PLUNGER LEAKAGE AND VISCOUS DRAG FOR ROD-DRAWN OIL WELL PUMPS

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SUMMARY

A new method for calculating plunger pump leakage in rod pumped wells is introduced. This method involves calculating a velocity profile for an annulus with the inner wall moving parallel to the outer wall. An average velocity is determined for the annular fluid flow, which in turn is used to calculate the fluid slippage. Eccentricity is also considered in the slippage calculation method. The results are evaluated against the historical field data and compare favorably to recent testing for smaller plunger clearances. Work remains to be done at larger clearances. A formula for calculating viscous plunger drag is also introduced.

INTRODUCTION

Many pump slippage formulas have been developed over the years. Most of these formulas over predict the pump slippage and a more accurate formula is needed. Slippage calculations are important to determine the amount of fluid required to lubricate the plunger-barrel while pumping with a plunger pump. A slippage formula is also required to help calculate the pump volumetric efficiency and plunger pump production in order to maximize production.

IMPORTANCE OF PROPER PUMP SLIPPAGE

Proper pump slippage is a balance between proper pump lubrication and pump volumetric efficiency. One definition of volumetric efficiency can be defined as the actual fluid displaced per stroke divided by the calculated displacement of the pump. On the upstroke, there is a large pressure difference across the plunger, which forces fluid to leak downward between the barrel and plunger. Fluid leakage between the pump barrel and pump plunger is called pump slippage and is one of the factors that affects pump lubrication and efficiency. Proper pump lubrication is necessary to extend the life of the pump. The clearance must be large enough to allow an appropriate amount of slippage to sufficiently lubricate the plunger. Sand and other particles need *to* pass between the barrel and plunger, which could otherwise jam the pump plunger in the pump barrel or the particles need to be excluded from the plunger-barrel area. The typical amount of slippage for lubrication is usually quoted to be 2 to 5% of the total production. When a plunger sticks on the down stroke, it compresses the rods above the plunger pump, which shortens the downward travel of the plunger. When a plunger sticks on the upstroke, tension is created on the rods above the pump, which shortens the upward travel of the plunger. A shorter travel of the plunger in either the upstroke or down stroke causes a loss of production. Plunger sticking or galling of metal can also shorten the life of the rods and the pump itself. Galling of the metal is when the pump plunger and pump barrel rub together without sufficient lubrication causing the metal to wear and become rough. In order to predict pump performance and design for proper lubrication, pump slippage needs to be accurately predicted.

COMPARISON OF HISTORICAL SLIPPAGE FORMULAS AND DATA

A comparison of some of the historical formulas is presented here. Figure 1 is a graph of Robinson's' data and the historical equations from Table 1. The physical constants for the formulas calculations are the same as those from Robinson's data and are presented in Table 2.

As illustrated in Figure 1, results from the three formulas do not agree. The ARCO-Harbison Fisher formula' predicts a slippage much less than either the Robinson and Reekstin equation[2] or the Davis and Steams equation[3,4]. The Davis and Stearns equation predicts a slippage about one half that predicted by Robinson and Reekstin

None of the historical formulas above consider plunger velocity or eccentricity in determining leakage. One researcher, Coberly [8], took this into consideration and developed an eccentric correction factor, Equation 1, which could by multiplied by any historical slippage formula.

$$1 + 1.5U^2 \qquad (1)$$

where U is the eccentricity factor, which varies from 0 to 1. For a fully eccentric plunger, U is 1 and the leakage correction factor is 2.5. This correction factor assumes the ratio of eccentric to concentric slippage is *2.5* for all eccentric plungers under any condition. There are a few theoretical derivations that show the ratio to be 2.5. One of these is Tao and Donovan [9] and another is shown in Chambliss [1].

FLUID VELOCITY PROFILE

As fluid flows between two plates, it forms a velocity profile. The velocity of the fluid at the surface of a plate is zero relative to the plate. If the plates are stationary, the velocity of the fluid is zero at the surface of the two plates and is at a maximum half way between the two plates. [10] Figure 2 shows an example velocity profile between two stationary plates.

To determine the flow between the two plates, an average velocity of the fluid is calculated. The average velocity is multiplied by the width of the two plates and the distance by which the two plates are separated. Higher pressure drops increase the amplitude of the velocity profile. If one or both of the plates move, Figure 2 is no longer valid. Since the fluid velocity is zero relative to the adjacent plate; the velocity of the fluid adjacent to the plate is the same as the velocity of the plate. This is valid for both a stationary and a moving plate. Figure 3 shows a velocity profile for a stationary plate and a moving plate where the moving plate moves in the same direction as the fluid. The maximum velocity is no longer midway between the plates, but shifted towards the moving plate.

If the moving plate moves against the flow, the fluid velocity adjacent to the plate is actually negative. Figure 4 shows an example of a velocity profile where the moving plate moves against the flow. The maximum velocity with one plate moving backward is no longer midway between the plates. It is shifted towards the stationary plate.

VELOCITY PROFILE APPLIED TO PLUNGER PUMP SLIPPAGE

To develop a slippage equation for a plunger pump, a velocity profile will be used. The flow between the plunger and barrel is not flow between two flat plates but flow between two curved surfaces. The velocity profile for fluid flow between two curved plates is similar to the velocity profile between two flat plates, with the exception that the maximum velocity is shifted slightly towards the inner curved plate, in this case the plunger. In a plunger pump with a stationary barrel and a moving plunger, the velocity profile between the barrel and plunger is calculated using Equation ***2***. The derivation of Equation 2 is presented by Chambliss [1].

$$V_z(r) = \frac{-P}{4\mu}\left[\left(r^2 - R_0^2\right) - \left(R_i^2 - R_0^2\right)\frac{\ln\frac{r}{R_0}}{\ln\frac{R_i}{R_0}}\right] + V_P\frac{\ln\frac{r}{R_0}}{\ln\frac{R_i}{R_0}} \qquad (2)$$

AVERAGE VELOCITY FOR ANNULAR FLOW

To determine the fluid slippage, the average velocity of the fluid is calculated using Equation 3 [1]:

$$\overline{V} = \frac{-P}{4\mu}\left[\frac{\left(R_0^4 - R_i^4\right)}{2\left(R_0^2 - R_i^2\right)} - R_0^2 - \frac{R_i^2\psi}{\left(R_0^2 - R_i^2\right)\ln\left(\frac{R_i}{R_0}\right)} + \frac{R_i^2\ln R_0}{\ln\left(\frac{R_i}{R_0}\right)} + \frac{R_0^2\psi}{\left(R_0^2 - R_i^2\right)\ln\left(\frac{R_i}{R_0}\right)}\right.$$

$$\left. - \frac{R_0^2\ln\left(R_0\right)}{\ln\left(\frac{R_i}{R_0}\right)}\right] + V_P\left[\frac{\psi}{\left(R_0^2 - R_i^2\right)\ln\left(\frac{R_i}{R_0}\right)} - \frac{\ln\left(R_0\right)}{\ln\left(\frac{R_i}{R_0}\right)}\right] \qquad (3)$$

In order to simplify, ψ was assigned as a value of:

$$\psi = R_0^2\left(\ln R_0 - \frac{1}{2}\right) - R_i^2\left(\ln R_i - \frac{1}{2}\right) \qquad (4)$$

Since a plunger usually does not travel at a constant speed, an profile of plunger velocity as a function of pumping unit rotation is required to calculate the slippage during one stroke. One way to do this is with a program that predicts plunger position by estimating rod stretch and other elements of a rod pumped system. A simple estimate of a velocity equation can be made by assuming the velocity to be sinusoidal, circular or even elliptical depending on the application. It should also be noted that the gearbox on a pumping unit does not turn at a constant speed, which affects the modeling of plunger velocity. **A** gearbox can slip as much as 13% with a NEMA D motor under normal load conditions.

Pump slippage is the amount of fluid that leaks around the plunger. This means that leakage is relative to the pump plunger, not the pump barrel. The plunger velocity should be added to the average fluid velocity found in Equation 3 for slippage relative to the plunger.

CONCENTRIC AND ECCENTRIC PLUNGERS

A concentric plunger is one that is centered in the pump barrel while an eccentric plunger is one that is not centered in the barrel. Barrel wear of used pump barrels suggest that plungers operate more eccentric than concentric. [5] Figures 5 and 6 depict a concentric cross section and an eccentric cross section, respectively.
The figures are exaggerated. In an actual pump, the plunger is only a few thousandths of an inch smaller than the barrel. Figure 5 and 6 depict the two extremes. The plunger is not limited to the two positions represented, but can be located anywhere in between. By using a term called the eccentricity factor, a position for the plunger can be assigned when calculating pump slippage. An eccentricity factor of one is fully eccentric, while an eccentricity factor of zero is concentric. **A** plunger will most likely not have a single eccentricity value, but vary over some range. Wells where the pump is inclined the plunger will most likely have an eccentricity factor of one.

CALCULATION OF FLUID SLIPPAGE FOR CONCENTRIC PLUNGERS

To calculate the fluid slippage for a concentric plunger, the calculated average fluid velocity is multiplied by the annular area and incremental travel time. Equation 5 is used to calculate the incremental travel time.

$$t_i = \frac{30}{RN,} \qquad (5)$$

Calculate a slippage for each stroke length increment and sum up all of the incremental slippages to calculate a total slippage for one stroke. For the slippage in units of barrels per minute, convert the slippage for one stroke from ft^3 to barrels and multiple by the pumping speed in stroke per minute.

CALCULATION OF FLUID SLIPPAGE FOR ECCENTRIC PLUNGERS

Calculation of fluid slippage for eccentric plungers is not unlike the calculations for the concentric plunger. The one major difference is for each increment of the stroke, the velocity profile is not constant across the plunger cross-section. To model this situation, stroke length can be divided into increments, thus determining the slippage for each increment of the stroke. Each increment of the stroke is further divided into elements, as illustrated in Figure 7 where there are eight. By summing all the individual slippages for a cross-section, a total slippage for an increment of the stroke can be determined. Due to symmetry, slippage for elements one through four can be evaluated and multiplied by two to solve for the total slippage of the cross-section. This method assumes the velocity profile of an element is not affected in any way by adjacent elements and as such is an approximation to some degree.

To calculate the slippage for an element, the average fluid velocity through that element is ascertained. Equation 6 calculates the distance from the center of the barrel to the edge of the plunger for the element as a function of the cross-sectional angel.

$$R_{CI} = -U(R_0 - R_i)\sin\theta + \frac{1}{2}\left[(2U(R_0 - R_i)\sin\theta)^2 - 4\left(R_i^2 + (U(R_0 - R_i))^2\right)\right]^{\frac{1}{2}} \tag{6}$$

The result from Equation *6* is used as the plunger radius in the average velocity equation (Equation 3). The barrel radius in Equation 3 is the same as in the concentric case. Once the average velocity of the element is found, it is multiplied by the width of the element, average clearance, and incremental travel time (Equation 5). The width of the element is calculated using Equation 7.

$$W = \frac{\pi D}{N_p} \tag{7}$$

Once the slippage is calculated for each element of the cross-section, the summation of all of the elements represents the slippage at that stroke increment cross-section. The procedure is then repeated for each stroke increment cross-section. Then each stroke increment cross-section is summed to determine the pump slippage per stroke. Following the above formulas and using the units listed in the nomenclature, slippage is calculated in ft^3 per stroke.

COMPARISON OF NEW FORMULAS TO HISTORICAL FORMULAS

Using the methods discussed above, data was calculated using a FORTRAN program and compared to the formulas in Table I. Table 3 lists the conditions used to determine the pump slippage, which are the same used in Robinson's data. Robinson did not list the pumping rate or stroke length in his paper, so an exact comparison between his data and the new slippage method cannot be made. The pumping rate and stroke length used are an assumption based on general statements in Robinson's paper. Two curves in Figure **8** are graphed using the new slippage calculation method. One curve is for concentric slippage while the other is for 100% eccentric slippage.

The new method agrees with the ARCO-Harbison Fisher equation in Figure **8,** which shows a maximum clearance of 0.005 inches. The concentric plunger slippage is less than the ARCO-Harbison Fisher, while the eccentric plunger slippage is more than the ARCO-Harbison Fisher equation as expected.

Figure 9 was determined using clearances from 0 to 0.020 inches, which is the range for which the ARCO-Harbison Fisher equation was derived. In Figure 9, the slippage calculated from the new method increases at a greater rate compared to the ARCO-Harbison Fisher equation. The new method assumes laminar flow. In the smaller clearances, the flow is laminar, but in larger clearances the flow may be turbulent, which would lower the slippage, making it closer to the values of the ARCO-Harbison Fisher equation. Additionally, the slippage values from the new method do not match the ARCO-Harbison Fisher at larger clearances. This could be due to entrance effects when the fluid first enters the clearance space, thus causing an initial drop in pressure.

Figure 10 is another graph of pump slippage for the conditions shown in Table 4. Again, the results are similar to those in Figure 9.

RATIO OF ECCENTRIC TO CONCENTRIC SLIPPAGE

Figure 11 is a graph representing the ratio of the eccentric leakage to the concentric leakage for Figure 9 and 10. The ratio starts out at one for a clearance of zero and increases to about two and a half at a clearance of 0.02 inches. From a clearance of 0.02 inches to 0.10 inches, the ratio remains close to 2.5, but increases gradually. Theoretically, the ratio should drop back down to one when the clearance equals the pump barrel diameter. Industry formulas have always assumed the ratio was *2.5* for all clearances.

PUMPING RATE EFFECTS ON SLIPPAGE

Figure 12 is a graph of a pump, pumped at two different pumping rates. The conditions used are listed in Table 5. The pumping rate has an effect on slippage, but a small effect. In Figure 12, doubling the pumping rate increases the slippage by about one half barrel per day for both the concentric and eccentric case. For pumps with larger clearances, this extra slippage is negligible, but for pumps with small clearances, the extra slippage in not negligible and should be taken into account during pump calculations.

Figure 13 is a graph of pump slippage versus pump clearance for the conditions listed in Table 5 and a pumping rate of **8** strokes per minute. Plunger velocity in two of the plots was set to zero to illustrate the effect of plunger movement on slippage. **As** can be seen, plunger velocity does not affect slippage as much as eccentricity affects slippage.

APPLICATION OF PUMP SLIPPAGE FORMULAS TO VARIOUS PUMPING UNITS

The historical slippage formulas presented in this paper are only valid for conventional pumping units. They all assume the down stroke travel time is equal to the upstroke travel time as is true with conventional pumping units. With adjusted geometry units, the upstroke travel time is longer than the down stroke travel time. In this case, the slippage calculated from the historical equation needs to be adjusted using Equation **8.**

$$B_{Adjusted} = \frac{Degrees_{UP}}{180} B_{Calculated} \qquad (8)$$

The number of degrees in which the pumping unit rotates during the upstroke is $Degrees_{t}$. For a Mark II unit, the upstroke is 195° of the rotation. In the new slippage method introduced in Chapter V, this is considered when deriving an equation to calculate plunger velocity

VISCOUS DRAG ON CONCENTRIC PLUNGERS

Viscous fluid between the pump barrel and pump plunger causes a drag force on the plunger when it moves parallel to the pump barrel. To calculate the drag force on a concentric plunger, Equation 9 is used. [11]

$$F = 2\pi\tau R_i L \qquad (9)$$

where t is Equation 10 used in Equation 9. The derivation of Equation 10 is shown by Chambliss [1]. For shear stress on the plunger $r = R_i$.

$$\tau = \left| -\frac{rP}{2} + \frac{PR_i^2}{4r\ln\frac{R_i}{R_0}} - \frac{PR_0^2}{4r\ln\frac{R_i}{R_0}} + V_P\frac{\mu}{r\ln\frac{R_i}{R_0}} \right| \qquad (10)$$

Figure 14 is a graph of viscous drag force as a function of plunger velocity. Table 6 lists the conditions used in the graph. **As** would be expected, viscous plunger drag increases with increased plunger velocity.

VISCOUS DRAG ON ECCENTRIC PLUNGERS

For the viscous drag on an eccentric plunger, divide the barrel circumference into a number of segments like in Figure 7 and calculate the drag for each segment. Calculate the plunger radius for each segment using Equation **6.** The sum of the forces from the individual segments is the total viscous drag on the plunger. In order to calculate the drag on each segment, the following equation is used:

$$F = \frac{2\pi\tau R_i L}{N_P} \qquad (11)$$

where t can be determined using Equation 10 is still valid. Figure 15 is a graph of viscous plunger drag as a function on eccentricity factor for the plunger conditions listed in Table 6 with a plunger velocity of one foot per second. The data point for an eccentricity factor of one is actually for an eccentricity factor of 0.9999. For a value of one, there is a division by zero error, which means the drag force is infinite. In reality the value is not infinite, it is zero because there is no fluid between the pump and barrel to create viscous drag. Therefore, Equation 10 is likely not valid for small clearances.

In Figure 15, the viscous plunger drag stays relatively constant from an eccentricity factor of 0.00 to 0.90. From an eccentricity factor of 0.90 to 1.00 the viscous drag increases and for eccentricities greater than approximately 0.98, the viscous plunger drag increases dramatically.

From Figure 15, the viscous drag is greater for very small clearances. With the eccentric plunger, the area adjacent to the side of the plunger, which touches the wall, has very small clearances. This area is responsible for the large increase in viscous drag for the completely eccentric plunger. Figure 15 was calculated using 360 segments. The eleven segments adjacent to the area of the plunger near the wall, account for **179** pounds-force of the total 215 pounds-force of drag on the plunger, or 83%.

CONCLUSIONS AND RECOMMENDATIONS

An attempt was made to develop analytical models to match new experimental data for pump slippage which does not agree with historical models. The new models to calculate slippage are valid up to a clearance of 0.005 inches when compared to the Patterson, et al. formula'. The Patterson et al. equation is essentially a curve fit of the new data he and others collected. From the new slippage method, two variables that affect slippage were studied, pumping rate and eccentricity.

Historical slippage formulas predict slippage as a function of pressure, diameter, clearance, viscosity and plunger length. None of the formulas took into consideration pumping rate to calculate slippage. Eccentricity was considered in some of the historical formulas, but was calculated using a multiplier that varied from 1 to 2.5. It was determined that the eccentric correction factor for a completely eccentric plunger was not 2.5 for all clearances.

Pumping rate does affect slippage to a great degree according to the models presented here. The effect is minimal though and slippage is increased only by about one half barrel per day when the pumping rate is doubled for the example pump used in this paper. Pumping rate can be neglected when performing general slippage calculations, but should be considered when possible, especially for small clearances.

Eccentricity has a much larger effect on slippage than the pumping rate, and should always be considered in slippage calculations. Eccentricity can affect the slippage by as much as a factor of 2.5 or more. Determining the eccentricity factor to use is difficult, if not impossible. Eccentricity can, and probably does vary on every stroke. On most wells, a range of slippage is more realistic than a single value.

Since eccentricity has such a large effect on slippage, additional research should be conducted. Any factor that may affect eccentricity should be studied. For example, one factor that affects eccentricity is pump inclination. Slippage data could be collected on plungers at various angles to determine what effect plunger inclination has on eccentricity. At a particular inclination, a plunger should always have an eccentricity factor of one. An additional factor is clearance, which may also affect eccentricity, or at least how eccentricity varies on a plunger.

Some of the past laboratory and field data is somewhat questionable and does not contain information about pumping rate and stroke length. Data could be collected to determine what effect pumping rate has experimentally on slippage.

Viscous plunger drag is also affected by eccentricity. The viscous plunger drag remains relatively constant for an eccentricity factor varying from 0.00 to 0.90. When the eccentricity factor is between 0.90 to 1.00, the viscous drag increases, and for eccentricities greater than about 0.98, the viscous plunger drag increases dramatically. The formulas used to calculate viscous drag may not be valid for eccentricity values close to one. Laboratory and field data should be collected on viscous plunger drag in order to determine how the drag behaves for eccentricity factors close to one and to determine what types of models are valid.

NOMENCLATURE

Symbol		Unit
B	Pump Slippage	bpd
C	Plunger-Barrel Clearance	in
D	Barrel Diameter	in

F	Drag Force	lb
L	Plunger Length	in
P	Pressure Over Plunger	psi
N_p	Number of Increments of Plunger Cross Section	Dimensionless
N_s	Number of Increments of Stroke	Dimensionless
$\boldsymbol{P}$	Pressure Gradient	psi/ft
R	Pumping Rate	spm
$\boldsymbol{r}$	Radius	in
R_{ci}	Incremental Plunger Radius	ft
$\boldsymbol{t}$	Pump Barrel Thickness	in
U	Eccentricity Factor	Dimensionless
V_p	Plunger Velocity	ft/s
V_z	Fluid Velocity as a Function of Radius	ft/s
W	Width of Plunger Element	ft
μ	Dynamic Fluid Viscosity	Centipoise

Subscripts

AH	ARCO-HF
DS	Davis and Stearns
RR	Robinson and Reekstin

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Table 1
Historical Slippage Formulas

Davis and Stearns [2, 3, 4]	$B_{DS} = 4.17 \times 10^{6} \dfrac{PC^{1.9}\left(d_2^2 - d_1^2\right)}{\mu^{0.1} r}$
Robinson and Reckstin (Corrected) [1, 2]	$B_{RR} = 1.80 \times 10^{8} \dfrac{PD^{0.7}C^{3.3}}{L\mu}$
ARCO-Harbison Fisher [5, 6, 7]	$B_{AH} = 870 \dfrac{DPC^{1.52}}{r}$

Table 2
Conditions from Robinson's Data

Plunger Size	1 ½ inches by 72 inches
Viscosity	7 centipoise
Pump Pressure	2230 psi

Table 3
Conditions Used in Figures 8 and 9

Pumping Rate	[illegible]
Stroke Length	55 inches
Plunger Size	1 1/2 inch by 72 inch
Viscosity	7 centipoise
Pump Pressure	2230 psi

Table 4
Conditions Used in Figure10

Pumping Rate	
Stroke Length	144 inches
Plunger Size	3.000 inch by 24 i...
Viscosity	15 centipoise
Pump Pressure	1600 psi

Table 5
Conditions Used in Figure 12

Pump	3 inches by 24 inches
Viscosity	7 centipoise
Pressure	[illegible] psi
Stroke Length	55 inches

Table 6
Conditions Used in Figures 14, 15 and 16

Barrel Diameter	3.000 inches
Clearance	0.003 inches
Plunger Length	36 inches
Pressure	2600 psi
Viscosity	7 centipoise

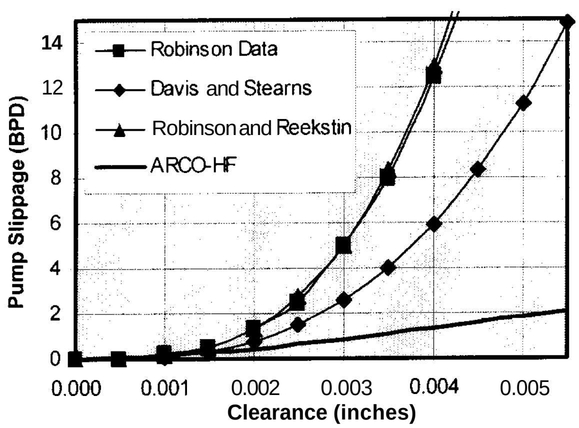

Figure 1 - Robinson's Data with Historical Slippage Formula

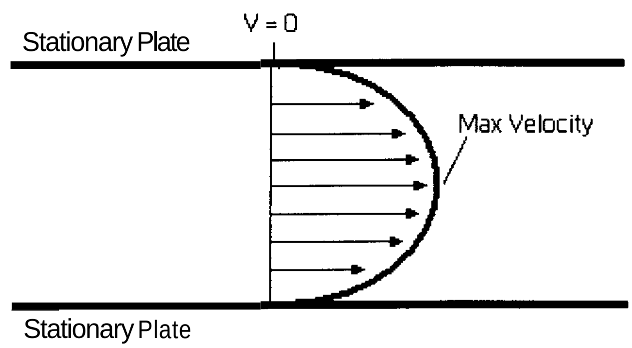

Figure 2 - Example Velocity Profile Between Two Stationay Plates

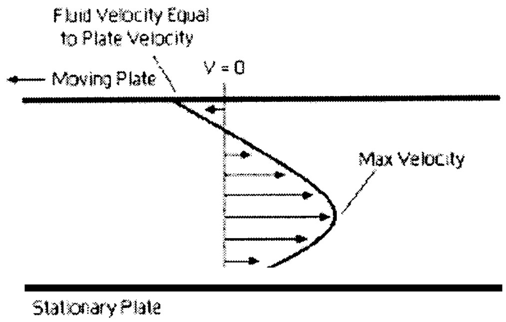

Figure 3 - Velocity Profile Between a
Stationary Plate and a Forward Moving Plate

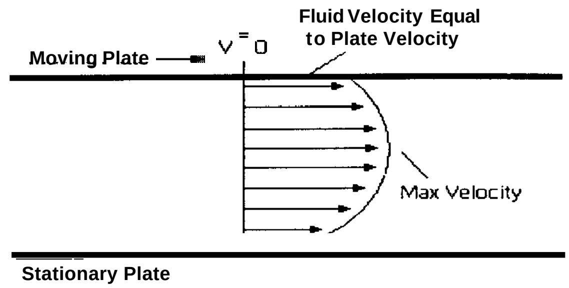

Figure 4 - Velocity Provile Between a Stationary Plate
and Backward Moving Plate with Flow Moving Foward

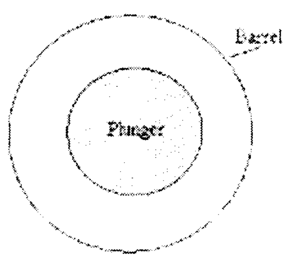

Figure 5 - Concentric Plunger Cross Section

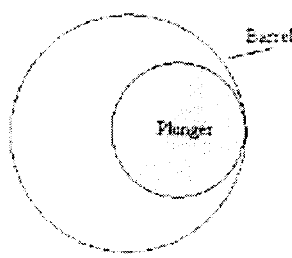

Figure 6 - Fully Eccentric Plunger
Cross Section

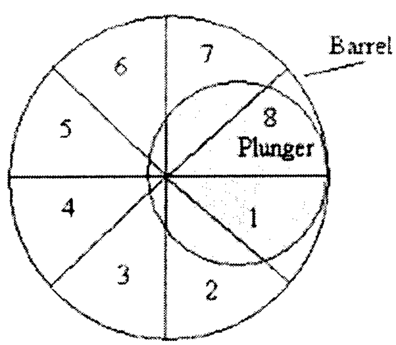

Figure 7 - Eccentric Plunger Divided Into 8 Elements

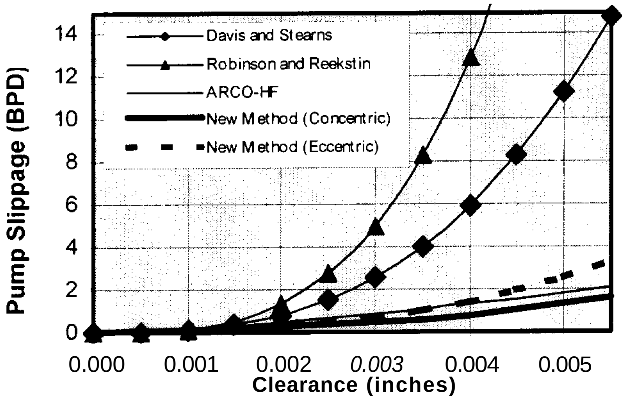

Figure 8 - Historical Slippage Forumulas with Data from New Method for Conditions Listed in Table 3

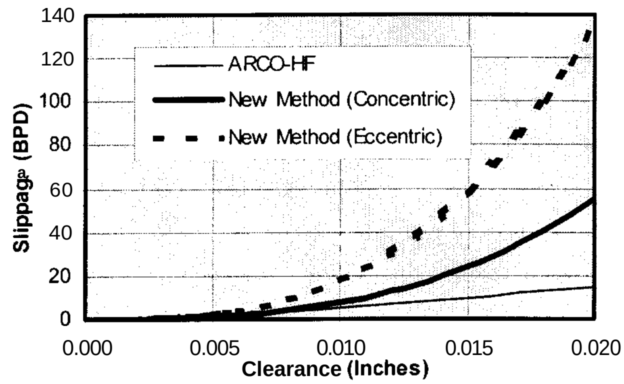

Figure 9 - Slippage for Conditions Listed in Table 3

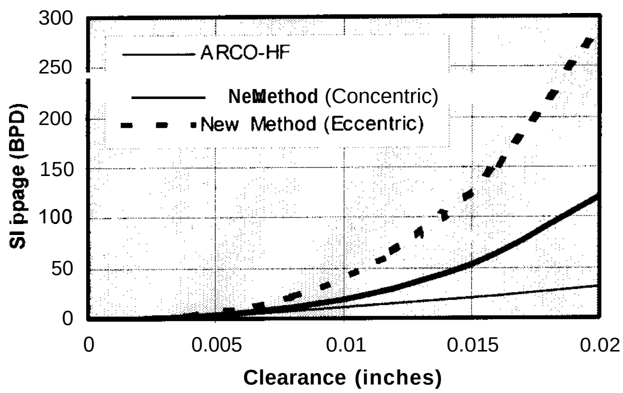

Figure 10 - Slippage for Conditions Listed in Table 4

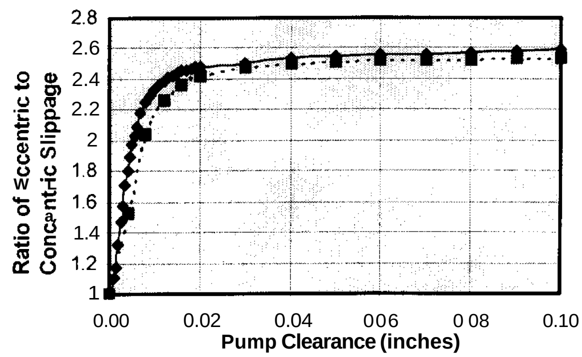

Figure 11 - Ratio of Eccentric Slippage to Concentric Slippage For Conditions Listed in Tables 3 and 4

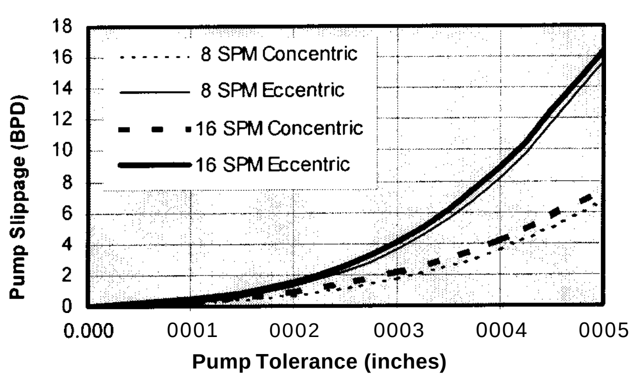

Figure 12 - Slippage for Conditions Listed in Table 5 Pumped at Two Different Pumping Rates

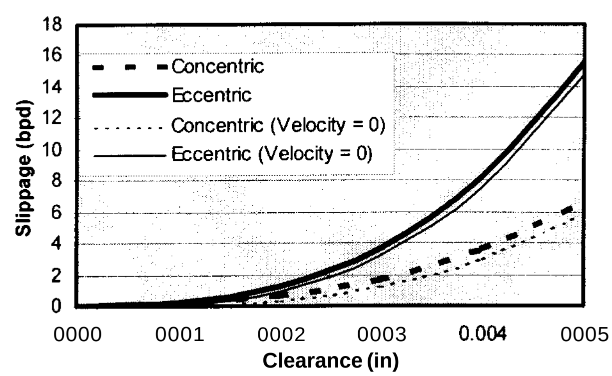

Figure 13 - Slippage for Conditions Listed in Table 5 To Show Effect of Moving Plunger

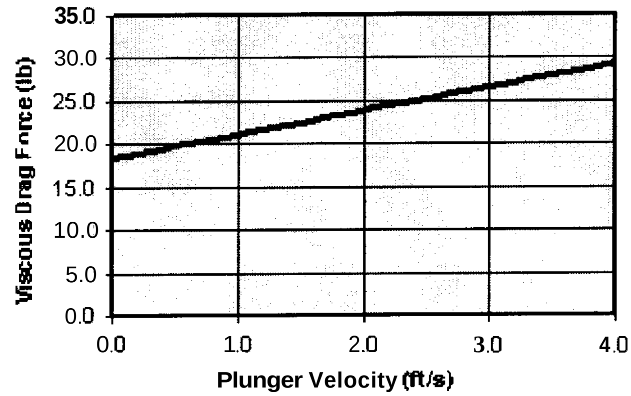

Figure 14 - Viscous Drag Force as a Function of
Plunger Velocity Using Conditions
Listed in Table 6

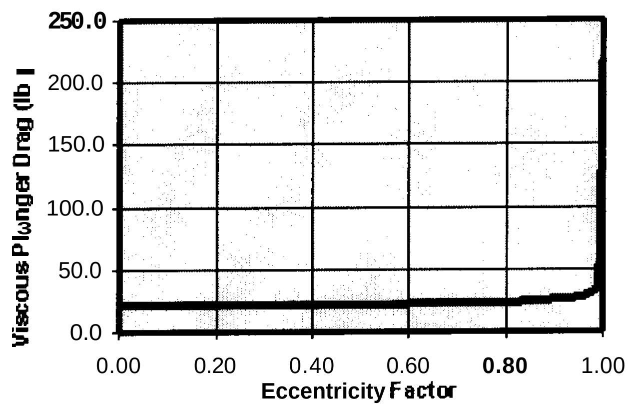

Figure 15 : Plunger Drag as a Function of Eccentricity
Factor for the Conditions Listed in Table 6 with a
Plunger Velocity of One Foot per Second