Permissible Load Diagrams For Pumping Units

By R. H. GAULT
Bethlehem Steel Company, Supply Division

INTRODUCTION

The function of a pumping unit is to lift a load comprised of rods and fluid a certain height, then lower the rods and pump back to bottom ready to lift another load of fluid.

To many of us it is a rather complicated piece of machinery. When we try to analyze it geometrically and find the meaning of torque factors, work, horsepower, etc., it seems especially difficult and complicated.

An attempt will be made to remove some of the confusion surrounding pumping unit calculations by a series of logical steps to show the basis of each calculation.

WORK

The beginning point should be work. We know that work by definition is a force acting through a certain distance. Therefore, if a one pound weight is raised five feet vertically, we have performed five foot-pounds of work. This is the function of a pumping unit. It raises a load a given distance. Because of the nature of the pumping load, it does not remain constant; so in order to find the work done on the unit upstroke, we must find what the average load is which is lifted on the upstroke.

This is accomplished with a planimeter which measures the area under the upstroke load as recorded by the dynamometer. Dividing this area by the base line length determines the average upstroke load. This load multiplied by the distance it is lifted (which is the stroke length) gives the work performed in one upstroke. On the unit downstroke the unit is lowering the rods back into the hole without the fluid load, so this amount of work is regained.

This downstroke load, again, is not constant, so we must find the average downstroke load. Measuring with a planimeter the area under the downstroke load and dividing this measured area by the length of the base line determines the average downstroke load. This load multiplied by the distance it is lowered gives the work returned on the downstroke load. The total work output of one stroke is the work on the upstroke minus the work on the downstroke.

Horsepower Requirements

Horsepower is the rate of doing work. One horsepower is 33,000 foot-pounds of work in one minute. The number of times this load is lifted and lowered (strokes per minute) determines how much horsepower is required at the polished rod. So polished rod horsepower is:

HP = (Strokes per Minute) [(Ave. load, upstroke) (Str. length) - (Ave. Load, Downstroke) (Str. Length)] divided by (33,000) (12)

Our next step should be to determine how these loads affect unit torque.

To illustrate the principles involved, we shall use the mechanical system shown in Fig. 1, with a load of one pound being lifted by a hoisting drum over a single sheave. The drum radius is one foot.

If we lift the load of one pound a height of 6.28 feet, we

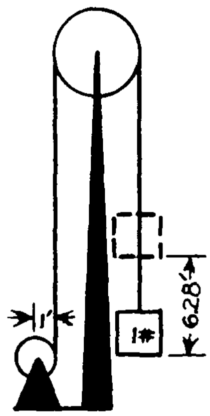

FIG. 1

have accomplished work in the amount of 6.28 foot-pounds. The pull on the opposite line must be one pound. Since we are lifting this on a drum with one foot radius, we must exert a torque of one foot-pound on the shaft of the drum to lift this load. This would represent a torque of 12 inch-pounds.

We might also wish to lift this load with a drum of larger diameter as in Fig. 2. You will observe that everything is the same except that the radius of the drum is now ten feet. Again a load of one pound is lifted 6.28 feet and 6.28 foot-pounds of work are performed except that now a torque requirement of ten foot-pounds, or 120 inch-pounds, is required.

Since the work done in both cases was the same, the torque requirements of a pumping unit are not proportional to the work done, but are rather a function of the effective length of the lifting arm. This lifting arm is the torque arm and in actual unit geometry the effective lifting arm will be called the "torque factor."

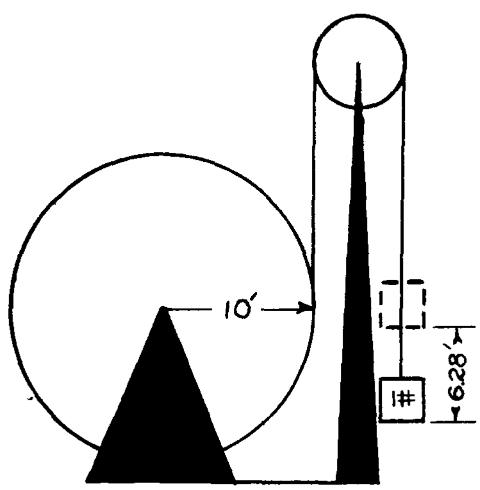

FIG. 2

TORQUE

To carry this illustration one step farther and determine how this torque affects the horsepower requirements, the one pound weight will be lifted 6.28 feet in one minute. Since the rope is winding around the drum, a point on the rim of each drum would necessarily move 6.28 feet. One revolution of the big drum would lift the weight (π) (20 feet) or 62.83 feet. The big drum would only move 1/10 of one revolution. One revolution of the small drum would lift the weight 6.28 feet, so the small drum would move one revolution. To convert torque to horsepower, the following formula is used.

$$HP = \frac{(2\pi)\ (\text{Torque, ft. lbs.})\ (\text{RPM})}{33{,}000}$$

Applying this formula to the above:

BIG DRUM:

$$HP = \frac{2\pi\,(10)\ (1/10)}{33{,}000} = \frac{6.283}{33{,}000}\ HP$$

SMALL DRUM:

$$HP = \frac{(2\pi)\ (1)\ (1)}{33{,}000} = \frac{6.283}{33{,}000}\ HP$$

Both work and horsepower remain exactly the same but the torque in one case is ten times the torque in the other case.

Bringing this illustration closer to the actual mechanical linkage of a unit, let us assume that instead of a rope winding around a hoisting drum it is anchored to a crank arm. To simplify this illustration, we will assume that the pulley sheave is high enough that the rope is always exactly vertical and neglect for the moment any angle of transmission. (Fig. 3)

When the crank arm is vertical, either up or down, no torque is required in the crank shaft. When the crank is horizontal, maximum torque must be exerted on the crank shaft. The vertical movement of the load, in this case, would be limited to the diameter of the circle described by the crank pin. In this illustration we will use a crank arm length of 50 inches and a total vertical lift of 100 inches with a load of 10,000 pounds and calculate torques required at 15 degree intervals, starting with 0 degrees when the crank is vertical up.

POSITION	CRANK TORQUE ARM (TF)	TORQUE INCH, POUNDS	POSITION OF LOAD
0	0	0	0
15	12.94"	129,400	2.7"
30	25.00"	250,000	6.7"
45	35.35"	353,500	14.65
60	43.30"	433,000	25.00
75	48.30"	483,000	37.06
90	50.00"	500,000	50.00
105	48.30"	483,000	62.94
120	43.30"	433,000	75.00
135	35.35"	353,500	85.35
150	25.00"	250,000	93.30
165	12.94"	129,400	98.30
180	0	0	100.00

The maximum torque would be 500,000 inch-pounds and the minimum torque would be 0 inch-pounds with average torque 318,300 inch-pounds. The total work done would be 1,000,000 inch-pounds of work. This would be the theoretical geometry for a pumping unit and would have the least torque possible with this linkage to do this work.

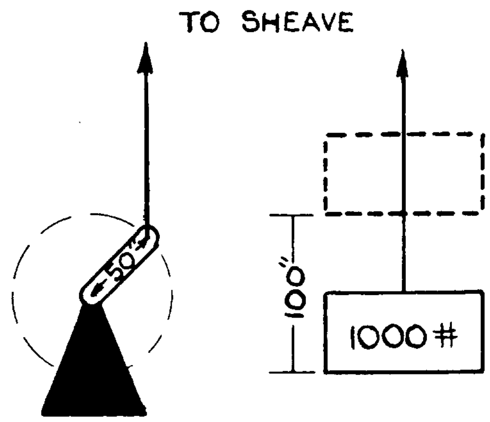

FIG 3

Using actual pumping unit geometry, we have an additional factor to consider. Since the crank arm is connected to the walking beam by an arm of finite or limited length, sometimes the load is lifted at an angle. The effective torque arm then is greater than the perfect torque arm would be because we are pulling at this angle, so more force must be exerted to lift the same load. (Fig. 4) Calculating torques required to lift this load we have:

POSITION	CRANK TORQUE ARM (TF)	TORQUE INCH, POUNDS	POSITION OF LOAD
0	- 9.187	- 91,870	.4
15	9.109	91,090	.5
30	27.266	272,660	4.9
45	42.265	422,650	14.1
60	51.234	512,340	26.2
75	53.719	537,190	39.7
90	51.188	511,880	53.4
105	45.046	450,460	65.5
120	38.078	380,780	76.4
135	30.656	306,560	84.9
150	22.750	227,500	91.9
165	14.906	149,060	96.6
180	6.375	63,750	99.4

We see that the maximum torque required to lift this identical load would then be 537,190 inch-pounds and the work performed 1,000,000 inch-pounds with an average torque of 318,300 inch-pounds.

Under actual pumping conditions, as has been previously mentioned, the pumping unit also lowers a reduced load back to bottom. This reduces the total work of one revolution by the amount of work put back into the system. As an example, if we lift 10,000 pounds a height of 100 inches

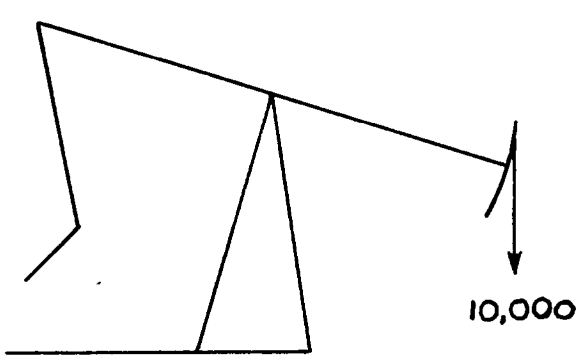

FIG. 4

we have done 1,000,000 inch-pounds of work. On the downstroke we lower a reduced load of 6000 pounds. The total work done is reduced by 600,000 inch-pounds and the net work done is 400,000 inch-pounds.

We have seen above that if we lift 10,000 pounds, we will have a maximum torque of 500,000 inch-pounds with theoretical geometry and 537,190 inch-pounds with actual geometry. To reduce these torques and spread work over the complete cycle, it is common practice to use counterbalance on the crank arm amounting to one-half of the total of the maximum load and the minimum load or, in the above example, 10,000 pounds plus 6000 pounds divided by two equals 8000 pounds. Using this 8000 pounds of counterbalance and determining results for both the theoretical and actual units through a complete cycle, we have:

CRANK POSITION	CRANK TORQUE ARM (TF)	TORQUE* FROM LOAD	TORQUE FROM* COUNTERBALANCE	NET TORQUE
0	0	0	0	0
15	12.94	129,400	-103,520	25,880
30	25.00	250,000	-200,000	50,000
45	35.35	353,500	-282,800	70,700
60	43.30	433,000	-346,400	86,600
75	48.30	483,000	-386,360	96,600
90	50.00	500,000	-400,000	100,000
105	48.30	483,000	-386,360	96,600
120	43.30	433,000	-346,400	86,600
135	35.35	353,500	-282,800	70,700
150	25.00	250,000	-200,000	50,000
165	12.94	129,400	-103,520	25,880
180	0	0	0	0
195	-12.94	- 77,640	103,520	25,880
210	-25.00	-150,000	200,000	50,000
225	-35.35	-212,100	282,800	70,700
240	-43.30	-259,800	346,400	86,600
255	-48.30	-289,800	386,360	96,600
270	-50.00	-300,000	400,000	100,000
285	-48.30	-289,800	386,360	96,600
300	-43.30	-259,800	346,400	86,600
315	-35.35	-212,100	282,800	70,700
330	-25.00	-150,000	200,000	50,000
345	-12.94	- 77,640	103,520	25,880
360	0	0	0	0

*Torque which causes the prime mover to exert force is positive, torque which exerts force tending to drive the prime mover is negative.

ACTUAL UNIT
100" Polished Rod Stroke, Rotation-down Post
10,000# Upstroke Load, 6000# Downstroke Load,
8000# Counterbalance.

CRANK POSITION	CRANK TORQUE ARM (TF)	TORQUE* FROM LOAD	TORQUE FROM* COUNTERBALANCE	NET TORQUE
0	- 9.187	- 91,870	0	- 91,870
15	9.109	91,090	-103,520	- 12,430
30	27.266	272,660	-200,000	72,660
45	42.265	422,650	-282,800	139,850
60	51.234	512,340	-346,400	165,940
75	53.719	537,190	-386,360	150,830
90	51.188	511,880	-400,000	111,880
105	45.046	450,460	-386,360	64,100
120	38.078	380,780	-346,400	34,380
135	30.656	306,560	-282,800	23,760
150	22.750	227,500	-200,000	27,500
165	14.906	149,060	-103,520	45,440
180	6.375	63,750	0	63,750
195	- 3.266	- 19,600	103,520	83,920
210	-13,859	- 83,150	200,000	116,850
225	-24.938	-149,630	282,800	133,170
240	-35.578	-213,470	346,400	132,930
255	-44.297	-265,780	386,360	120,580
270	-50.062	-300,370	400,000	100,000
285	-52.125	-312,750	386,360	73,610
300	-50.625	-303,750	346,400	42,650
315	-45.703	-274,220	282,800	8,580
330	-37.109	-222,650	200,000	- 22,650
345	-24.640	-147,840	103,520	- 44,320
360	- 9.187	- 55,120	0	- 55,120

Comparison of the above should point out a number of things. First of all, maximum torque factors are not an accurate yardstick of maximum torques which a unit must exert in lifting a given load. Consideration must be given to the effect of the counterbalance while this load is being lifted. The second thing is that because of the relationship of the overall geometry of the unit to the counterbalance effect, we must exert 165,940 divided by 100,000, or 165.94 per cent of maximum theoretical torque to lift this load. This compares to 537,190 divided by 500,000, or 107.44 per cent, when we are lifting the load without benefit of counterbalance.

The actual unit has a maximum torque on the upstroke of 165,940 inch-pounds and a maximum torque on the downstroke of 133,170 inch-pounds on the upstroke. This proves counterbalancing by equal areas above and below measured counterbalance line is in error. Proper counterbalance calculations can only come from the actual torque calculations.

The maximum upstroke torque occurs at 60 degrees and the maximum downstroke torque occurs at 225 degrees. Additional counterbalance torque would reduce all net torques on the upstroke and increase all net torques on the downstroke. At 60 degrees we would realize only .866 of the maximum additional torque and at 225 degrees we would realize only .707 of the maximum additional torque. For equal torques in each position we would use the following formula.

$$165{,}940 - X \sin 60^\circ = 133{,}170 + X \sin 45^\circ$$
$$165{,}940 - .866\,X = 133{,}170 + .707X$$
$$X = \frac{32{,}770}{1.573} \approx 20{,}832 \text{ inch-pounds}$$

The maximum torque on both up and downstroke with the additional counterbalance would be 147,900 inch-pounds.

This comparison of a theoretical unit with an actual unit shows the need for a better method of evaluating unit performance than merely observing torque factors. We have long assumed that when we are buying an API unit we were buying the same load lifting capacity; but, as has just been illustrated, we certainly are not. The actual capacity that a given API size unit will lift varies widely.

Permissible Load Diagram

To give an accurate method of evaluating exactly how much load a given unit will carry without overloading the gear reducer with a given counterbalance torque, I have originated a parameter which shall be called the "Permissible Load Diagram." It is a diagram of the loads that a unit will carry with a given counterbalance without exceeding the rating of the gear reducer.

A "Permissible Load Diagram" is very simple to construct. We know from the previous calculations that the net torque on the reducer is determined by multiplying the torque factor times the load being lifted to find the torque from load and subtracting the torque from the counterbalance. Stated mathematically, this would read:

$$\text{TORQUE} = \overline{TF}\,(W-B) - M \sin \theta. \quad \text{Where:}$$

$\overline{TF}$ = Torque Factor, inches
W = Load being lifted, pounds
B = Unit unbalance, pounds
M = Counterbalance moment, inch-pounds
θ = Position of crank, degrees.

To find the load that can be carried without exceeding the torque of the unit, assume the torque on the gear reducer will remain constant at its maximum. To this rated torque we will add algebraically the torque from the counterbalance and divide by the torque factor at each position and add the unbalanced load. This will give the load which the unit can carry without exceeding its torque rating. Expressed mathematically, this would be:

$$W = \frac{(\text{TORQUE} + M \sin \theta)}{TF} + B$$

These loads would be plotted against relative polished rod position and a diagram constructed which would show the maximum loads the unit could carry at all positions.

Using the theoretical unit in the example, and neglecting the unbalanced load, we have:

THEORETICAL UNIT: FIG. 5
100" Stroke, 160,000 inch-pound torque,
400,000 inch-pound maximum counterbalance Torque

POSITION	TORQUE OF UNIT	COUNTERBALANCE TORQUE	TORQUE FACTOR	PERMISSIBLE LOAD	POLISHED ROD POS.
0	+160,000	0	0	0	0
15	+160,000	+ 103,500	12.94	21,960#	2.7
30	+160,000	+ 200,000	25.00	14,400#	6.7
45	+160,000	+ 282,800	35.35	12,526#	14.65
60	+160,000	+ 343,600	43.30	11,630	25.00
75	+160,000	+ 386,800	48.30	11,313	37.06
90	+160,000	+ 400,000	50.00	11,200	50.00
105	+160,000	+ 386,400	48.30	11,313	67.94
120	+160,000	+ 346,400	43.30	11,630	75.00
135	+160,000	+ 282,800	35.35	12,526	85.35
150	+160,000	+ 200,000	25.00	14,400	93.30
165	+160,000	+ 103,500	12.94	21,900	98.30
180	+160,000	0	0	0	100.00
195	+160,000	-103,520	-12.94	- 4,366	98.30
210	+160,000	-200,000	-25.00	800	93.30
225	+160,000	-282,800	-35.35	2,605	85.35
240	+160,000	-346,400	-43.30	4,305	75.00
255	+160,000	-386,800	-48.30	4,690	62.94
270	+160,000	-400,000	-50.00	4,800	50.00
285	+160,000	-386,400	-48.30	4,690	37.06
300	+160,000	-346,400	-43.30	4,305	25.00
315	+160,000	-282,800	-35.35	2,605	14.65
330	+160,000	-200,000	-25.00	800	6.70
345	+160,000	-103,500	-12.94	- 4,366	2.70
360	+160,000	0	0	0	0

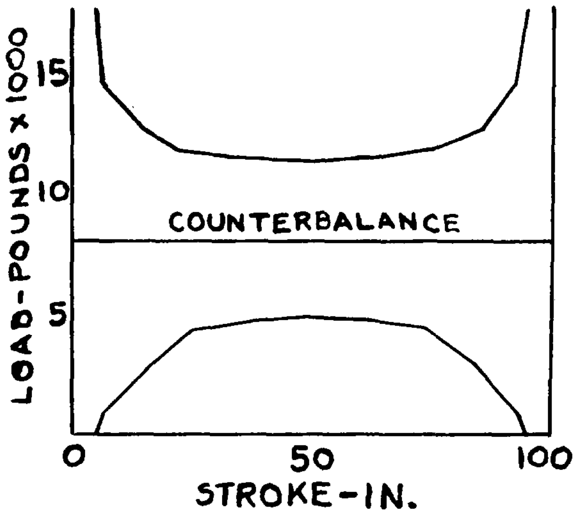

FIG. 5

ACTUAL UNIT: FIG. 6
160,000 inch-pounds Torque, 100" Stroke,
400,000 inch-pounds maximum CB Torque, Rotation Down Post

POSITION	TORQUE OF UNIT	COUNTERBALANCE TORQUE	TORQUE FACTOR	PERMISSIBLE LOAD	POLISHED ROD POS.
0	+ 160,000	0	- 9.187	Max.=-1,740	.004
15	+ 160,000	± 103,500	9.109	28,930	.005
30	+ 160,000	+ 200,000	27.27	13,201	.049
45	+ 160,000	+ 282,800	42.27	10,475	.141
60	+ 160,000	+ 343,600	51.23	9,830	.262
75	+ 160,000	+ 386,400	51.719	10,560	.397
90	+ 160,000	+ 400,000	51.188	10,940	.534
105	+ 160,000	+ 386,400	45.046	12,405	.655
120	+ 160,000	+ 346,400	38.078	13,300	.764
135	+ 160,000	+ 282,800	30.656	14,440	.849
150	+ 160,000	+ 200,000	22.750	15,820	.919
165	+ 160,000	+ 103,500	14.906	17,680	.966
180	+ 160,000	0	6.375	25,100	.994
195	+ 160,000	-103,500	- 3.266	-17,300	.999
210	+ 160,000	-200,000	-13.859	2,886	.977
225	+ 160,000	-282,800	-24.938	4,920	.924
240	+ 160,000	-346,400	-35.578	5,240	.845
255	+ 160,000	-386,400	-44.297	5,100	.743
270	+ 160,000	-400,000	-50.062	4,790	.622
285	+ 160,000	-386,400	-52.125	4,340	.489
300	+ 160,000	-346,400	-50.625	3,680	.358
315	+ 160,000	-282,800	-45.703	2,687	.232
330	+ 160,000	-200,000	-37.109	1,080	.128
345	+ 160,000	-103,500	-21.640	- 2,293	.048
360	+ 160,000	0	- 9.187	-17,400	.004

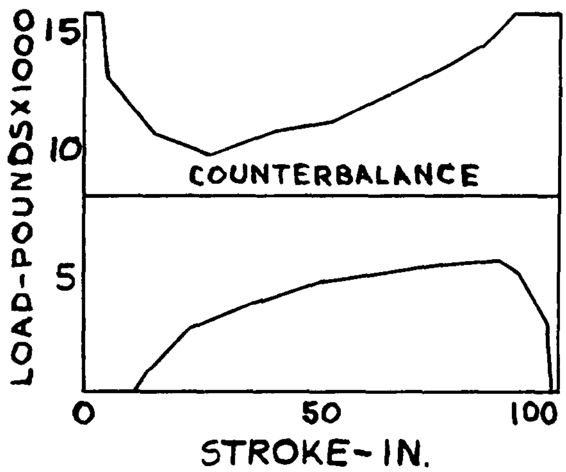

FIG. 6

Additional work could be performed by the theoretical unit with a constant upstroke and a constant downstroke load without overloading the unit. The theoretical unit could do (11,200 - 4800) (100) = 640,000 inch-pounds of work if the dynamometer card were a parallogram and the actual unit could only do (9,830-5240)(100) = 459,000 inch-pounds of work. The theoretical unit could do 139.4 per cent of the work performed by the actual unit if the dynamometer card were an exact parallelogram.

We find this same situation is true in different brands of units. This depends on the actual unit construction and counterbalance phase angle. It should be pointed out that this does not mean that unit efficiency will be improved. Additional amounts of input work must be generated, plus, of course, mechanical friction to obtain the additional output work.

Comparisons of the following standard 160,000 inch-pounds torque units, with 64 inch stroke and 480,000 inch-pounds maximum counterbalance torque. Counterbalance of 15,000 pounds has been subtracted from actual permissible loads so they may be used with other amounts of counterbalance near 15,000 pounds without going through new calculations. These permissible loads, as shown, would be added to effective counterbalance for plotting permissible load diagrams.

Permissible Loads:
160,000 inch-pounds reducer rating, 64 inch stroke length, clockwise rotation, 480,000 inch-pounds counterbalance.

	UNIT "A"			UNIT "B"		
θ	POS. OF RODS	TORQUE FACTOR	PERMISSIBLE LOAD	POS. OF RODS	TORQUE FACTOR	PERMISSIBLE LOAD
0	.004	- 5.88	-15,300	.001	- 1.65	-15,400
15	.005	5.83	34,050	.016	9.53	14,500
30	.049	17.45	8,250	.078	20.02	4,600
45	.141	27.05	3,750	.177	28.04	2,400
60	.262	32.79	2,850	.302	32.44	2,300
75	.397	34.38	3,450	.438	33.16	3,400
90	.534	32.76	4,850	.569	31.08	5,200
105	.655	28.83	6,950	.688	27.34	7,400
120	.764	24.37	8,950	.792	22.86	9,700
135	.849	19.62	10,750	.877	18.01	12,300
150	.919	14.56	12,850	.939	12.90	15,600
165	.966	9.54	15,150	.981	7.44	22,800
180	.994	4.08	25,550	1.00	1.50	91,600
195	.999	- 2.09	-15,300	.993	- 4.99	15,400
210	.977	- 8.87	- 5,600	.958	-11.90	- 8,700
225	.924	-15.96	- 3,500	.896	-18.71	- 5,800
240	.845	-22.77	- 3,500	.806	-24.69	- 5,000
255	.743	-28.35	- 4,000	.695	-29.18	- 5,000
270	.622	-32.04	- 4,700	.570	-31.75	- 5,300
285	.489	-33.36	- 5,600	.438	-32.22	- 6,000
300	.358	-32.40	- 6,800	.309	-30.49	- 7,900
315	.232	-29.25	- 8,600	.192	-26.52	- 8,600
330	.128	-23.75	-11,300	.095	-20.31	-11,500
345	.049	-15.77	-15,300	.029	-11.90	-15,400

	UNIT "C"			UNIT "D"		
θ	POS. OF RODS	TORQUE FACTOR	PERMISSIBLE LOAD	POS. OF RODS	TORQUE FACTOR	PERMISSIBLE LOAD
0	.000	- 2.38	-15,400	.004	6.08	-14,500
15	.016	9.90	13,100	.002	5.44	38,100
30	.078	24.8	700	.055	16.96	8,900
45	.183	30.3	1,100	.138	27.20	3,900
60	.319	32.4	2,400	.260	33.28	2,800
75	.455	34.6	2,600	.400	35.52	3,100
90	.594	31.1	5,200	.540	33.92	4,300
105	.713	26.5	8,200	.671	30.11	6,700
120	.808	21.8	11,000	.782	25.02	8,600
135	.887	16.6	14,700	.868	19.62	10,900
150	.940	12.0	18,000	.936	13.82	14,500
165	.984	6.75	26,700	.978	8.96	17,900
180	1.000	1.35	103,100	1.000	1.92	66,200
195	.992	- 4.85	-15,400	.994	- 4.48	-14,500
210	.958	-11.6	- 8,500	.963	10.88	- 7,100
225	.898	-18.6	- 5,800	.906	16.96	- 4,000
240	.812	-23.2	- 4,400	.827	22.72	- 3,300
255	.704	-29.0	- 5,000	.726	27.20	- 3,400
270	.556	-31.6	- 5,300	.610	30.72	- 4,000
285	.445	-32.3	- 6,000	.477	32.00	- 5,100
300	.319	-30.6	- 7,000	.350	31.36	- 6,400
315	.198	-27.3	- 8,800	.234	28.80	- 8,200
330	.104	-21.4	-11,700	.142	23.36	-11,100
345	.013	-13.0	-15,400	.055	16.00	-14,500

Applying a constant load on the upstroke equal to the minimum permissible load, and a constant load on the downstroke equal to the minimum negative permissible load, we will construct a parallelogram dynamometer card. Here is a comparison of the various units by the amount of work in this parallelogram:

Unit "A": (2850 + 3500)(64) = 406,400 inch-pounds of work per stroke.

Unit "B": (2300 + 5000)(64) = 467,200 inch-pounds of work per stroke.

Unit "C": (700 + 4400)(64) = 326,400 inch-pounds of work per stroke.

Unit "D": (2800 + 3300)(64) = 390,400 inch-pounds of work per stroke.

Theoretical Unit: (5000 + 5000)(64) = 640,000 inch-pounds work per stroke.

CONCLUSION

Of course, dynamometer cards do not form regular rectangular patterns and it is necessary to compare the actual permissible load diagram to evaluate each unit against any other. This does show the wide variation among units of the same API size in the amounts of work that can be done without exceeding the torque rating of the gear box.