PREDICTION OF THE LOCATION AND MOVEMENT OF FLUID INTERFACES IN A FRACTURE

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INTRODUCTION

Recent developments in stimulation practice have resulted in two or more fluid systems being used continuously during a treatment. Fracture acidizing, for one, has involved the use of a highly efficient pad fluid preceding the acid into the fracture. In the use of the crosslinked gels, it is common practice to precede the gel-sand slurry into the fracture with a pad fluid of an entirely different character.

Where these techniques are used, it is useful to know the rate at which one fluid is displacing another and the location of the interface between these two fluids. The importance of this information is as follows:

1. In fracture acidizing, the location of the fluid interface indicates the farthest point that the acid has reached into the fracture.
2. In using the crosslinked gels with proppants, the location of the interface pinpoints the leading edge of sand-laden fluid and hence, propped fracture length. It can also provide valuable information concerning sand scheduling.
3. Where overflush volumes are used behind high strength acid treatments, this information is valuable in sizing the volume to be used as overflush.

The gross, averaging techniques used in the past have not allowed the location of the interface between two fluids or stages of the same fluid. In order to develop a method to accurately predict the interface location it was necessary to go back to the basic fluid mechanics of fracturing. In this analysis a greatly simplified expression was developed which predicts fracture area within 10% (or less) of the Howard, Fast, and Carter equation.[1] This simple equation has greatly simplified fracturing and fluid displacement analysis without introducing significant errors in accuracy.

SIZING PAD VOLUMES

The selection of pad volume to precede a given stimulation volume will use the following criteria. The pad volume will be selected so that the pad will be completely absorbed by the leak-off when the selected stimulation volume has been pumped.

The following formula can be used to compute the pad volume:

$$V'_p = \left(\frac{W' - W}{W}\right) V_s \tag{1}$$

In addition:

$$V'_p = V_p \quad \text{when} \quad C_s = C_p$$

Where:

V_s = stimulation fluid volume, gal.
V'_p = pseudo pad fluid volume, gal.
V_p = actual pad fluid volume, gal.
W = average pumping width, in.

$$W' = W + 0.787 \frac{V_{sp}}{A_f} \tag{2}$$

V_{sp} = spurt volume, cm^3
A_f = area of the filter medium, cm^2
C_s = stimulation fluid leak-off coefficient, $\frac{ft}{\sqrt{min}}$
C_p = pad fluid leak-off coefficient, $\frac{ft}{\sqrt{min}}$

Note that the actual pad volume, V_p, can be arrived at directly, by Eq. (1), only when the pad and stimulation leak-off coefficients are identical.

Where the leak-off coefficients of the pad and stimulation fluid are not the same, then a different approach must be used. This problem can be solved either graphically or mathematically.

Graphical Solution

In this approach, a simplified form of the Howard, Fast, and Carter[1] fracture area equation will be used:

$$A = \frac{67.32\, Q t}{W' + 34\, C \sqrt{t}} \qquad (3)$$

Where:

A = fracture area, ft^2
Q = injection rate, BPM
C = combined leak-off coefficient, $\frac{ft}{\sqrt{min}}$

W′ = pseudo width, preveiously defined.
t = pumping time, min

The following procedure can now be used to find the required pad volume:

1. Compute V'_p using Eq. (1).
2. Construct a curve of volume versus area for both the pad fluid and stimulation fluid using Eq. (3) and the correct leak-off coefficients (C_s and C_p). See Fig. 1.
3. Enter V'_p on the volume axis and move vertically to the stimulation fluid curve. This would be the area created by the pad fluid if the leak-off coefficients were identical.
4. Move horizontally until the pad fluid curve is intercepted and note the volume of pad required. This is the pad volume required for total displacement by the selected stimulation volume. The volume of pad found will create the same fracture area as V'_p would create if the leak-off coefficients were identical.

To find the fracture area created by both the pad fluid volume and the stimulation fluid volume:

1. Find V′ where:

$$V' = V'_p + V_s \qquad (4)$$

2. Find the area created by V′ on the chart.

Note that this technique can be used to find the area created by any pad volume and stimulation volume selected as follows:

1. Find the area created by the selected pad volume.

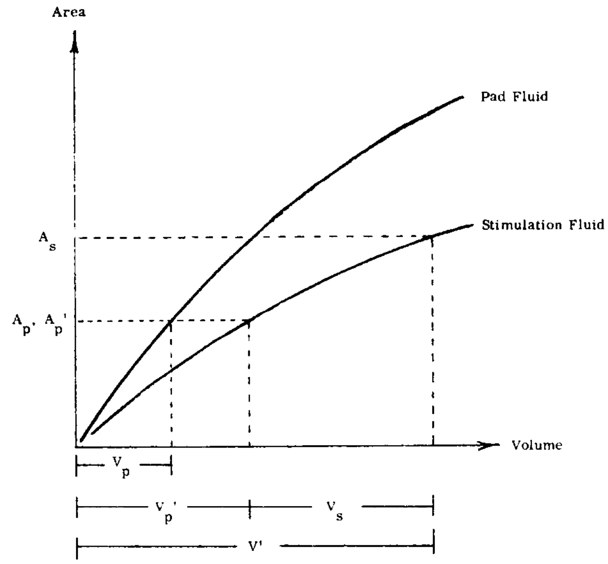

FIG. 1—VOLUME VS AREA

2. Using Fig. 1, find the volume of stimulation fluid that would be required to create an equivalent area.
3. Add the volume found in (2) above to the selected stimulation volume (V_s) and look up the area that would be created by this combined volume.

Mathematical Solution

1. Find V'_p from Formula 1.
2. Now find pseudo-time t'_p:

$$t'_p = \frac{V'_p}{Q} \qquad (5)$$

3. Calculate the fracture area created by V'_p from Eq. (3).

$$A = \frac{67.32\, Q t'_p}{W' + 34\, C_s \sqrt{t'_p}}$$

4. Now the actual pad volume can be calculated from:

$$V_p = 42Q \left\{ 0.252\, \frac{C_p A}{Q} + \left[0.0632 \left(\frac{C_p A}{Q} \right)^2 + \frac{A W'}{67.5 Q} \right]^{1/2} \right\}^2 \qquad (6)$$

It should be remembered that Eqs. (1) and (3) are only valid for pads that are wall-building fluids and exhibit spurt loss. There are methods for handling pads without fluid loss agents; however, the solution is more complex and beyond the scope of this paper.

The basic technique does not depend on the use of Eq. (3). The usual Howard, Fast, and Carter equations[1] can be used in conjunction with Eqs. (1), (4), (5) and (6). It is practical for use with almost any conventional computer program that predicts fracture areas.

LOCATION OF FLUID INTERFACES

This section deals with locating the interface between the pad fluid and the stimulation fluid at any time after displacement starts until completion. In any instance where displacement is incomplete, the location of the interface marks the furthest advance of the stimulation fluid into the formation.

To find the location of the pad fluid-stimulation fluid interface at any time before displacement is complete, the following equations can be used:

$$L_t = \frac{33.72\ Qt'}{W'h + 34h\ C_s\sqrt{t'}} \qquad (7)$$

Where:

- L_t = total fracture length, ft
- h = fracture height, ft
- t' = $\theta + t'_p$, min
- θ = stimulation fluid pumping time at which the interface is desired, min

In addition:

$$L_i = \frac{33.72\ Q\theta}{Wh + 34h\ C_s\ \theta/\sqrt{t'}} \qquad (8)$$

Where:

- L_i = distance from the wellbore to the pad fluid-stimulation fluid interface, ft

Figure 2 shows the movement of the interface during pad displacement for a typical case. It also shows the growth of the leading edge of the fracture.

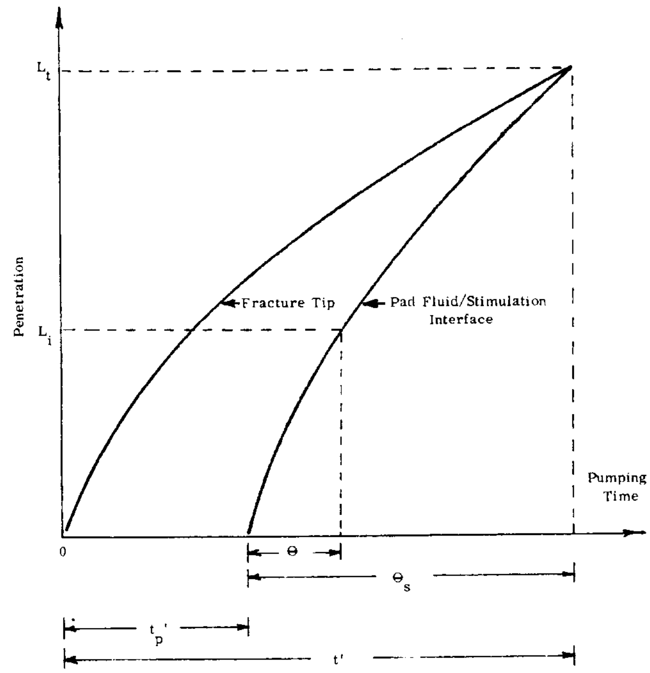

FIG. 2—MOVEMENT OF INTERFACE AND FRACTURE GROWTH DURING PAD DISPLACEMENT

For the cases presented in Figs. 1 and 2 the pad fluid is more efficient than the stimulation fluid ($C_p < C_s$). This is representative of most acid fracturing design problems. The technique would work perfectly well where C_s is smaller than C_p.

Appendices A, B, and C show the analytical development of the equations used in the text. Appendix D contains two example problems that illustrate the use of the techniques presented.

REFERENCES

1. Howard, G.C. and Fast, C.R.: Optimum Fluid Characteristics for Fracture Extension, *Drlg. and Prod. Prac.*, API 1957, p. 261.
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APPENDIX A

SIMPLIFIED FRACTURE AREA EQUATION

An expression for the area of a hydraulically-induced fracture is developed by applying the conservation of mass technique to the fracture control volume as illustrated in Fig. A1.

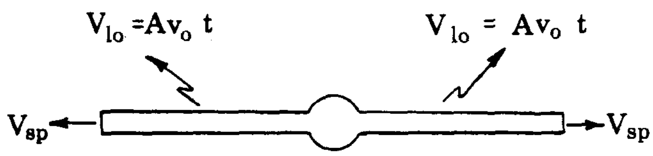

FIG. A1—FRACTURE CONTROL VOLUME

A mass balance of this system shows that the total fracture volume is equal to the volume of fluid used to generate the area less the volume lost due to leak-off and spurt:

$$U_t = Q_t - 2v_oAt - 2V_{sp} \tag{A1}$$

Noting that total fracture volume can be expressed as the product of fracture area and fracture width:

$$U_t = WA \tag{A2}$$

and that the spurt volume, V_{sp}, can be accounted for by using an effective fracture width defined as:

$$W'A = WA + 2V_{sp} \tag{A3}$$

Substituting Eqs. (A2) and (A3) into Eq. (A1) and rearranging in terms of fracture area yields:

$$A = \frac{Qt}{W' + 2v_ot} \tag{A4}$$

Now it remains to develop an expression which relates the leak-off velocity to time. The instantaneous leak-off velocity, v_{oi}, at any given time is given by:

$$v_{oi} = \frac{C}{\sqrt{t}} \tag{A5}$$

Noting that a good approximation of the leak-off velocity with time is obtained by using a time averaged leak-off velocity, v_o:

$$v_o = \frac{C}{\sqrt{(t/2)}} \tag{A6}$$

where: C = combined fluid leak-off coefficient
t = total pumping time

the following greatly simplified area equation is obtained:

$$A = \frac{Qt}{W' + \sqrt{8C}\ \sqrt{t}} \tag{A7}$$

This equation, which provides a closed form expression for calculating fracture area, is an extremely powerful tool for use in the calculation of fluid displacement. Comparing the results of Eq. (A7) with the more complex Howard, Fast, and Carter expression, it is found to give remarkably accurate results. Fracturing efficiencies calculated using Eq. (A7) and the Howard, Fast, and Carter equation are compared in Table (A1).

TABLE A1—COMPARISON OF FRACTURE EFFICIENCY EXPRESSIONS

X*	Efficiency from Eq. (A7)	Efficiency from Howard, Fast, and Carter
0.0	1.0000	1.0000
0.2	0.8620	0.8680
0.4	0.7581	0.7638
0.6	0.6763	0.6800
0.8	0.6104	0.6122
1.0	0.5562	0.5560
2.0	0.3852	0.3780
4.0	0.2386	0.2282
6.0	0.1728	0.1630
10.0	0.1114	0.1035

* "X" function is defined as:

$$X = \frac{2C\sqrt{\pi t}}{W'}$$

APPENDIX B

FLUID DISPLACEMENT THEORY

In developing this theory, it will be assumed that both the pad and stimulation fluids have the same leak-off characteristics for clarity. It should be noted that it will work equally well for fluids having different coefficients by using appropriate "C" factors and pseudo times.

The time required to pump a given volume of pad fluid can be found as follows:

$$t_p = \frac{V_p}{Q} \tag{B1}$$

The resulting fracture volume created is found as:

$$U_p = Qt_p \text{ Eff}(t_p) \tag{B2}$$

$$U_p = Qt_p \left[\frac{W}{W' + \sqrt{8}\, C \sqrt{t_p}} \right] \tag{B3}$$

At this point, an expression must be developed for the efficiency of the stimulation fluid when it is preceded by another fluid (pad).

The time required to pump a given volume of stimulation fluid can be found as follows:

$$\Theta_s = \frac{V_s}{Q} \tag{B4}$$

Note: Θ is used to represent stimulation fluid pumping time as opposed to total time since the start of the job.

The fracture volume occupied by the stimulation fluid at any time. t, can be found as:

$$U_s = Q\,(t-t_p) \text{ Eff}(t-t_p) \tag{B5}$$

where: $t_p \leq t \leq t_p + \Theta_s$

The expression for Eff $(t-t_p)$ can be developed using the method outlined in Appendix A as follows:

$$U_s = Q\,(t-t_p) - 2\,v_o\,A\,(t-t_p) \tag{B6}$$

and for this case:

$$v_o = \frac{C}{\sqrt{t/2}} \tag{B7}$$

$$U_s = A_s W \tag{B8}$$

Combining Eqs. (B6), (B7), and (B8) yields:

$$A_s = \frac{Q\,(t-t_p)}{W + \sqrt{8}\; C\;(t-t_p)/\sqrt{t}} \tag{B9}$$

which can be rearranged in terms of the stimulation fluid efficiency to give:

$$E\,(t-t_p) = \frac{W}{W + \sqrt{8}\,C\,(t-t_p)/\sqrt{t}} \tag{B10}$$

Since the area is equal to the fracture height times penetration, the stimulation fluid/pad fluid interface may be located at any time,t, by using Eq. (B9).

The special case may now be considered where the pad fluid in the fracture has been completely absorbed by leak-off and spurt precisely at the end of the job.

The fluid volume balance in the fracture may be written as:

$$U_t = U_s + U_p \tag{B11}$$

Noting that $t_s = t_p + \Theta_s$

and $U_t = Qt_s \dfrac{W}{W' + \sqrt{8}\,C\sqrt{t_s}}$

and the fact that U_p approaches zero as t approaches t_s, Eq. (B11) may be rewritten as:

$$Qt_s \left[\frac{W}{W' + \sqrt{8}\,C\;\sqrt{t_s}} \right] = Q\Theta_s \left[\frac{W}{W + \sqrt{8}\;C\Theta_s/\sqrt{t_s}} \right] \tag{B12}$$

Solving Eq. (B12) for t_p in terms of Θ_s yields:

$$t_p = \frac{W' - W}{W}\,\Theta_s \tag{B13}$$

or assuming a constant injection rate:

$$V_p = \frac{W' - W}{W}\,V_s \tag{B14}$$

Eq. (B14) can now be used to determine how much pad fluid is required for a given stimulation fluid volume.

APPENDIX C

PREDICTING FRACTURE GROWTH AND FLUID DISPLACEMENT WITH TWO-COMPONENT FLUID SYSTEMS WHICH EXHIBIT DIFFERENT FLUID LOSS CHARACTERISTICS

In Appendix B, an analytical method of predicting the displacement of one fluid by another of like fracturing characteristics is developed. However, most often we are faced with two-component fluid systems which have considerably different fracturing characteristics. The method described below provides a straightforward way to overcome this problem.

The main problem encountered when the pad fluid and stimulation fluid have different leak-off characteristics involves the definition of the pumping times to be used in the efficiency equations.

Referring to Fig. 1 in the text, it can be seen that there is a difference in the pumping time required to generate a given area depending on which loss coefficient is used.

To account for this difference, a pseudo time is defined.

Expressed analytically, the fracture area created by the pad fluid is:

$$A_p = Qt_p\left[\frac{1}{W' + \sqrt{8}\,C_p\sqrt{t_p}}\right] \tag{C1}$$

And the equivalent area created by the pad fluid if it had leak-off characteristics similar to the stimulation fluid is:

$$A'_p = Qt'_p\left[\frac{1}{W' + \sqrt{8}\,C_s\sqrt{t'_p}}\right] \tag{C2}$$

Since the areas given by Eqs. (C1) and (C2) must be equal, the relationship between actual pad pumping time and pseudo pad pumping time can be established.

Setting the right side of Eq. (C1) equal to A'_p and introducing the grouping expressions:

$$B = \frac{\sqrt{8}\,C_pA'_p}{Q} \tag{C3}$$

$$Z = \frac{A'_p\,W'}{Q} \tag{C4}$$

Eq. (C1) can be rearranged to:

$$t_p - B\sqrt{t_p} - Z = 0 \tag{C5}$$

Eq. (C5) is a quadratic in the $\sqrt{t_p}$ and it can be shown that only one root (the maximum) is applicable in this case. The solution for t_p becomes:

$$t_p = \left\{\frac{B}{2} + \frac{1}{2}\left[B^2 + 4Z\right]^{.5}\right\}^2 \tag{C6}$$

Reintroducing Eqs. (C3) and (C4) into Eq. (C6) yields:

$$t_p = \left\{\frac{\sqrt{2}\,C_pA'_p}{Q} + \left[2\left(\frac{C_pA'_p}{Q}\right)^2 + \frac{W'A'_p}{Q}\right]^{.5}\right\}^2 \tag{C7}$$

or in terms of the pad fluid volume:

$$V_p = Q\left\{\frac{\sqrt{2}\,C_pA'_p}{Q} + \left[2\left(\frac{C_pA'_p}{Q}\right)^2 + \frac{W'A'_p}{Q}\right]^{.5}\right\}^2 \tag{C8}$$

NOMENCLATURE FOR APPENDICES A, B, C*

A - fracture area, ft^2
A_p - fracture area exposed to pad fluid, ft^2
A'_p - defined by Eq. (C2), ft^2
A_s - fracture area exposed to stimulation, ft^2
B - grouping term defined by Eq. (C3)
C - fluid loss coefficient, $ft/\sqrt{min}$
C_p - pad fluid loss coefficient, $ft/\sqrt{min}$
C_s - stimulation fluid loss coefficient, $ft/\sqrt{min}$
H - fracture height, ft
L - fracture length, ft
Q - pumping rate, ft^3/min
t - pumping time, min
t_p - pad fluid pumping time, min
t'_p - pseudo pad fluid pumping time, min
t_s - total job time ($t_p + \Theta_s$), min
U_p - fracture volume created by pad fluid, ft^3
U_s - fracture volume created by stimulation fluid, ft^3
U_t - fracture volume, ft^3
V' - pseudo fluid volume, ft^3
V_{lo} - fluid lost due to leak-off, ft^3
v_o - fluid leak-off velocity, ft/min
v_{oi} - instantaneous leak-off velocity, ft/min
V_p - actual pad fluid volume, ft^3
V_s - actual stimulation fluid volume, ft^3
V_{sp} - spurt volume, cc
W - average fracture width, ft
W' - fracture width accounting for spurt, ft
Z - grouping term defined by Eq. (C4)
Θ_s - stimulation fluid pumping time, min

* Nomenclature for the text is contained within the text.

APPENDIX D
SAMPLE PROBLEM CALCULATIONS

Pad fluid and stimulation fluid having the same fluid loss characteristics:

Assume:

V_s = 10,000 gal.
W = 0.15 in.
A_f = 22.8 cm^2
V_{sp} = 3 cc
C_p = .003 ft/$\sqrt{min}$
C_s = .003 ft/$\sqrt{min}$

Find the volume of pad fluid required so that it will be completely absorbed at the end of the job.

(1) Compute W′ from Eq. (A3):

$$W' = W + \frac{0.787\ V_{sp}}{A_f} = 0.2536 \text{ in.}$$

(2) Compute V_p from Eq. (B14):

$$V_p = \frac{0.2536 - 0.15}{0.15}\ (10{,}000) = 6900 \text{ gal.}$$

Pad fluid and stimulation fluid having different fluid loss characteristics:

Assume:

V_s = 10,000 gal.
W = 0.15 in.
A_f = 22.8 cm^2
V_{sp} = 3 cc
C_p = .003 ft/$\sqrt{min}$
C_s = .007 ft/$\sqrt{min}$
Q = 10 BPM

Find the volume of pad fluid required so that it will be completely absorbed at the end of the job.

(1) Compute W′ from Eq. (A3):

$$W' = W + \frac{0.787\ V_{sp}}{A_f} = 0.2536 \text{ in.}$$

(2) Compute V'_p from Eq. (B14):

$$V'_p = \frac{0.2536 - 0.15}{0.15}\ (10{,}000) = 6900 \text{ gal.}$$

(3) Calculate t'_p:

$$t'_p = \frac{V'_p}{Q} = \frac{6900}{42(10)} = 16.4 \text{ min}$$

(4) Calculate the fracture area created by V'_p from Eq. (C2):

$$A'_p = \frac{Qt'_p}{W' + \sqrt{8}\ C_s\ \sqrt{t'_p}}$$

$$= \frac{5.62\ (10)\ (16.4)}{\frac{0.2536}{12} + \sqrt{8}\ (.007)\ \sqrt{16.4}}$$

$$= 9094 \text{ ft}^2$$

(5) Calculate the actual pad fluid pumping time using Eq. (C7):

$$t_p = \left\{ \frac{\sqrt{2}\,C_p A'_p}{Q} + \left[2 \left(\frac{C_p A'_p}{Q} \right)^2 + \frac{A_p W'}{Q} \right]^{.5} \right\}^2 = \left\{ \frac{\sqrt{2}\,(.003)\ (9094)}{5.62\ (10)} + \left[2 \left(\frac{.003\ (9094)}{5.62\ (10)} \right)^2 + \frac{9094\ (0.2536)}{12(5.62)\ (10)} \right]^{.5} \right\}^2 = 7.08 \text{ min}$$

(6) Compute actual pad fluid volume:

$$V_p = Qt_p$$

$$= 42\ (10)\ (7.08) = 2974 \text{ gal.}$$