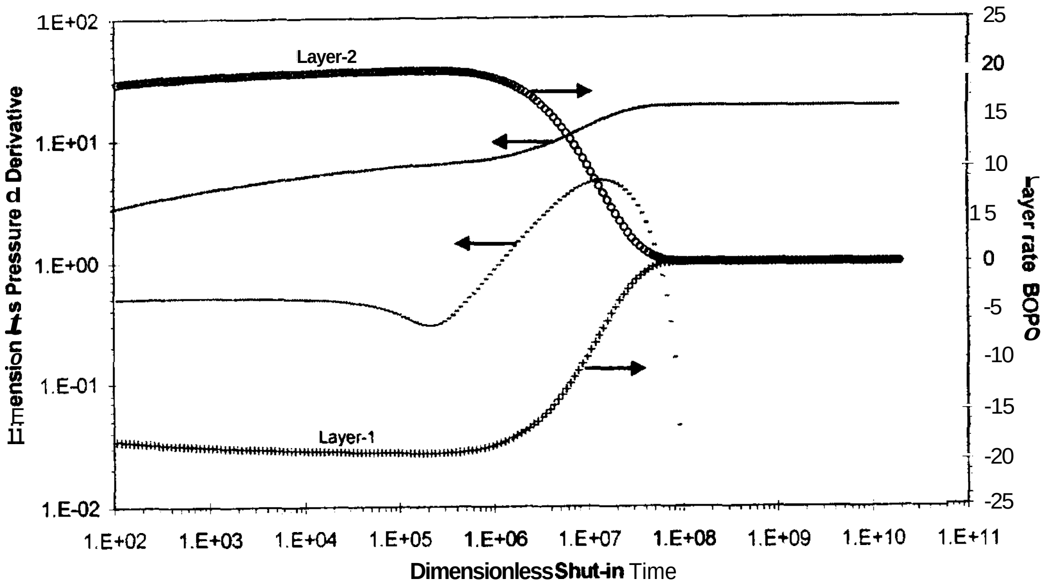


Figure 8 - Layer Rate Behaviors During Shut-In Period
Two-layer, constant-rate buildup type curves $\Omega = \gamma = 1$, $C_D = 0$, $s = 0$

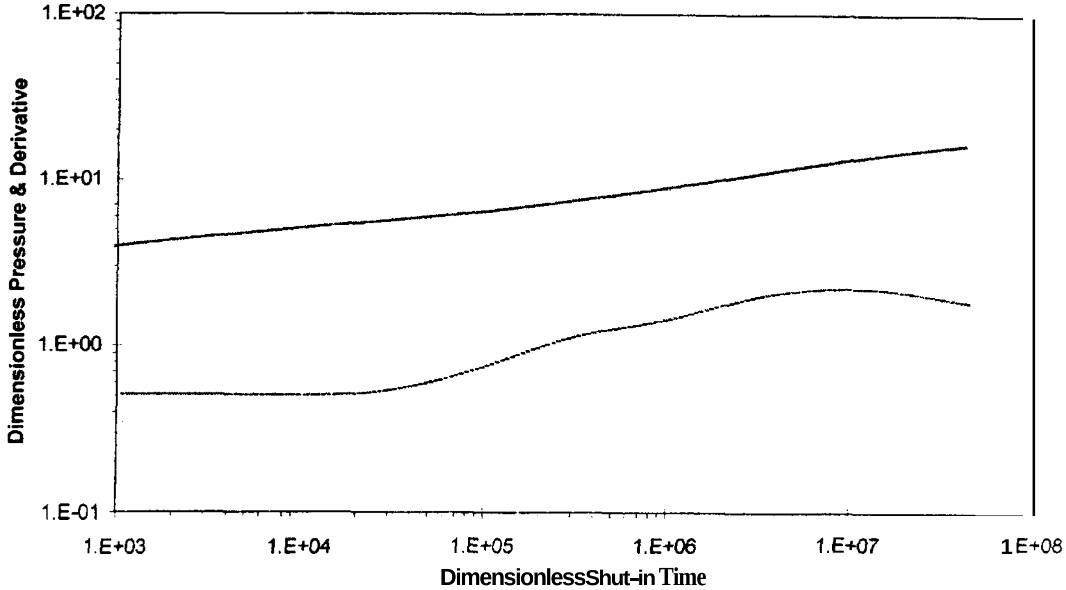
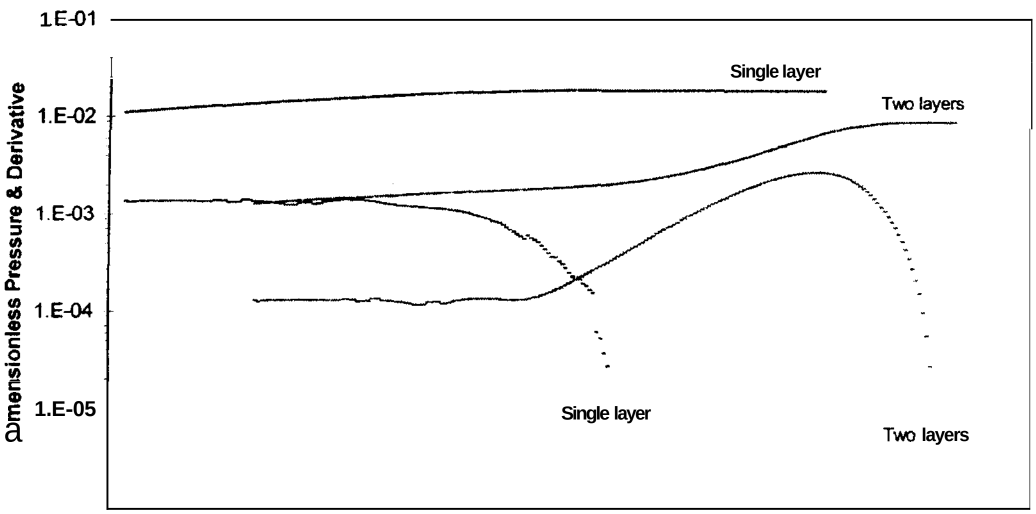


Figure 9 - Six-Layer, Constant-Rate Buildup Type Curves
Varying layer properties $C_D = 0$, $s = 0$



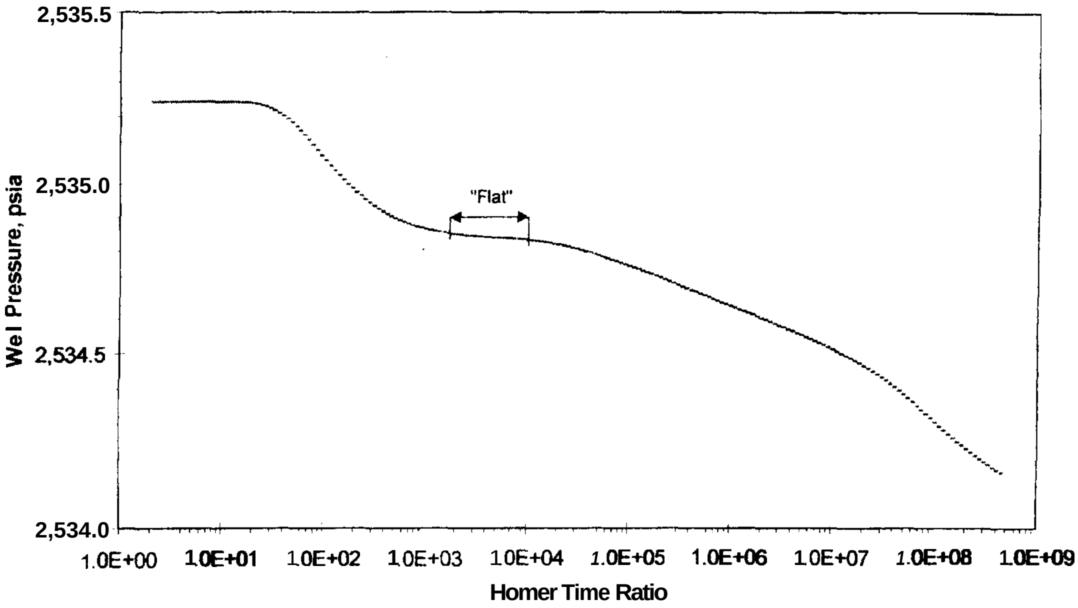


Figure 11 - Two-Layer Characteristics in Horner Plot
Varying layer permeability and drainage area $\Omega = 10, \lambda = 100, C_D = 50, s = 0$

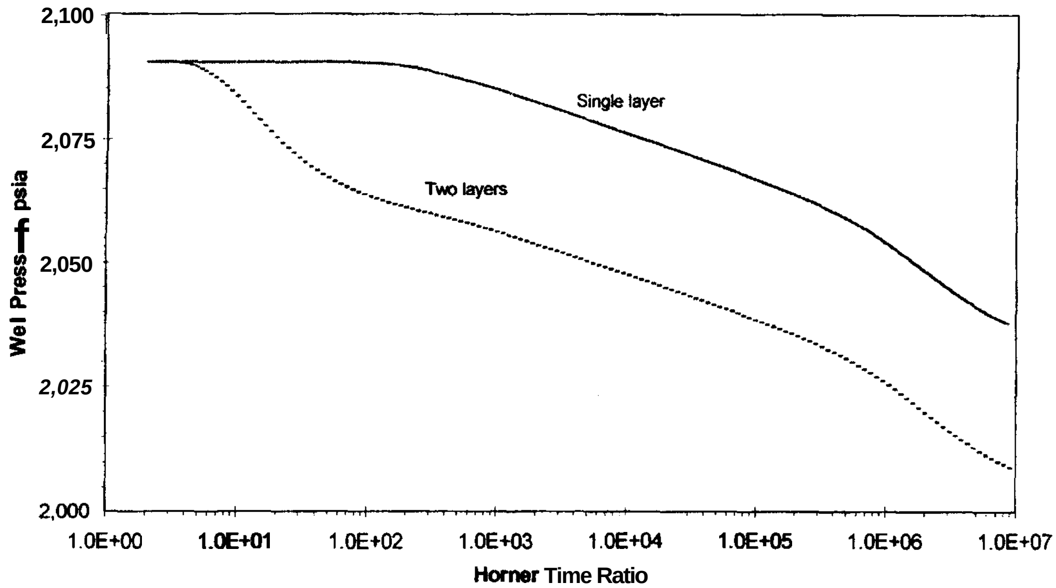


Figure 12 - Two-Layer vs. Single-Layer Characteristics
Constant-rate buildup tests $C_D = 50, s = 0$

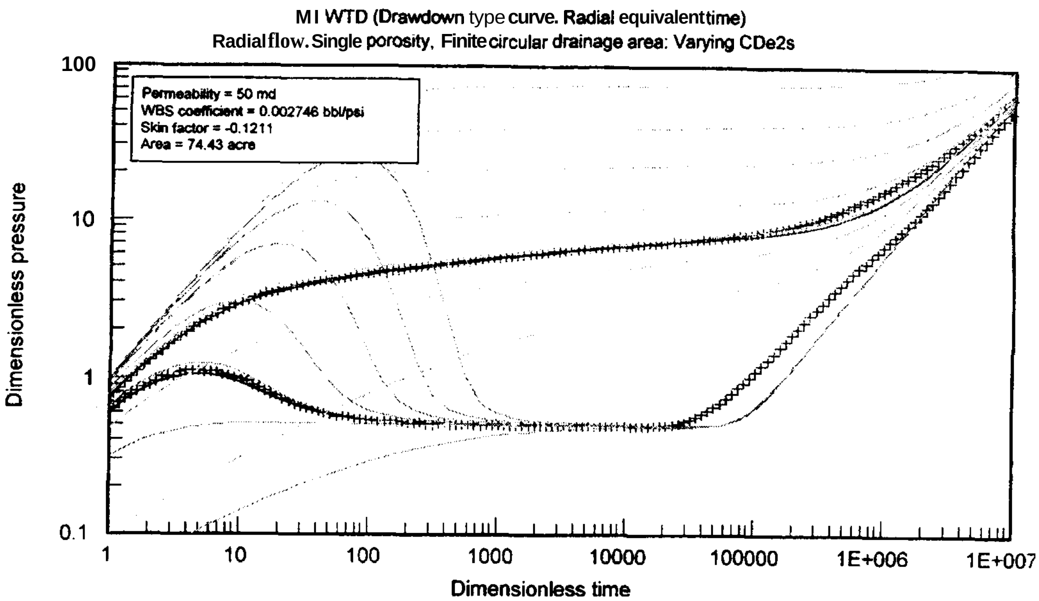


Figure 13 - Welltest™ Analysis of the Drawdown Data from a Two-Layer Model
Varying layer drainage area **A**= 130/26 acres, C_D= SO, $s = 0$

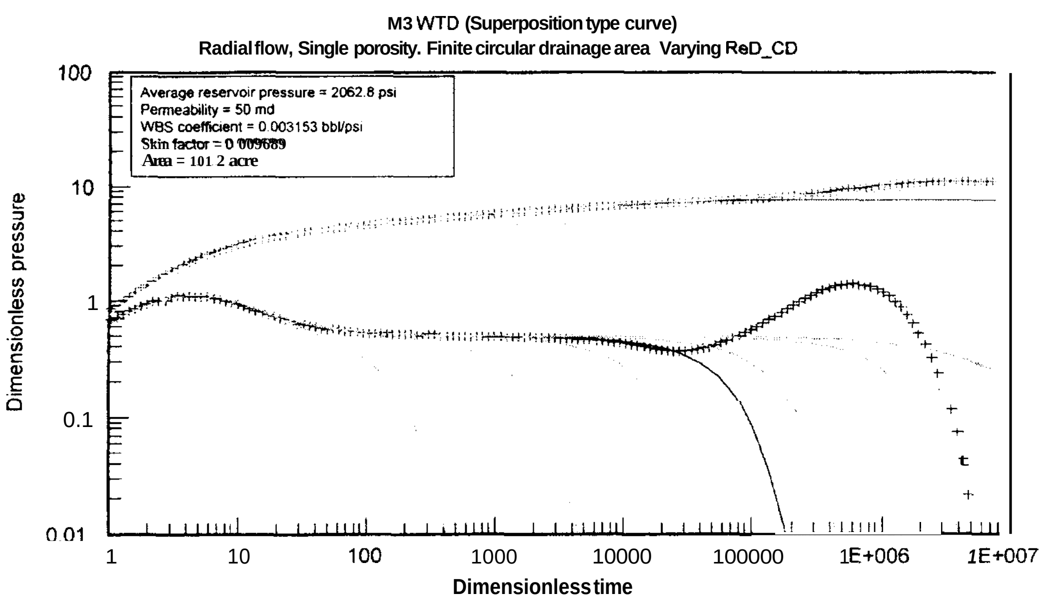


Figure 14 - Welltest™ -Finite Single Porosity - Buildup Analysis
Two-Layer model with varying layer drainage area **A**= 130/26 acres, C_D= SO, $s = 0$

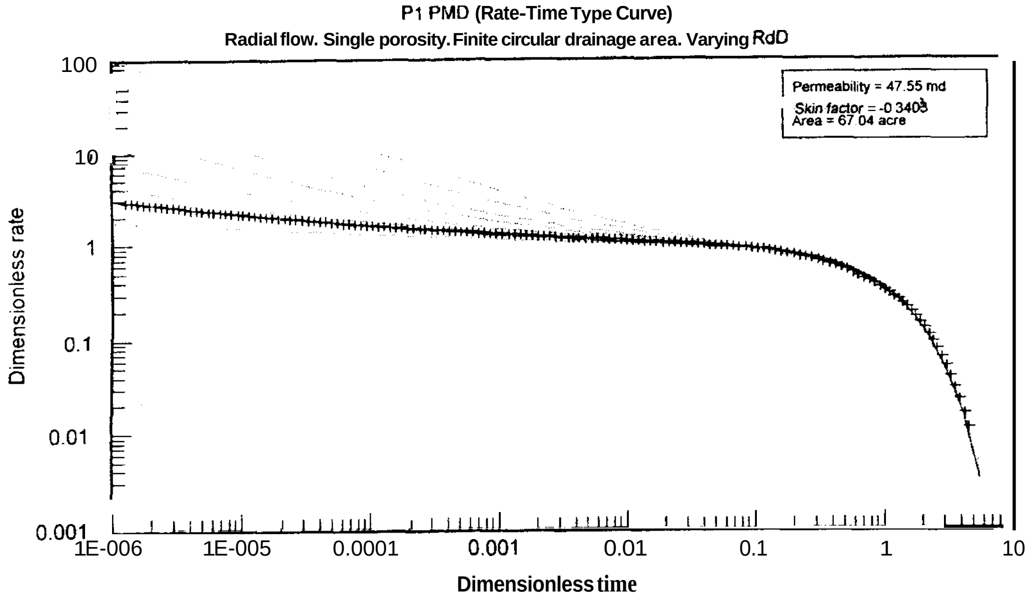


Figure 15 - Promat™ Analysis of the Drawdown Data from a Two-Layer Model
Varying layer drainage area *A=* 130/26 acres, $C_D = 50$, $s = 0$

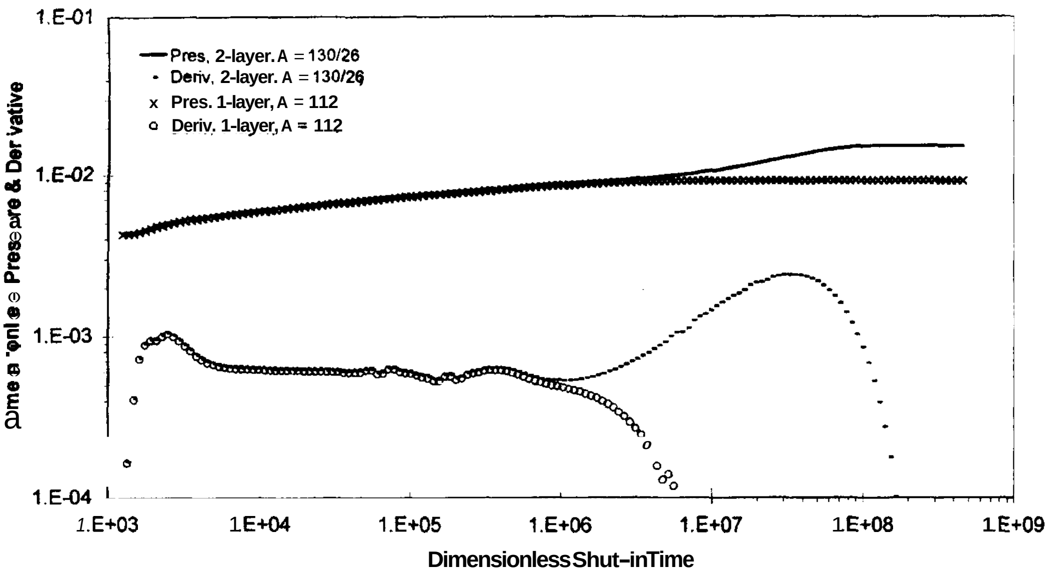


Figure 16 - Single-Layer Analysis of the Buildup Data from a Two-Layer Model
Varying layer drainage area *A=* 130/26 acres, $C_D = 50$, $s = 0$

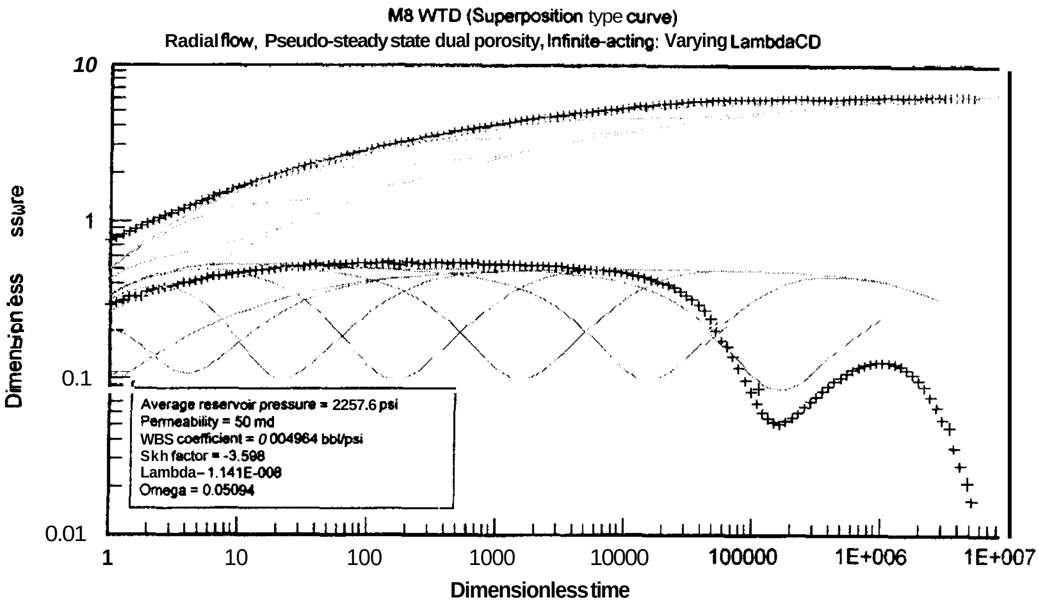


Figure 17 - Welltest™ - Pseudo-Steady State Dual Porosity - Buildup Analysis
Two-Layer model with varying layer skin $s = 0/-3$, $C_D = 50$,

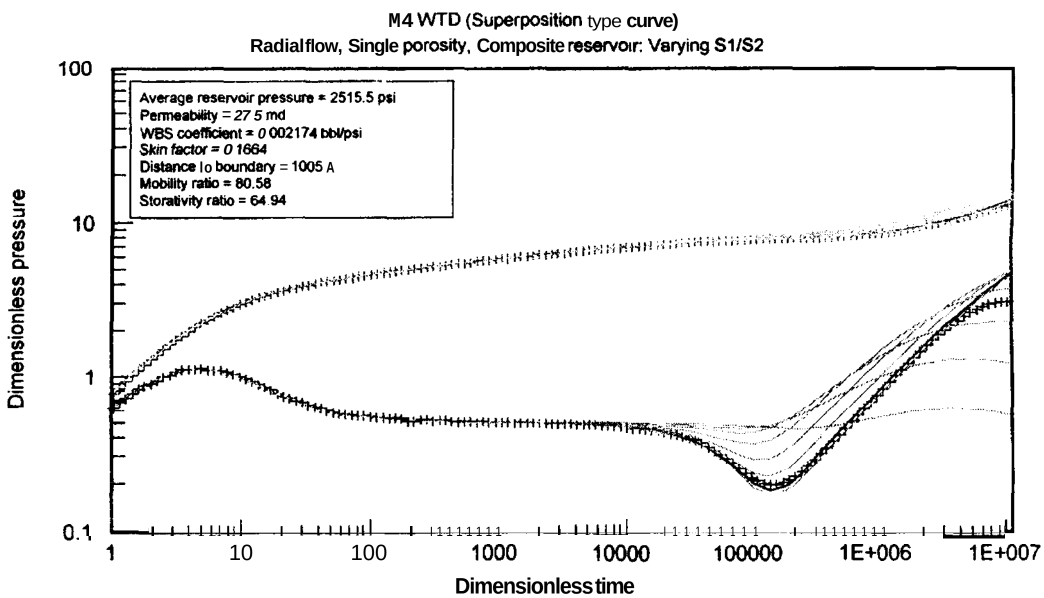


Figure 18 - Welltest™ - Composite Single Porosity - Buildup Analysis
Two-layer model with varying layer permeability $k = 50/5$, $C_D = 50, s = 0$

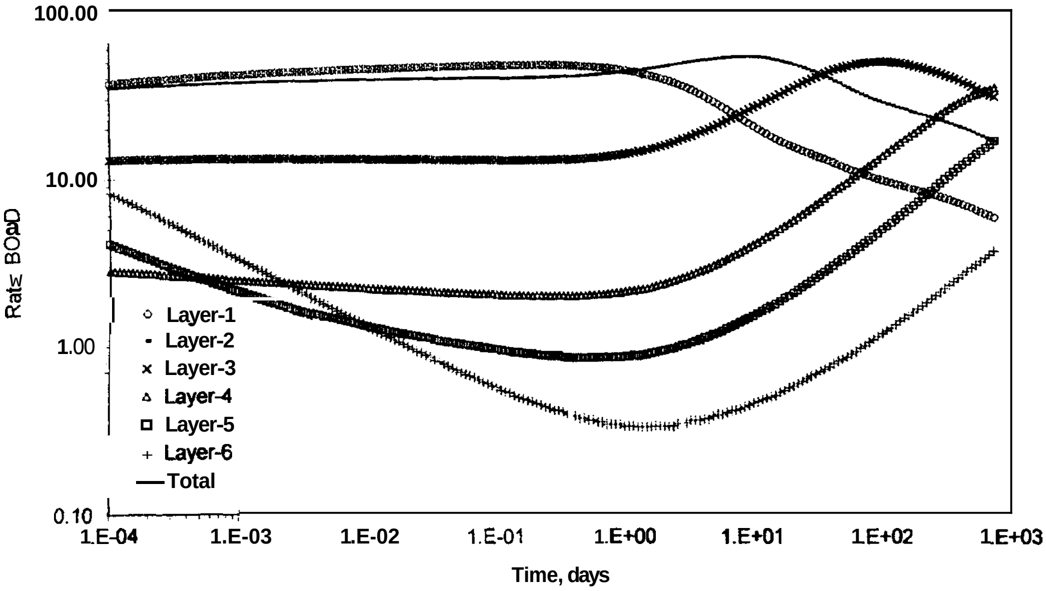


Figure 19 - Layer Rate History During Drawdown Test
Model Is, $C_D = 0$, $s \neq 0$

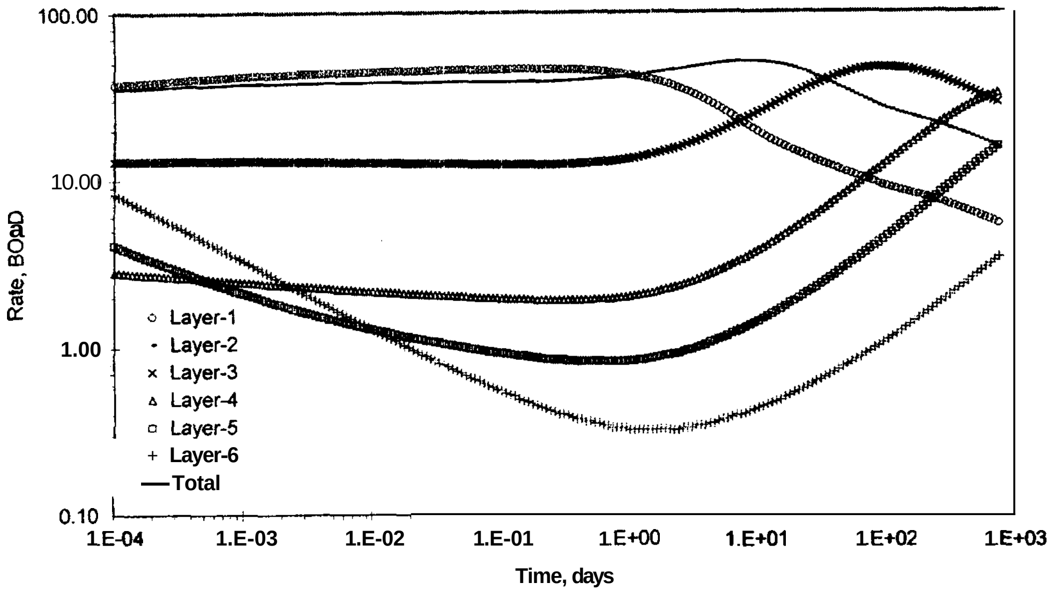


Figure 20 - Layer Rate History During Drawdown Test
Model 2s, $C_D = 0$, $s \neq 0$

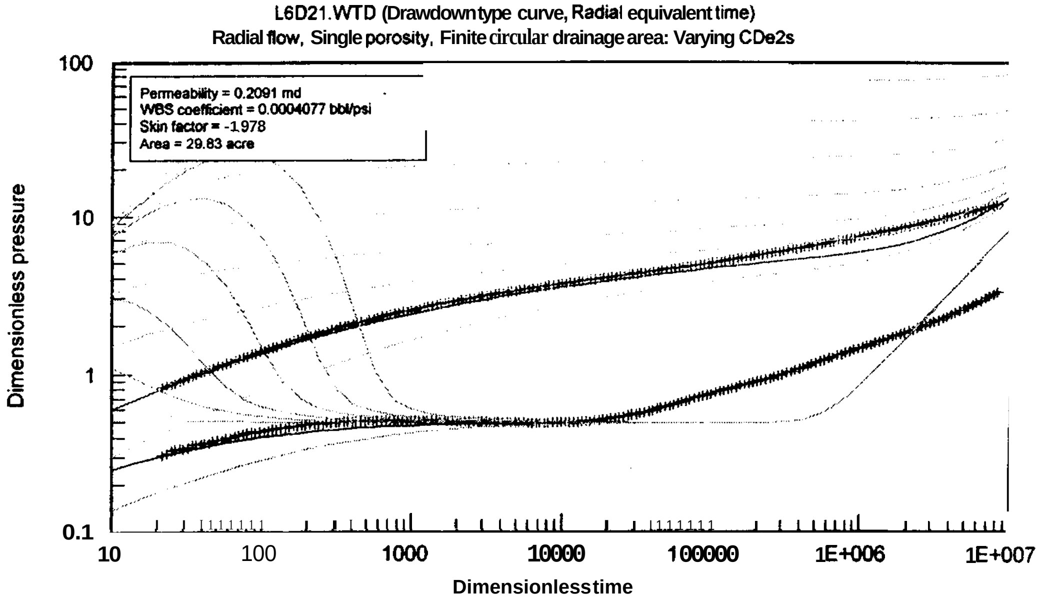


Figure 21 - Welltest™ Analysis of the Drawdown from a Six-Layer Model
Model 2s, $C_D = 0$, s # 0

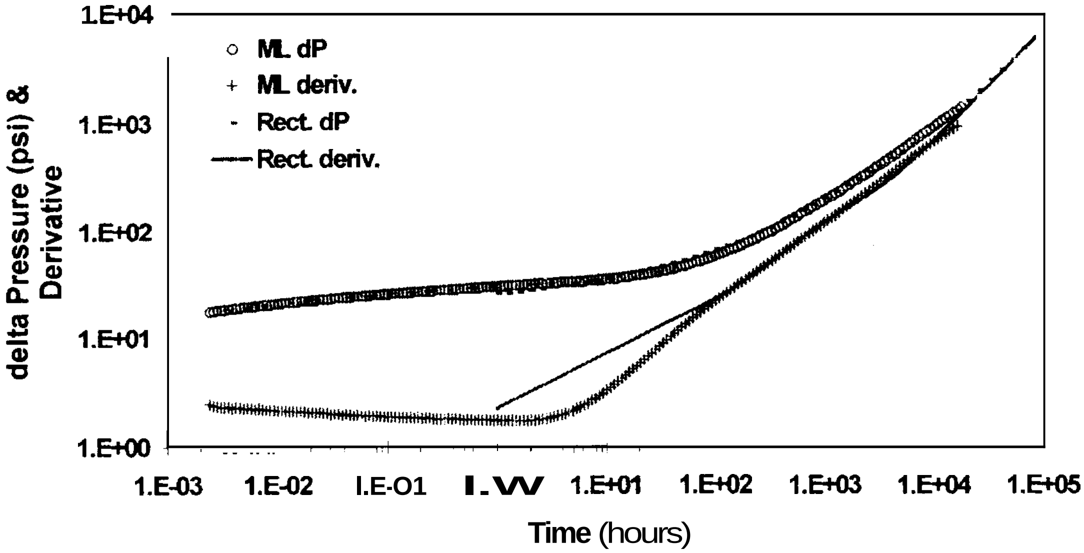


Figure 22 - Six-Layer (ML) vs. Saphir™ Rectangular Model
Model Is, drawdown analysis $C_D = 0$, $s \neq 0$

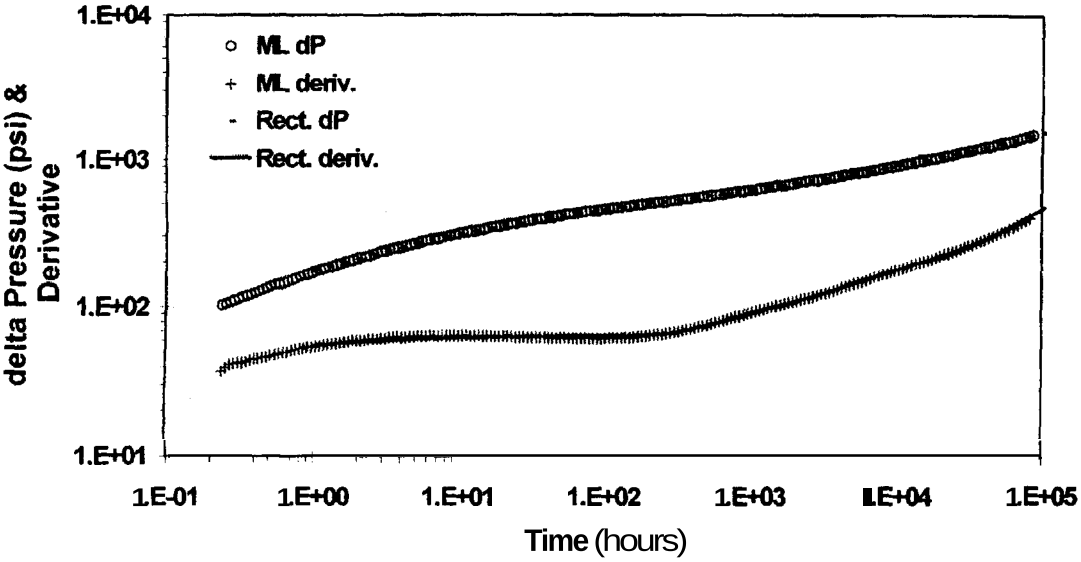


Figure 23 - Six-Layer (ML) vs. Saphir™ Rectangular Model
Model 2s, drawdown analysis $C_D = 0$, $s \neq 0$

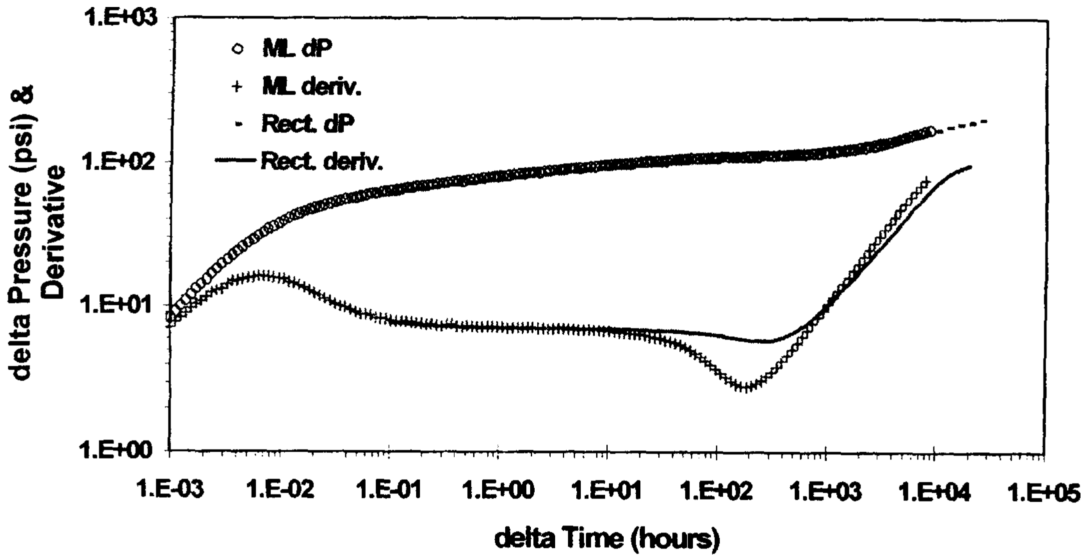


Figure 24 - Six-Layer (ML) vs. Saphir™ Rectangular Model
Buildup analysis. Varying layer permeability, k = 50/5 md

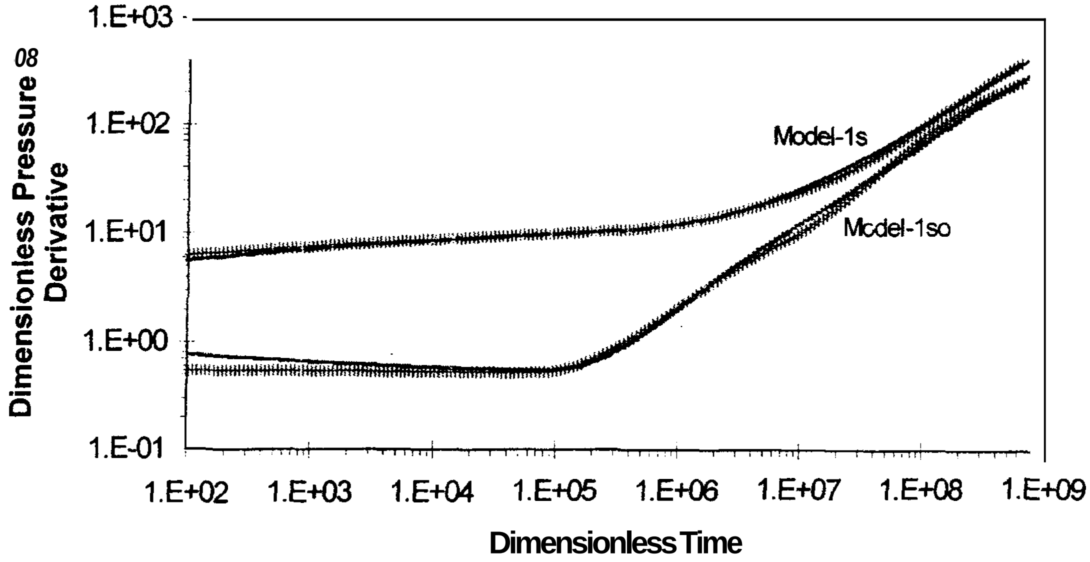


Figure 25 - Six-Layer Model 1s vs. Three-Layer Model 1so
Constant-rate drawdown case with skin effects.

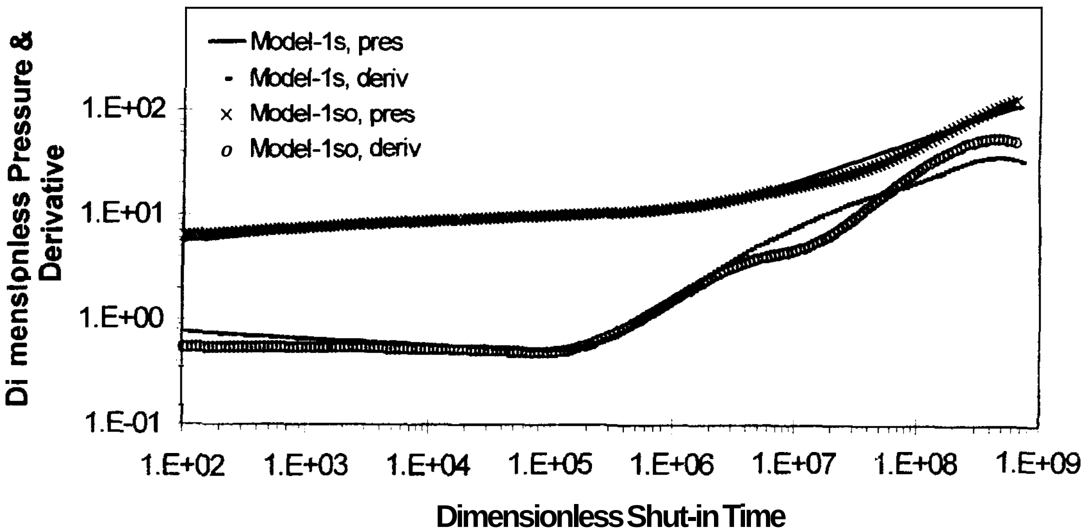


Figure 26 - Six-Layer Model 1s vs. Three-Layer Model 1so
Constant-rate buildup case with skin effects.

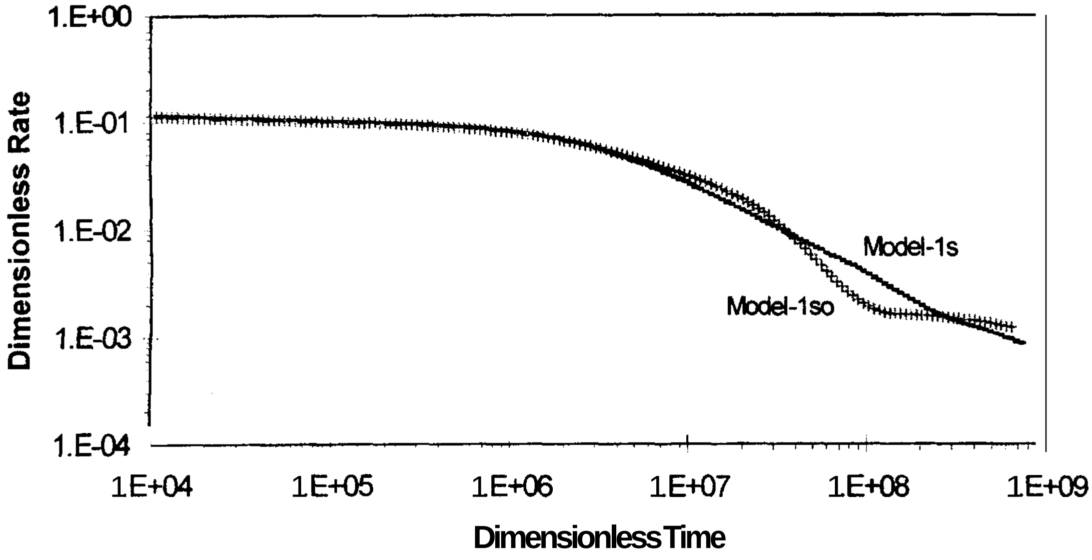


Figure 27 - Six-Layer Model Is vs. Three-Layer Model 1so
Constant-pressure drawdown case with skin effects.

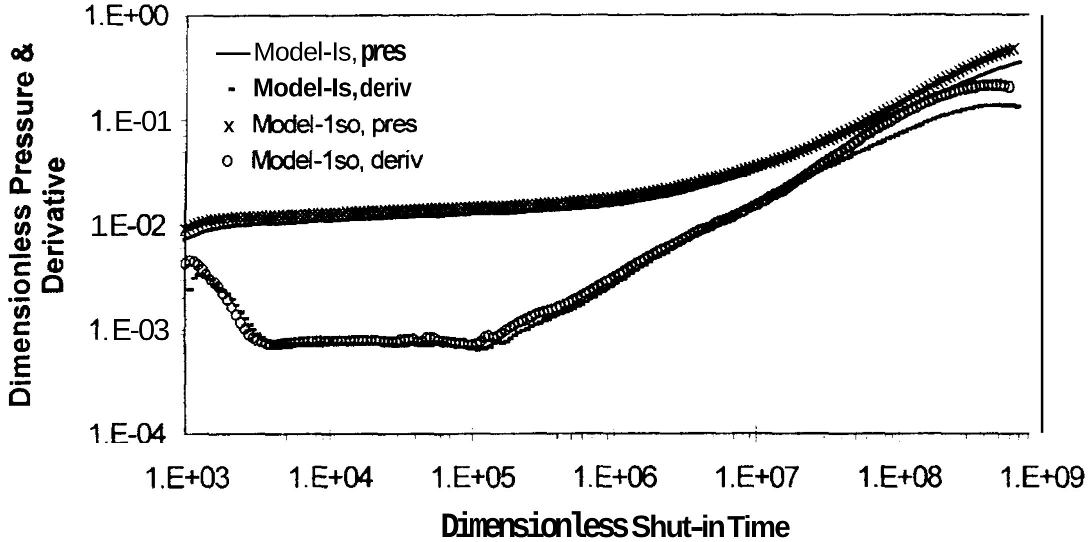


Figure 28 - Six-Layer Model 1s vs. Three-Layer Model 1so
Constant-pressure buildup case with skin effects.

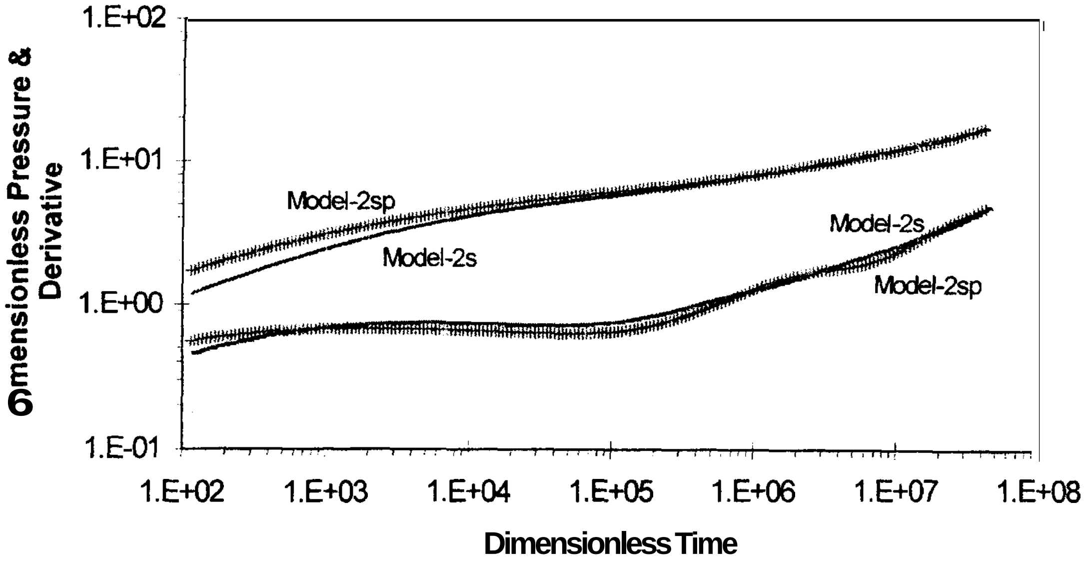


Figure 29 - Six-Layer Model 2s vs. Three-Layer Model 2sp
Constant-rate drawdown case with skin effects.

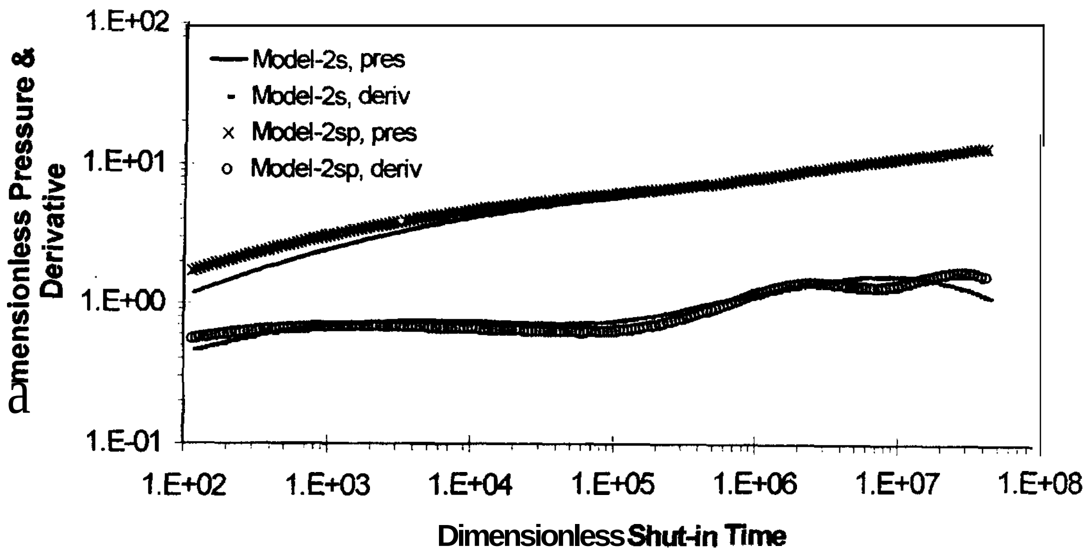


Figure 30 - Six-Layer Model 2s vs. Three-Layer Model 2sp
Constant-pressure buildup case with skin effects.

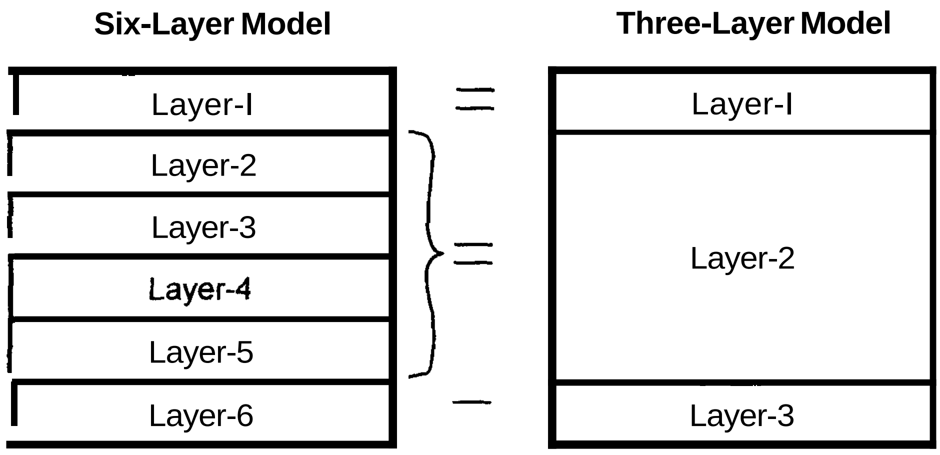


Figure 31 - Layer Property Assignments: Six-Layer vs. Three-Layer Model

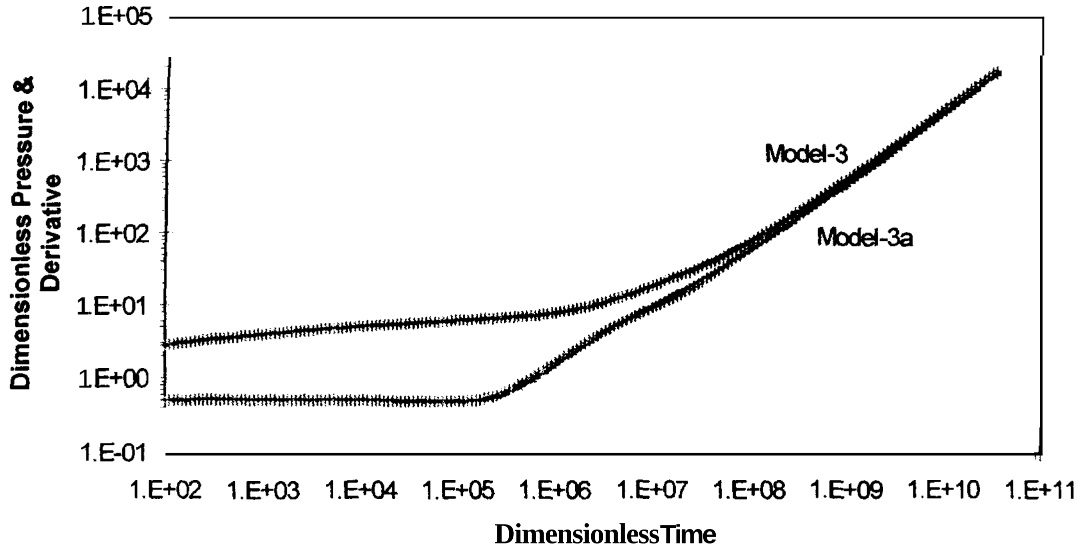


Figure 32 - Six-Layer Model 3 vs. Three-Layer Model 3a
Constant-rate drawdown case with varying layer permeability.

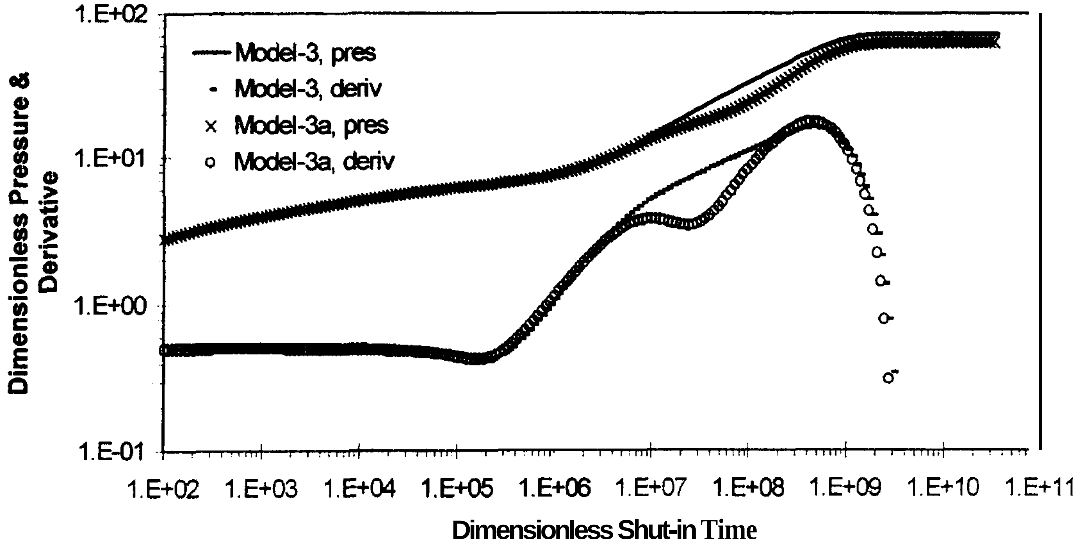


Figure 33 - Six-Layer Model 3 vs. Three-Layer Model 3a
Constant-rate buildup case with varying layer permeability.

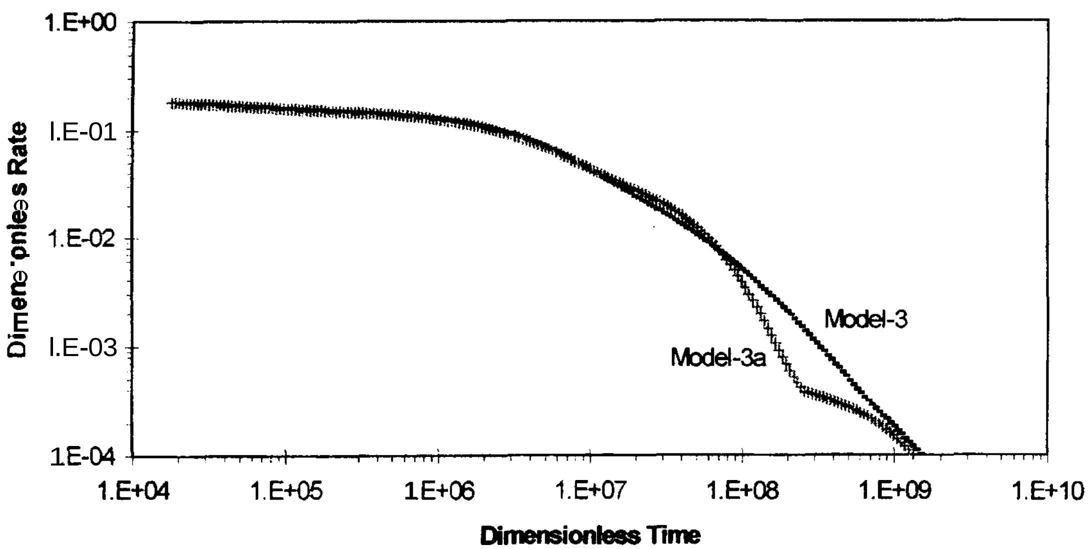


Figure **34** - Six-Layer Model 3 vs. Three-Layer Model 3a
Constant-pressure drawdown case with varying layer permeability.

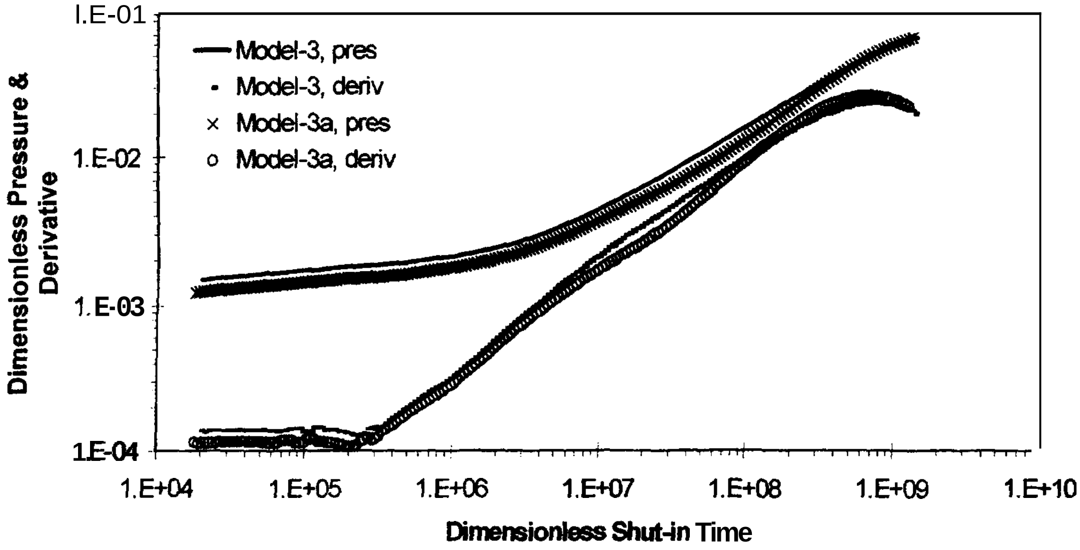


Figure 35 - Six-Layer Mode 3 vs. Three-Layer Model 3a
Constant-pressure buildup case with vary layer permeability.

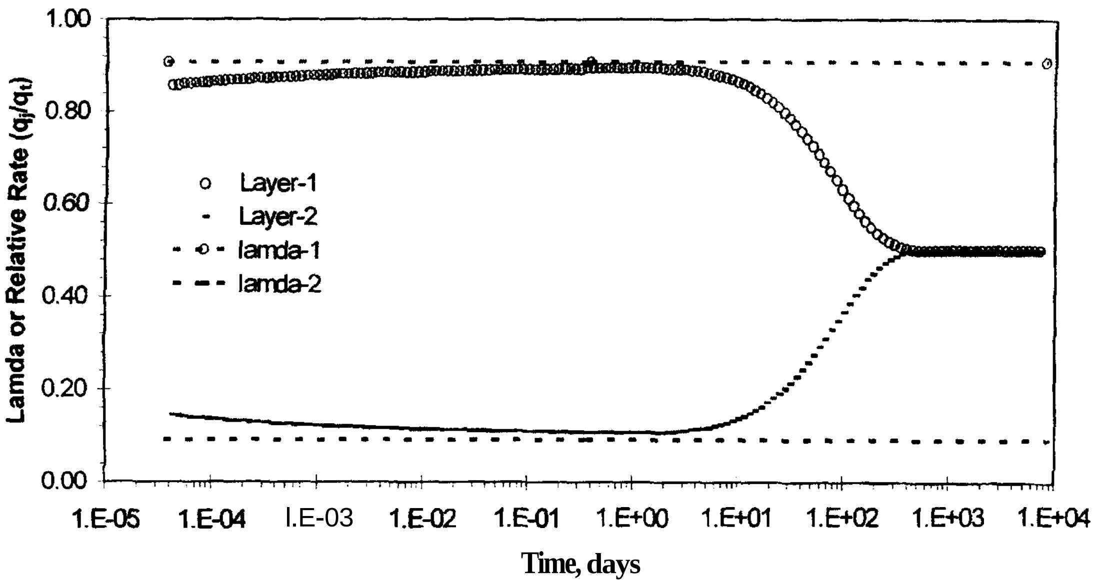


Figure 36 - Two-Layer Model: Relative Rate vs. λ
Constant-rate drawdown case with varying layer permeability.

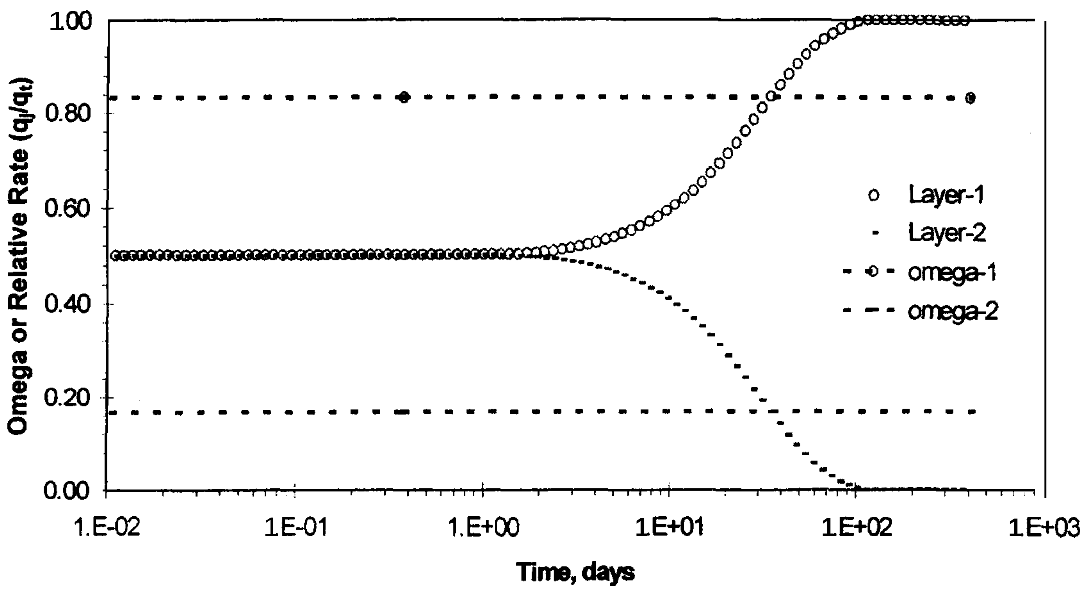


Figure 37 - Two-Layer Model: Relative Rate vs. Ω
Constant-pressure drawdown case with varying layer drainage area.

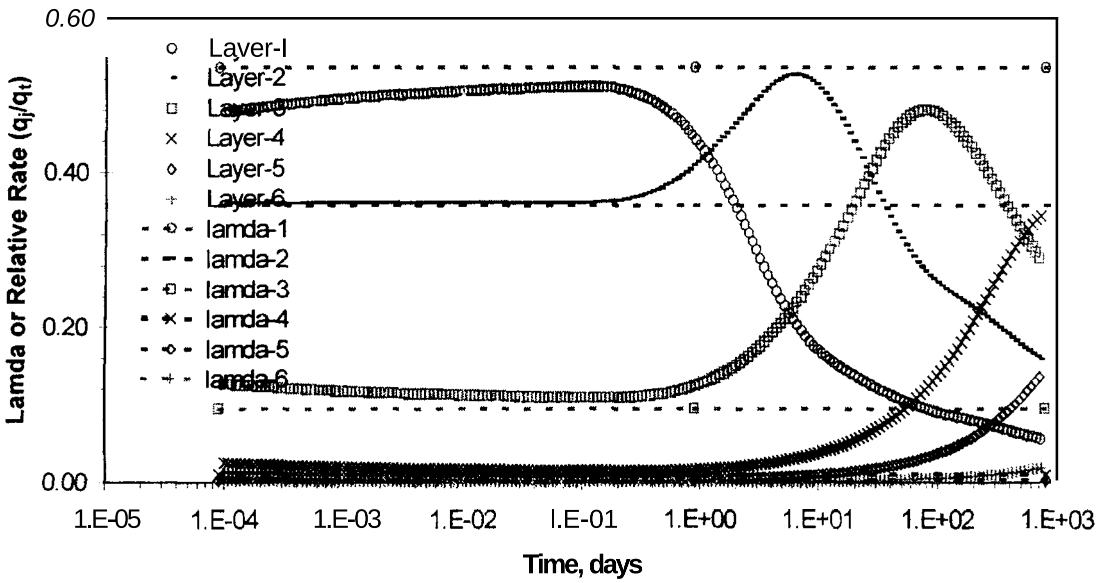


Figure 38 - Six-Layer Model 1: Relative Rate vs. λ
Constant-rate drawdown case with varying layer properties.

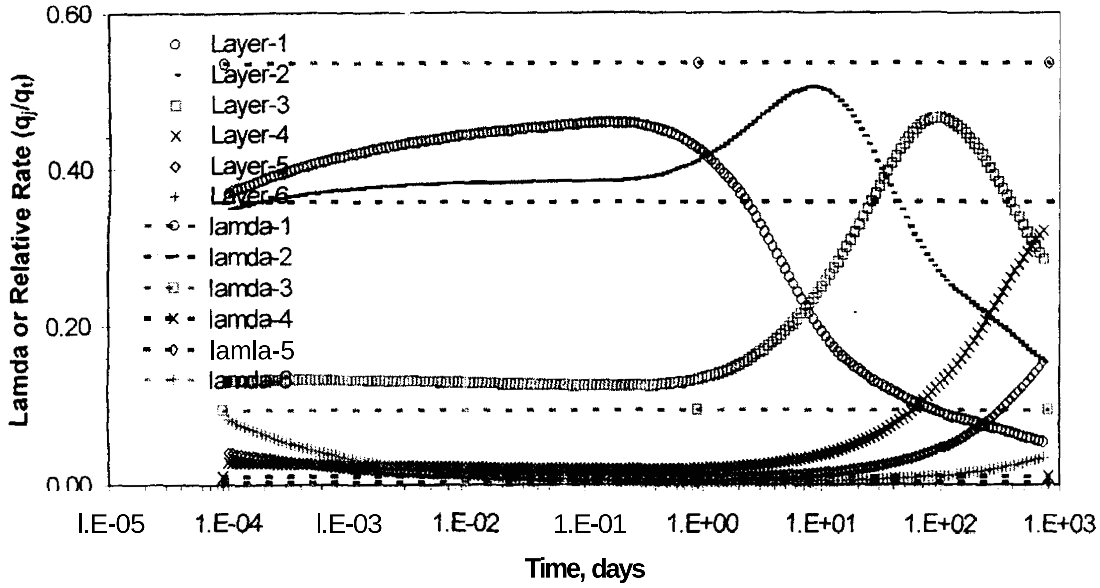


Figure 39 - Six-Layer Model 1s: Relative Rate vs. λ
Constant-rate drawdown case with varying layer properties and **skin**