PERSONAL COMPUTER APPLICATION OF THE
ANALYSIS OF SUCKER ROD PUMPING WELLS

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INTRODUCTION

One purpose of obtaining a surface dynamometer card is to learn how well the downhole pump is functioning. Several methods have been used to develop a finite mathematical model of a simplified pumping system to predict or evaluate pump behavior given dynamometer polished rod loads and displacement at the surface. The purpose of these methods was to develop a computerized method of analyzing surface dynamometer cards. The alternative is to evaluate the card by visual inspection, which is very subjective, requires a skilled analyst, and yields results which are strictly qualitative in nature. The work done by Gibbs, in particular, has apparently been successful, considering the expense involved in using or obtaining one of his patented programs.[1] Other than the expense, the programs are generally designed for applications with main frame computers, and the development of the equations used in the programs seems to have been left purposefully ambiguous.

The purpose of this paper is to present a relatively comprehensive and user-friendly program for use with micro-computers based on a finite difference method initially developed by Roy Knapp at the University of Kansas.[2]

STATEMENT OF THE PROBLEM

A sucker-rod pumping system consists of the basic units (See Fig. 1 for a diagram of the system.):

(1) prime mover
(2) surface pumping unit
(3) downhole pump
(4) tubing string
(5) the column of fluid being pumped

Several simplifications are neccessary to be able to determine the dynamic characteristics of the combination of all the elements:

(1) The prime mover is assumed to move at a constant angular velocity at the crank.
(2) The inertial effects of counter weights and pumping unit were neglected.
(3) The solution of the problem is restricted to a single diameter rod string.

The key to the analysis of the performance of a sucker rod pumping system is the method by which the forces at the polished rod are evaluated by an approximation of the one-dimensional wave equation. This process of evaluation is based on a differential equation and a set of boundary conditions. This differential equation, which describes the motion of a long slender rod is the one-dimensional wave equation, with a viscous damping term added to simulate the damping of the rod vibrations by the pump. The problem is that of approximating the wave equation and boundary conditions with a set of mathematical equations which are suitable for use within a computer program.

DEVELOPMENT OF THE EQUATIONS

The one-dimensional wave equation is used to simulate the behavior of the rod string. The boundary conditions, which are neccessary to obtain a solution of the wave equation, describe the motion of the polished rod, the downhole pump operation, and the initial stress and velocity of the rod string. The wave equation, along with the conditions which describe the pump operation and polished rod motion are the three factors which must be modeled mathematically to solve the problem. In this section the boundary conditions will be discussed and then the wave equation will be modeled for the whole system.

Polished Rod Motion

The motion of the polished rod is determined by the geometry of the surface pumping unit. Five linear dimensions are used to describe the motion of the polished rod in terms of the crank angle (illustrated in Fig. 2). The equation used to calculate the polished rod position is given by:[3]

$$S(\theta) = UT\ \{\ L5\ [\ \mathrm{ARCOS}\left(\frac{C1 + C2\ \ COS\ (\theta\ R + D)}{C3 + 2\ \ C4\ \ COS\ (\theta\ R + D)}\right)\ ]\}$$

$$+\ \mathrm{ARCSIN}\left(\frac{C4\ \ SIN\ (\theta\ R + D)}{C3 + 2\ \ C4\ \ COS\ (\theta\ R + D)}\right)$$

where:

$C1 = (L1^2 + L2^2 + L3^2 - L4^2)\ /\ (2\ L2\ L3)$
$C2 = L1\ /\ L3$
$C3 = 1 + (L1\ /\ L2)^2$
$C4 = L1\ /\ L2$
$C5 = (L2^2 + L3^2 - (L4 - L1)^2)\ /\ (2\ L2\ L3)$

$D = (\pi\ /\ 2)\ \ (1 + UT) - \left(\frac{L2 - L3 + (L4 + UT\ L1)}{2\ \ L2\ \ (L4 + UT\ \ L1)}\right)$
L1 = crank length
L2 = fixed bar length
L3 = driving bar length
L4 = pitman length
L5 = driven bar length
UT = 1 for a Class I unit and -1 for a Class II unit
R = 1 for normal rotation and -1 for reverse rotation
θ = crank angle in radians

The crank speed (or angular velocity) is assumed to remain constant.

Wave Equation

The wave eqation describes the longitudinal vibrations of a long slender rod. In a sucker-rod pumping system, viscous and nonviscous damping are forces present. However, nonviscous effects, such as hysteresis loss and coulomb friction are considered very small compared viscous damping and can be neglected. The damped wave equation is given by:[1]

$$\frac{\partial^2 U\ (x,t)}{\partial t^2} = E\ \ Ar\ \ \frac{\partial^2 U\ (x,t)}{\partial x^2} - C\ \ \frac{\partial U\ (x,t)}{\partial t} \qquad (1)$$

acceleration term = rod stretch term - damping term

$\partial^2 U(x,t)$ = displacement from the equilibrium position of the sucker rod
E = modulus of elasticity
Ar = cross-sectional area of the rods
C = damping coefficient

After adding the effect of gravity on the rod load, the equation becomes:[4]

$$\frac{\rho Ar \partial^2 U}{g \partial t^2} = \frac{E\ Ar\ \partial^2 U}{\partial x^2} - \frac{C\ \partial U}{\partial t} + Wb$$

∂U = $\partial U\ (x,t)$
ρ = density of the rod material
ρAr = weight per ft. of the rods in air
g = force of gravity
Wb = weight of the rods in fluid in lb./ft.
(used to subtract the weight of the rods for each Δx)

After rearranging, the above equation becomes:

$$\frac{\partial^2 U}{\partial x^2} = \frac{Wa}{E\ Ar\ g}\ \frac{\partial^2 U}{\partial t^2} + \frac{C}{E\ Ar}\ \frac{\partial U}{\partial t} - \frac{Wb}{E\ Ar}$$

To approximate the function a finite difference analogy must be used. The finite difference representation used here is a truncated Taylor Series expansion:[5]

$$\frac{\partial^2 U(x,t)}{\partial x^2} \cong \frac{U(x = \Delta x,t) - 2\ \ U(x,t) + U(x - \Delta x,t)}{\Delta x}$$

$$\frac{\partial^2 U(x,t)}{\partial t^2} \cong \frac{U(x,t + t) - 2\ \ U(x,t) + U(x,t - t)}{\Delta t}$$

$$\frac{\partial U(x,t)}{\partial t} \cong \frac{U(x,t + t) - U(x,t)}{\Delta t}$$

Where the first two terms are centered difference approximations, and the last term is a backward difference approximation.

After rearranging the equation and making the following substitutions:

i for x
i + 1 for x + x
i - 1 for x - x
j for t
j + 1 for t + t
j - 1 for t - t

The equation now becomes:

$$U(i+1,j) = 2\,U(i,j) - U(i-1,j) + (\Delta x^2/\Delta t^2)\ [Wa/(E\ Ar\ g)]\ [U(i,j+1) - 2\,U(i,j) + U(i,j-1)] + (C\,\Delta x^2)/(E\ Ar\ \Delta t)\ [U(i,j+1) - U(i,j)] - \Delta x^2[Wb/(E\ Ar)]$$

This is the form of the main equation that is used in the program. The number of data points, which represent the number of values of polished rod load versus displacement chosen from the dynamometer card, are denoted by (j) in the above equation.

$U(1,j)$ = the values of displacement from the dynamometer card or as calculated by the displacement condition at the surface (as described above)
$U(2,j) = U(1,j) - \Delta x/(E\ Ar)PRL(j)$
where:
$PRL(j)$ = load at the polished rod and
$\Delta x/(E\ Ar)PRL(j)$ = load (lbs.) converted to strain (ft.)

The terms U(1,j) and U(2,j) represent the two surface boundary conditions, and form the first two rows of a two dimensional array, with (i) rows and (j) columns, where (j) represents the data points and (i) represents the sections delta x (Δx) in length. The number of sections of x is one less than the number data points.

Equation No. 2 is used to calculate the subsequent rows of the array. The value of the element U(3,2) is calculated by using the values of the elements U(1,2), U(2,1), U(2,2) and U(2,3). A wrap-around condition is used to calculate the special cases of the elements of the first and last columns. For example, U(3,1) is calculated by using the values of U(1,1), U(2,Npts), U(2,1), and U(2,2). The last row of values in the array represents the displacement of the pump.

The rod stretch due to the weight of the rods added to these values yields the pump displacement relative to the polished rod displacement. The pump load is the dynamic load at the bottom of the rod string. A Taylor series expansion is used to calculate the load using the last three rows of the displacement array (U).

For example, the pump load at position (1) is given by:

$$\text{Pump Load at } (1) = E\ Ar/\Delta x(1.5\,U(i,1) - 2\,U(i-1,1) + .5\,U(i-2,1))$$

This completes the solution of the problem. The equations are now in the form needed for programming into the computer.

RESULTS AND CONCLUSIONS

For very slow pumping speeds (less than about one stroke per minute), a polished rod dynamometer card is in the shape of a parallelogram, as shown in Figure 3. The card is rectangular and the difference between the constant peak polished rod load and minimum polished rod load is the same as the difference between the peak and minimum pump load.

This difference represents the weight of the fluid lifted. Also, the difference in displacement between the polished rod and the pump is equivalent to the stretch of the rod string due to the weight of the fluid lifted. The computer program yields results for very slow pumping speeds which are within one percent of the theoretical values. The only difference between very slow and normal pumping speeds in the calculations is that the damping coefficient is approximately zero for very slow pumping speeds.

For normal pumping speeds the results are promising but not as precise or accurate as those for very slow pumping speeds (Figure 4). The problem is that the damping factor must be empirically derived and probably will not be exactly the same for any two systems. It is easy to conclude that the damping term varies with the velocity of the sucker rod string, since it is negligible at very slow polished rod speeds as show above, but must be included when pumping at normal speeds. According to Gibbs, the damping term is related to the pumping speed, the dimensions of the rod string, and the difference between the polished rod and pump horsepower, which means that it is related to the polished rod velocity. However, according to Gibbs[6] and Lea[4], the damping factor is relatively small and varies over a narrow range for normal pumping systems, and can be approximated by a constant. The best results seem to be obtained by using a constant damping factor for the upstroke, and a constant factor for the downstroke.

For more accurate results, other than defining the value of the damping coefficient more precisely, the variations in prime mover speed should be included when calculating the polished rod displacement. The program would also be applicable to a wider variety of pumping systems if the option of a change in cross-sectional area of the rod string was considered. This can be done by including the area term in the solution of the wave equation, instead of treating it as a constant as was done here.

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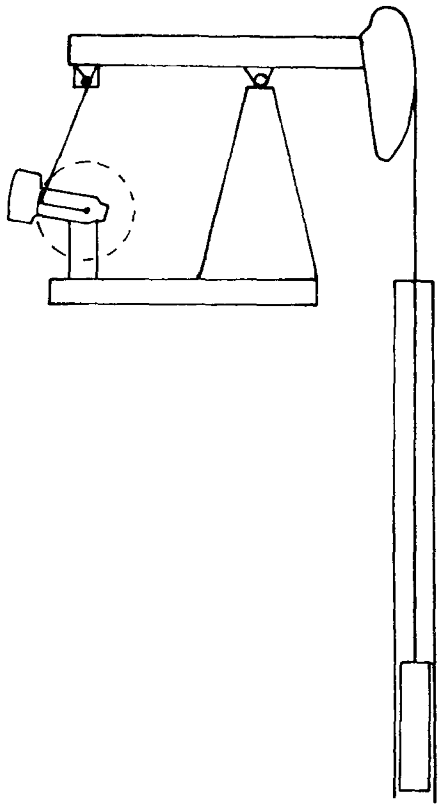

Figure 1

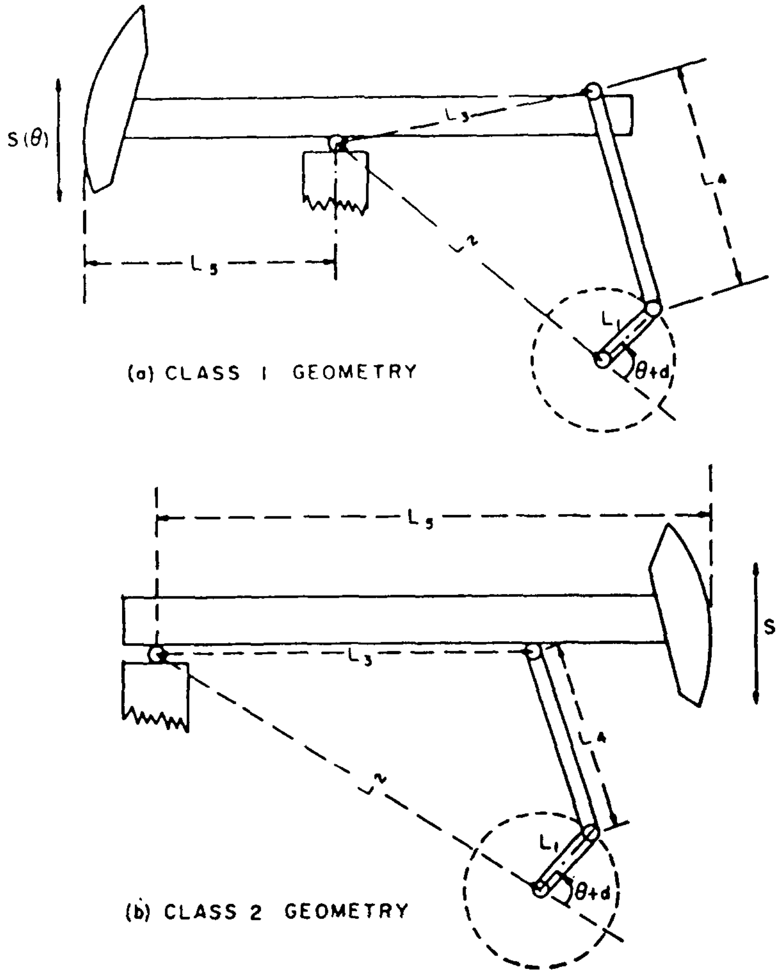

Figure 2 - Unit geometry for beam pumping units

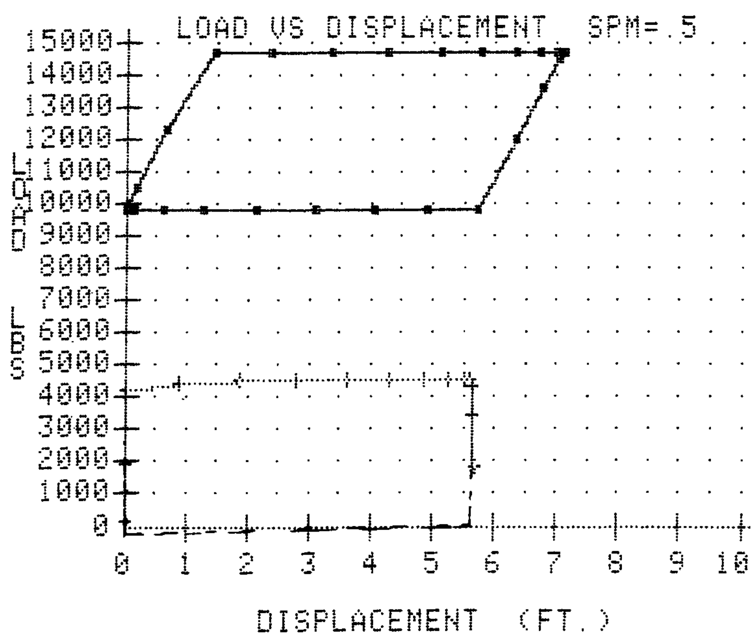

Figure 3

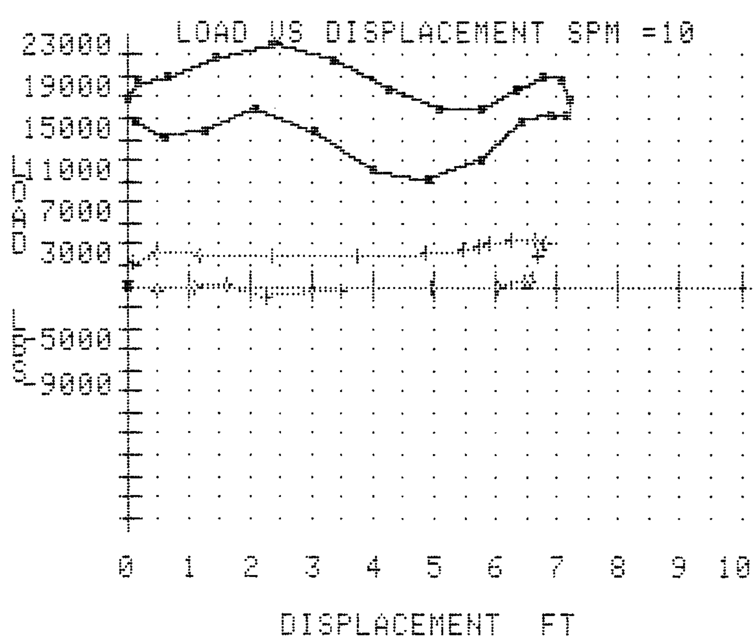

Figure 4